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Fluid Mechanics, Fluid Machines and Open Channel Flow

Civil Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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Fluid Mechanics, Fluid Machines and Open Channel Flow

First Edition: 2019

Reprint : 2020

Reprint : 2021

Second Edition: 2022

Reprint : 2023

Reprint : 2024

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Fluid Mechanics

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CHAPTER

Fluid Properties

1.1 INTRODUCTION

- A fluid is a substance which deforms continuously under the influence of shearing forces no matter how small the forces may be.
- Fluids are substance capable of flowing and they conforms to the shape of the containing vessel.
- This property of continuous deformation in technical terms is known as 'flow property', whereas this property is absent in solids.
- If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.
- Fluids are classified as ideal fluids and practical or real fluids.
- Ideal fluids are those fluids which have neither viscosity nor surface tension and they are incompressible. In nature, the ideal fluids do not exist and therefore, they are only imaginary fluids.
- Practical or real fluids are those fluids which possess viscosity, surface tension and compressibility.
- Fluids are considered to be continuum i.e., a continuous distribution of matter with no voids or empty spaces.
- Difference between fluid and solid is that solid can resist a shear stress by static deflection but fluid cannot resist it.

1.2 FLUID MECHANICS

- Fluid mechanics is study of fluids either at rest or in motion, i.e., if deals with the static, kinematic and dynamic aspects of fluids.
- The steady of fluids at rest is known as fluid statics.
- The steady of fluids in motion, without considering the pressure forces is known as fluid kinematics.
- When pressure forces are also considered for fluids in motion, that branch of science is known as fluid dynamics.

1.3 FLUID AS CONTINUUM

- Since fluids are aggregations of molecules widely spread for gas and closely spaced for a liquid. The distance between molecules is very large compared to molecular diameter.
- The molecules are not fixed in lattice but move about freely. Thus fluid density or mass per unit volume has no practical meaning because the numbers of molecule occupying a given volume continuously changes.
- But if chosen unit volume is too large there could be noticeable variation in the bulk aggregation of particle. So density can be written as

$$\rho = \lim_{\delta v \to \delta v'} \frac{\delta m}{\delta v}$$

- Since most engineering problems are connected with larger sample volume, so density being a point function and other fluid properties can be thought of as varying continually in space. Such a fluid is called a continuum, which simply means that its variation in properties is so smooth that differential calculus can be used to analyse the substance.

1.4 FLUID PROPERTIES

- Any characteristic of a fluid system is called a fluid property.
- Fluid properties are of two types:
  - (i) **Intensive Properties:** Intensive properties are those that are independent of the size of the system or the amount of material in it.

    Example: Temperature, pressure, density etc.
  - (ii) **Extensive Properties:** Extensive properties are those whose values depend on the size or extent of the system.

    Example: Total mass, total volume, total momentum etc.
- Following are some of the intensive and extensive properties of a fluid system.

  (i) Viscosity (ii) Surface tension (iii) Vapour pressure (iv) Compressibility and elasticity

1.4.1 Some Important Properties

1. **Density or Mass Density :** It is the mass of fluid per unit volume.
   - Mass density, $\rho = \dfrac{\text{Mass of fluid}}{\text{Volume of fluid}}$
   - Unit in SI system : kg/m^3
2. **Specific weight or weight density :** It is defined as the weight of fluid per unit volume.

$$\text{Specific weight, } w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}} \quad \text{...}(g = \text{acceleration due to gravity})$$

$$\therefore \quad w = \rho \times g$$

   - Unit in SI system : N/m^3
   - Both mass density and specific weight depend upon temperature and pressure.

3. **Specific volume :** It is the volume of fluid per unit mass.

So, $$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume of fluid}}}$$

$$\therefore \quad \text{Specific volume} = \frac{1}{\rho}$$

- Unit in SI system : m^3/km

4. **Specific Gravity :** Specific gravity (S) is the ratio of specific weight (or mass density) of a fluid to the specific weight (or mass density) of a standard fluid. The standard fluid chosen for comparison is pure water at 4°C.

$$\text{Specific gravity of liquid } (S) = \frac{\text{Specific weight of liquid}}{\text{Specific weight of water}} = \frac{\text{Specific weight of liquid}}{9810 \text{ N/m}^3}.$$

$\Rightarrow$ Specific weight of liquid = $S \times 9.81 \times 1000$ N/m^3

$\Rightarrow$ Density of a liquid $\times\, g = S \times 9.81 \times 1000$

$\Rightarrow$ Density of a liquid = $S \times 1000$ kg/m^3

- For gases, air is taken as the standard fluid.

5. **Relative Density (R.D.) :** It is defined as ratio of density of one substance with respect to other substance.

$$\rho_{1/2} = \frac{\rho_1}{\rho_2}$$

where, $\rho_{1/2}$ = Relative density of substance '1' with respect to substance '2'.

EXAMPLE - 1.1

Three litres of petrol weigh 23.7 N. Calculate the mass density, specific weight, specific volume and specific gravity of petrol.

Solution:

Mass density of petrol, $$\rho_p = \frac{M}{V} = \frac{W/g}{V} = \frac{W}{gV} = \frac{23.7}{9.81 \times 3} = 0.805 \text{ kg/litre} = 805 \text{ kg/m}^3$$

Mass density of water, $$\rho_w = 1000 \text{ kg/m}^3$$

$$\text{Specific gravity of petrol} = \frac{\rho_p}{\rho_w} = \frac{805}{1000} = 0.805$$

$$\text{Specific weight of petrol} = \text{weight per unit volume}$$
$$= \frac{23.7}{3.0} = 7.9 \text{ N/litre} = 7.9 \text{ kN/m}^3$$

$$\text{Specific volume} = \text{volume per unit mass}$$
$$= \frac{1}{\rho_p} = \frac{1}{805} = 1.242 \times 10^{-3} \text{ m}^3\text{/kg}$$

1.5 VISCOSITY

- Viscosity is a property of the fluids by virtue of which they offer resistance to shear or angular deformation.
- It is primarily due to cohesion and molecular momentum exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers.

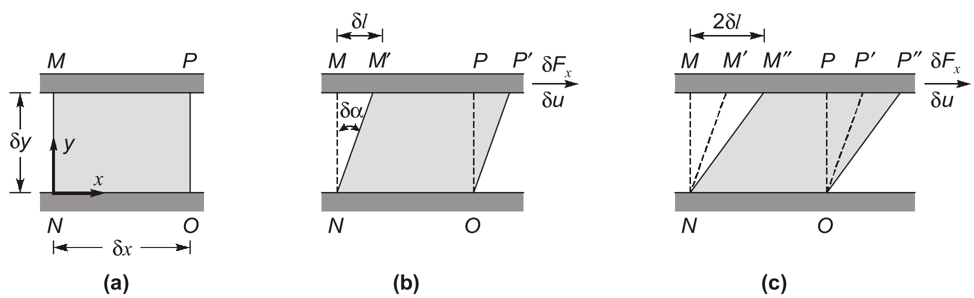


Fig. *(a) Fluid element at time t, (b) deformation of fluid element at time t + δt, and (c) deformation of fluid element at time t + 2δt.*

- Consider the behavior of a fluid element between the two infinite plates as shown in Fig. (a).
  The rectangular fluid element is initially at rest at time t. Let us now suppose a constant rightward force δF_x is applied to the upper plate so that it is dragged across the fluid at constant velocity δu. The relative shearing action of the plates produces a shear stress, τ_{yx}, which acts on the fluid element and is given by $\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$, where δA_y is the area of contact of the fluid element with the plate and δF_x is the force exerted by the plate on that element.
  Various positions of the fluid element, shown in Fig. illustrate the deformation of the fluid element from position *MNOP* at time *t*, to *M′NOP′* at time $t + \delta t$, to *M″NOP″* at time $t + 2\delta t$, due to the imposed shear stress. The deformation of the fluid is given by

$$\text{Deformation rate} = \lim_{\delta t \to 0} \frac{\delta\alpha}{\delta t} = \frac{d\alpha}{dt}$$

Distance between the points *M* and *M′* is given by,

$$\delta l = \delta u\, \delta t \qquad \text{...(i)}$$

Alternatively, for small angles, $\delta l = \delta y \delta\alpha$ ...(ii)

Equating Eq. (i) and (ii), we get, $\delta u \delta t = \delta y \delta\alpha$

or

$$\frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y}$$

Taking the limits of both sides

$$\lim_{\delta t \to 0} \frac{\delta\alpha}{\delta t} = \lim_{\delta t \to 0} \frac{\delta u}{\delta y}$$

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$

Thus, the rate of angular deformation is equal to velocity gradient across the flow.

- On the basis of relation between the applied shear stresses and the flow or rate of deformation, fluids can be categorized as Newtonian and non-Newtonian fluids.

1.6 TYPE OF FLUIDS

- The fluids are classified into following five types:
  (i) Newtonian fluid (ii) Non Newtonian fluid (iii) Ideal fluid
  (iv) Real fluid (v) Ideal plastic fluid

1.6.1 Newtonian Fluids

- Fluids which obey Newton's law of viscosity are known as Newtonian fluids.
- According to Newton's law of viscosity, shear stress is directly proportional to the rate of deformation or velocity gradient across the flow.

  Thus, $$\tau \propto \frac{du}{dy}$$

  or $$\tau = \mu \frac{du}{dy}$$

  where, μ = Coefficient of dynamic viscosity
- Water, air, and gasoline are Newtonian fluids under normal conditions.

Dynamic Viscosity (μ)

- From above, $\mu = \dfrac{\tau}{\frac{du}{dy}}$

  So, viscosity is the shear stress required to produce unit rate of shear strain.
- Dimension of $\mu = [M L^{-1} T^{-1}]$
- Unit of μ = N-s/m^2 or Pa.s
- In c. g. s. units, μ is expressed as 'poise', 1 poise = 0.1 N-s/m^2
- (μ) water $\approx 10^{-3}$ N-s/m^2;
  (μ) air $\approx 1.81 \times 10^{-5}$ N-s/m^2 (Both at 20°C and at standard atmospheric pressure)

NOTE: Water is nearly 55 times viscous than air.

Kinematic Viscosity (ν)

- The kinematic viscosity (ν) is defined as the ratio of dynamic viscosity to mass density of the fluid therefore, $\nu = \mu/\rho$
- It is called kinematic because the mass unit cancel, leaving only the dimensions.
- Dimension of $\nu = [L^2 T^{-1}]$
- Unit of ν = m^2/s or cm^2/s (stoke)
- 1 stoke = 10^{-4} m^2/s
- At 20°C and atmospheric pressure $\nu_{water} = 1.0 \times 10^{-6}$ m^2/s, $\nu_{air} = 15 \times 10^{-6}$ m^2/s

NOTE: Kinematic viscosity of air is about 15 times greater than the corresponding value of water.

Effect of Temperature on Viscosity

- It is necessary to understand the factors contributing to viscosity to analyse temperature effect.
- In liquid, viscosity is caused by intermolecular attraction force which weaken as temperature rises so viscosity decreases.
- In gases, viscosity is caused by the random motion of particle/ molecules. Due to increase in temperature, randomness increases causing increase in viscosity.
- For liquids viscosity decreases with temperature and it is roughly exponential as

$$\mu = ae^{-bT}$$

where *a* and *b* are constant for a particular liquid.

For water $a = -1.94, \quad b = -4.80$

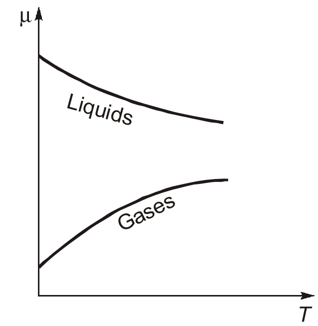


Fig. *Variation of Viscosity with Temperature*

1.6.2 Non-Newtonian Fluids

- Fluids for which shear stress is not directly proportional to velocity gradient are known as non-Newtonian fluids. Toothpaste and paint are the examples of non-Newtonian fluids.
- Non-Newtonian fluids are commonly classified as having time-independent or time-dependent behavior.
- Relation between shear stress and rate of deformation for non-Newtonian fluid can be represented as

$$\tau = k\left(\frac{du}{dy}\right)^n$$

where, n = flow behavior index; k = consistency index

For Newtonian fluid, $n = 1; \quad k = \mu$

above equation can also be represented as

$$\tau = k\left(\frac{du}{dy}\right)^{n-1}\left(\frac{du}{dy}\right) = \eta\frac{du}{dy}$$

where, $\eta = k\left(\frac{du}{dy}\right)^{n-1}$ is referred as the apparent viscosity

NOTE: Dynamic viscosity (μ) is constant (except for temperature effects) while apparent viscosity (η) depends on the shear rate.

- Various types of non-Newtonian fluids are :
  1. **Pseudoplastic** : Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$) are called pseudoplastic fluids (or shear thinning). Most non-Newtonian fluids fall into this group.

     Example: Polymer solutions, colloidal suspensions, milk, blood and paper pulp in water.
  2. **Dilatant :** If the apparent viscosity increases with increasing deformation rate ($n > 1$), the fluid is termed as dilatant (or shear thickening).

     Example: Suspensions of starch, saturated sugar solution.
  3. **Thixotropic :** Apparent viscosity (η) for thixotropic fluids decreases with time under a constant applied shear stress.

     Example: Paints, printer inks
  4. **Rheopectic :** Apparent viscosity (η) for rheopectic fluids increases with time under constant shear stress.

     Example: Gypsum pastes.

1.6.3 Ideal Fluid

- It is a fluid which is incompressible and has no viscosity. It is an imaginary fluid as all the existing fluids haves some viscosity.

1.6.4 Real Fluid

- It is a fluid which possess viscosity. So, practically all fluids are real fluids.

1.6.5 Ideal plastic fluid

- It is a fluid which behave as a solid until a minimum yield stress τ_y and flow after crossing this limit. It is also known as Bingham plastic.

$$\tau = \tau_y + \frac{\mu dx}{dy}$$

Example: Clay suspension, drilling muds, creams and toothpaste.

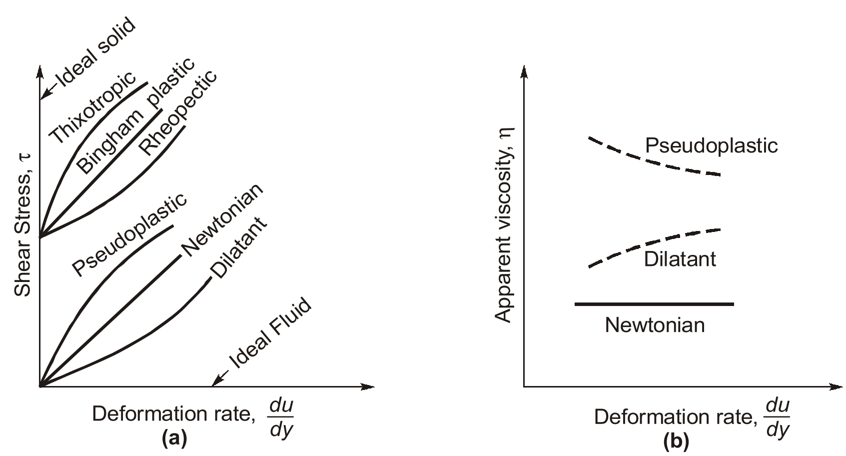


Fig. *(a) Shear stress, τ and (b) Apparent viscosity, η*

NOTE

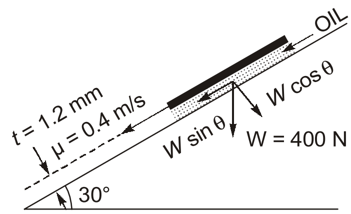
(i) There is no relative movement between fluid attached to the solid boundary and solid boundary i.e. the fluid layer just adjacent to the solid surface will have same velocity as of the solid surface.

(ii) Viscoelastic : Fluids which after some deformation partially return to their original shape when the applied stress is released such fluids are called viscoelastic.

(iii) Rheology : Branch of science which deals with the studies of different types of fluid behaviours.

EXAMPLE - 1.2

An oil film of thickness 1.2 mm is used for lubricating the space between an inclined place with angle of indication 30° and a square plate of size 1.2 m × 1.2 m. The square plate, having a weight of 400 N, slides down the inclined plane with a uniform velocity of 0.4 m/s. Calculate the dynamic viscosity of the oil used.



Solution:

Velocity of plate, $u = 0.4$ m/s

Thickness of oil film, $t = 1.2$ mm

Area of plate, $A = 1.2 \times 1.2 = 1.44\ \text{m}^2$

Angle of inclination of plate, $\theta = 30°$

Weight of plate, $W = 400$ N

Let the coefficient of dynamic viscosity of the oil be *u*.

$$\text{Component of weight along the plane} = W\sin\theta = W\sin 30° = 400 \times \sin 30° = 200\ \text{N}$$

So, shear force acting on the surface of the plate = 200 N

Now, $$\text{Shear stress} = \frac{F}{A} = \frac{200}{1.44} = 138.89\ \text{N/m}^2$$

We know that, $$\tau = \frac{\mu du}{dy}$$

Where, du = change in velocity between 2 surfaces

= velocity of plate – Velocity of inclined plane

$= u - 0 = 0.4 - 0$

$= 0.4$ m/s

and $dy = t = 1.2\ \text{mm} = 1.2 \times 10^{-3}\ \text{m}$

So, $$138.89 = \mu \times \frac{0.4}{1.2 \times 10^{-3}}$$

$$\Rightarrow \quad \mu = 138.89 \times \frac{1.2 \times 10^{-3}}{0.4} = 0.4167\ \text{Ns/m}^2$$

$$\Rightarrow \quad \mu = 4.167\ \text{poise}$$

EXAMPLE - 1.3

Find the kinematic viscosity of a liquid having density 981 kg/m^3. The velocity gradient at a point is 0.5 per second and the shear stress acting at the point is 0.3 N/m^2.

Solution:

Given, Density or mass density, $\rho = 981\ \text{kg/m}^3$

Shear stress, $\tau = 0.3\ \text{N/m}^2$

Velocity gradient, $\frac{du}{dy} = 0.5$

Now, we have $\tau = \mu \frac{du}{dy}$

$\Rightarrow \quad 0.3 = \mu \times 0.5$

$\Rightarrow \quad \mu = \frac{0.3}{0.5} = 0.6 \text{ Ns/m}^2$

Now, kinematic viscosity,

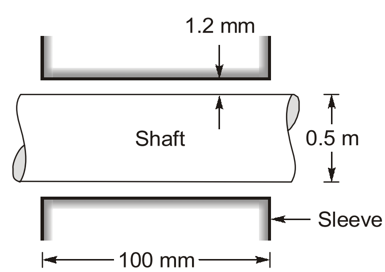
$$\upsilon = \frac{\mu}{\rho} = \frac{0.6}{981} = 6.116 \times 10^{-4} \text{ m}^2\text{/sec}$$

$$= 6.116 \times 10^{-4} \times 10^{4} \text{ cm}^2\text{/sec}$$

$$= 6.116 \text{ stoke}$$

EXAMPLE - 1.4

A shaft of diameter 0.5 is rotating at 200 rpm. An oil is used for lubrication between the shaft and sleeve. The thickness of oil film is 1.2 mm and the dynamic viscosity of oil is 5 poise. Calculate the power lost in the breaking for a sleeve length of 100 mm.



Solution:

Given, Dynamic viscosity, $\mu = 5 \text{ poise} = 0.5 \text{ Ns/m}^2$

Thickness of oil film, $t = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}$

Diameter of shaft, $D = 0.5 \text{ m}$

Speed of shaft, $N = 200 \text{ rpm}$

Sleeve length, $L = 100 \text{ mm}$

Now, tangential velocity of shaft, $u = \frac{\pi DN}{60} = \frac{\pi \times 0.5 \times 200}{60} = 5.236 \text{ m/s}$

Shear stress on shaft, $\tau = \mu \frac{du}{dy} = 0.5 \times \frac{5.236}{1.2 \times 10^{-3}} = 2181.66 \text{ N/m}^2$

$\therefore$ Shear force on shaft, $F = \text{Shear stress} \times \text{Area}$

$$= \tau \times \pi DL$$

$$= 2181.66 \times \pi \times 0.5 \times (100 \times 10^{-3})$$

$$= 342.69 \text{ N}$$

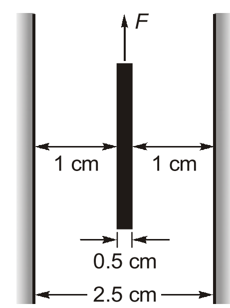
Torque on the shaft, $T = F \times \frac{D}{2} = 342.69 \times \frac{0.5}{2} = 85.67$ Nm

So, Power lost, $P = t \times \omega$ Where, ω = angular velocity = $\frac{2\pi N}{60}$

$\Rightarrow$ $P = \frac{2\pi NT}{60} = \frac{2A \times 200 \times 85.67}{60} = 1794.34$ W

EXAMPLE - 1.5

A plate 1.5 m × 1.5 m × 0.5 cm is lifted up with a constant velocity of 0.2 m/sec through an infinite vertical gap. The width of gap is 2.5 cm and it contains a fluid of viscosity 15 poise and specific gravity 0.8. If the plate is in the middle of the gap and has a weight of 100 N, find out the force required to lift the plate.



Solution:

Given: Width of gap = 2.5 cm

Viscosity, μ = 15 poise

= 1.5 Ns/m^2

Specific gravity of fluid = 0.8

$\therefore$ Mass density of fluid = 0.8 × 1000 kg/m^3

and weight density of fluid = 0.8 × 1000 × 9.81 N/m^2

= 7848 N/m^2

Volume of plate = 1.5 m × 1.5 m × 0.5 cm

= $1.5 \times 1.5 \times (0.5 \times 10^{-2})$

= 0.01125 m^3

Thickness of plate = 0.5 cm

Velocity of plate = 0.2 m/sec

Weight of plate = 100 N

As plate is in the middle of gap, so distance of plate form either vertical surface of gap

$$= \left(\frac{\text{Width of gap-thickness of plate}}{2}\right)$$

$$= \left(\frac{2.5 - 0.5}{2}\right) = 1 \text{ cm} = 0.01 \text{ m}$$

Now, shear force on either side of the plate, (because the plate is symmetrically, so $F_1 = F_2$)

$F_1 = F_2$ = shear stress × Area

$$= \mu\left(\frac{du}{dy}\right) \times A = 1.5 \times \left(\frac{0.2}{0.01}\right) \times 1.5 \times 1.5 = 67.5\text{N}$$

$\therefore$ Total shear force = $F_1 + F_2 = 67.5 + 67.5$

= 135 N

> **NOTE** If the plate is dragged horizontally, then total shear force is equal to the force required to drag the plate. But here, as the plate is lifted up vertically, so weight of the plate and upthrust also acts in vertical direction. So, these are also to be considered for calculating the net force required to lift up the plate.

Now, Upward thrust = Weight of fluid displaced

= (Weight density of fluid) × (Vol. of fluid displaced by plate)

= 7848×0.01125

= 88.29 N

The net force acting in the downward direction due to weight of the plate and upward thrust

= Weight of plate – Upward thrust

= 100 – 88.29

= 11.71 N

So, total force required to lift the plate up

= Total shear force + 11.71

= 135 + 11.71

= 146.71 N

EXAMPLE - 1.6

Calculate the velocity gradient at distance 0, 100, 150 mm from the boundary if the velocity is a parabola with vortex 150 mm from boundary, where velocity is 1 m/s. Also calculate the shear stress at these points if the fluids has a viscosity of 0.804 Ns/m^2.

Solution:

Let the equation of velocity profile

$$u = Ay^2 + By + C$$

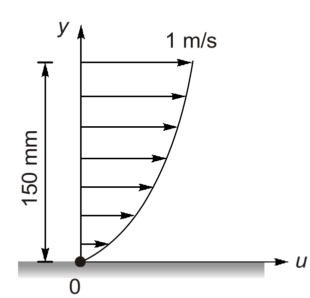
Now apply boundary condition

(i) $u = 0$ at $y = 0 \Rightarrow c = 0$

(ii) $u = 1$ m/s at $y = 0.15$ m

$1 = 0.15^2 \times A + 0.15\,B$ ...(ii)

(iii) at $y = 0.15$ m at $\frac{du}{dy} = 0$



$$\frac{du}{dy} = 2Ay + B$$

$2A \times 0.15 + B = 0$ ...(iii)

From Eq. (ii) and (iii), we get,

$A = -44.4$; $B = 13.33$

So velocity profile will be given as

$u = -44.4\,y^2 + 13.33\,y$

So, $\frac{du}{dy} = -2 \times 44.4y + 13.33$

(a) at $y = 0$ mm $\qquad \dfrac{du}{dy} = -2 \times 44.4 \times 0 + 13.33 = 13.33 \text{ sec}^{-1}$

Shear stress, $\qquad \tau = \mu \dfrac{du}{dy} = 0.804 \times 13.33 = 10.8 \text{ N/m}^2$

(b) at $y = 100$ mm $\qquad \dfrac{du}{dy} = -2 \times 44.4 \times 0.1 + 13.33 = 4.45 \text{ sec}^{-1}$

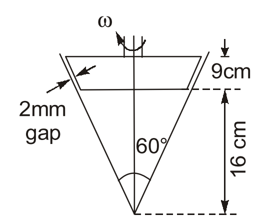
$$\tau = \mu \frac{du}{dy} = 0.804 \times 4.45 = 3.575 \text{ N/m}^2$$

(c) at $y = 150$ mm $\qquad \dfrac{du}{dy} = -2 \times 44.4 \times 0.15 + 13.33 = 0$

$$\tau = 0$$

EXAMPLE - 1.7

For the truncated cone as shown in fig. calculate the torque required if the cone is rotate at 200 rpm. Viscosity of oil in the 2 mm gap between the cone and the housing is 2 poise.

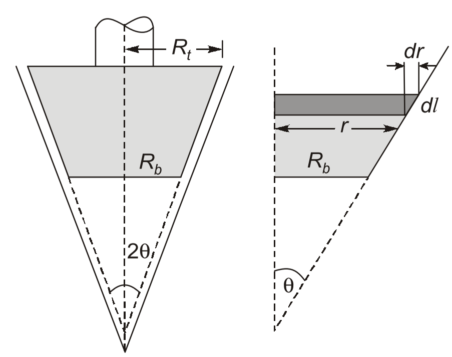


Solution:

Consider the truncated cone as shown in Fig., let R_t and R_b be the radii at top and bottom of the cone of the vertex angle 2θ. Let the cone is rotated at angular speed of ω rad/s and the thickness of the gap is t.

Consider an elementary strip of the cone with radius r.

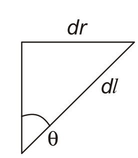
Shear stress on the sloping wall of the strip



$$\tau = \mu\left(\frac{du}{dy}\right)$$

$$\tau = \mu\left(\frac{u}{t}\right) = \mu\left(\frac{r\omega}{t}\right)$$

Area of sloping wall of strip $= 2\pi r\,(dl) = 2\pi r\left(\dfrac{dr}{\sin\theta}\right)$



$\therefore$ Shear force on strip, $F = \tau \times \text{Area} = \left(\dfrac{\mu r\omega}{t}\right) \times \left(\dfrac{2\pi r dr}{\sin\theta}\right) = \left(\dfrac{2\pi\mu\omega}{t\sin\theta}\right) r^2 dr$

Torque about central axis due to shear force on the strip, $dT = Fr$

$$dT = \left(\frac{2\pi\mu\omega}{t\sin\theta}\right) r^3 dr$$

$\therefore$ Total torque, $T = \displaystyle\int_{R_b}^{R_t} dT = \frac{2\pi\mu\omega}{t\sin\theta}\int_{R_b}^{R_t} r^3 dr$

$\therefore$ $$T = \frac{2\pi\mu\omega}{t\sin\theta}\left[\frac{R_t^4}{4} - \frac{R_b^4}{4}\right]$$

$$T = \frac{\pi\mu\omega}{2t\sin\theta}\left[R_t^4 - R_b^4\right] \qquad \text{...(i)}$$

$$R_t = (16 + 9)\tan 30° = 25 \tan 30° = 14.43 \text{ cm} = 0.144 \text{ m}$$

and $$R_b = 16 \tan 30° = 9.24 \text{ cm} = 0.0924 \text{ m}$$

$\therefore$ $$\mu = 2 \text{ poise} = \frac{2}{10} \text{ Ns/m}^2 = 0.2 \text{ Ns/m}^2,\ t = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

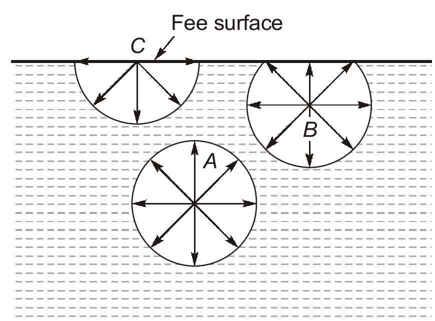
$$\theta = 30°,\ \omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/sec}$$

$\therefore$ Putting these values in Eq. (i), we get,

$$T = \frac{\pi\mu\omega}{2t\sin\theta}\left[R_t^4 - R_b^4\right]$$

$$= \frac{\pi \times 0.2 \times 20.94}{2 \times 2 \times 10^{-3} \sin 30°}\left[(0.144)^4 - (0.0924)^4\right]$$

$$= 2.35 \text{ N.m}$$



EXAMPLE - 1.8

A circular disc of diameter *d* is slowly rotated in a liquid of large viscosity 'μ' at a small distance '*h*' from fixed surface. Derive expression for torque '*T*' necessary to maintain an angular velocity 'ω'.

Solution:

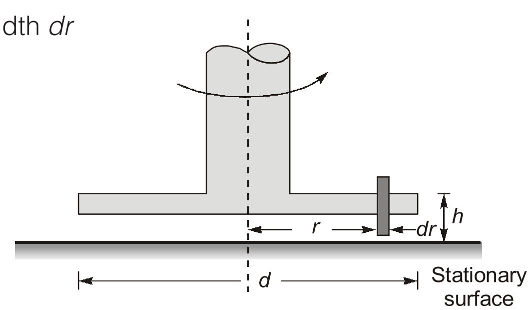
Consider an element of disc at radius *r* and having a width *dr*

Linear velocity at this radius

$$V = r\omega$$

$$\text{Torque} = \text{Shear stress} \times \text{Area} \times r$$

$$\text{Shear stress, } \tau = \frac{\mu du}{dy}$$



Torque required for small ring, *dT*

$\therefore$ $$dT = \frac{\mu du}{dy} \times 2\pi r \cdot dr \cdot r$$

Now assuming that *h* is very small and velocity distribution is linear. So,

$$\frac{du}{dy} = \frac{r\omega}{h}$$

$\therefore$ $$dT = \frac{\mu r\omega}{h} \times 2\pi r^2 dr$$

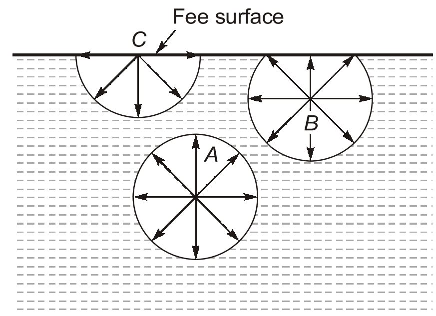
$$= \frac{2\pi\mu\omega}{h} r^3 dr$$

So total torque required,

$$T = \int_0^{d/2} dT = \frac{2\pi\mu\omega}{h}\left[\frac{r^4}{4}\right]_0^{d/2}$$

$$T = \frac{\mu\pi d^4\omega}{32h}$$

1.7 SURFACE TENSION



- It is a force which exists on the surface of a liquid when it is in contact with another fluid or a solid boundary. Its magnitude depends upon the relative magnitude of cohesive and adhesive forces.
- Surface tension is a force in the liquid surface and acts normal to a line of unit length drawn imaginarily on the surface. Thus it is a line force.
- It represents surface energy per unit area. It has dimension MT^{-2} and SI unit is N/m.
- Whenever a liquid is in contact with other liquids or gases the interface develops that acts like a stretched elastic membrane, creating surface tension.

1.7.1 Effect of Temperature

- As the surface tension depends directly upon the intermolecular cohesion and since this cohesion is known to decrease with temperature rise the surface tension decreases with rise of temperature.
- Its value for water-air contact (free-surface of water) reduces from 0.0731 N/m at 17.8°C to 0.0585 N/m at 100°C.

1.7.2 Droplet and Jet

- When a droplet is separated initially from the surface of the main body of liquid, then due to surface tension there is a net inward force exerted over the entire surface of the droplet which causes the surface of the droplet to contract from all the sides and results in increasing the internal pressure within the droplet.
- The contraction of the droplet continues till the inward force due to surface tension is in balance with the internal pressure and the droplet forms into sphere which is the shape for minimum surface area.
- The internal pressure within a jet of liquid is also increased due to surface tension.
- The internal pressure intensity within a droplet and a jet of liquid in excess of the outside pressure intensity may be determined by the expressions derived below.

(i) **Pressure intensity inside a droplet :** Consider a spherical droplet (Fig. (a)) of radius r having internal pressure intensity p in excess of the outside pressure intensity. If the droplet is cut into two halves, then the forces acting on one half (Fig. (b)) will be those due to pressure intensity (p) on the projected area (πr^2) and the tensile force due to surface tension (σ) acting around the circumference ($2\pi r$). These two forces will be equal and opposite

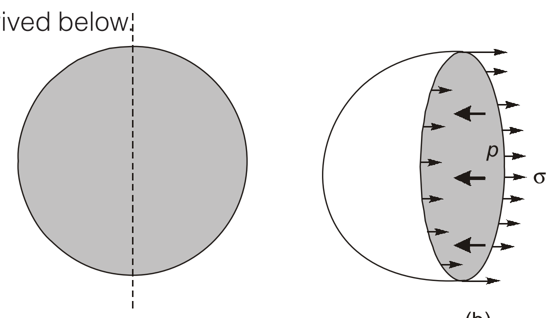


Fig. *Surface Tension (σ) and Internal Pressure (p) in a droplet* for equilibrium and hence we have

$$p(\pi r^2) = \sigma(2\pi r)$$

or
$$p = \frac{2\sigma}{r}$$

NOTE: Above equation indicates that the internal pressure intensity increase with the decrease in the size of droplet.

(ii) **Pressure intensity inside a soap bubble :** A spherical soap bubble has two surfaces in contact with air, one inside and the other outside, each one of which contributes the same amount of tensile force due to surface tension (Fig.). As such on a hemispherical section of a soap bubble of radius r, the tensile force due to surface tension is equal to $2\sigma(2\pi r)$. However, the pressure force acting on the hemispherical section of the soap bubble is same as in the case of a droplet and it is equal to $p(\pi r^2)$. Thus equating these two forces for equilibrium, we have

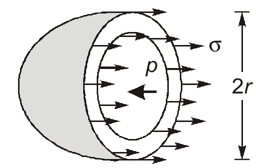


Fig. *Soap Bubble*

$$p(\pi r^2) = 2\sigma(2\pi r)$$

or
$$p = \frac{4\sigma}{r}$$

(iii) **Pressure intensity inside a liquid jet :** Consider a jet of liquid of radius r, length l and having internal pressure intensity p in excess of outside pressure intensity. If the jet is cut into two halves, then the forces acting on one half will be those due to pressure intensity (p) on the projected area ($2rl$) and the tensile force due to surface tension (σ) acting along the two sides ($2l$). These two forces will be equal and opposite for equilibrium and hence we have (Fig.),

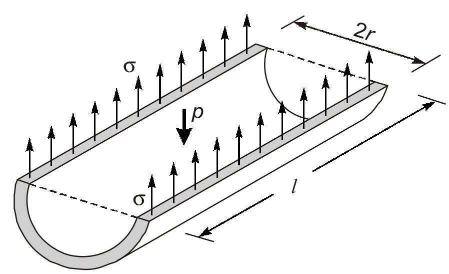


Fig. *Liquid Jet*

$$p(2rl) = \sigma(2l)$$

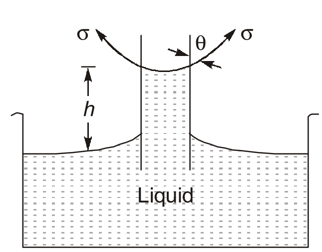
or
$$p = \frac{\sigma}{r}$$

1.8 CAPILLARITY

- When a small tube is held vertically in a liquid, the liquid surface in it falls or rises relative to the adjacent level of liquid. This phenomenon, which occurs due surface tension, is known as capillarity.

Expression for capillary rise:

- Adjoining figure shows a glass tube of small diameter 'd' in which liquid rises up to a height 'h' due to capillarity.



Under equilibrium,

$$\text{Weight of liquid of height } h \text{ in tube} = \text{Vertical component of surface tensile force}$$

$$\Rightarrow \quad (\text{Area of tube} \times h) \times \rho \times g = (\sigma \times \text{circumference}) \times \cos\theta$$

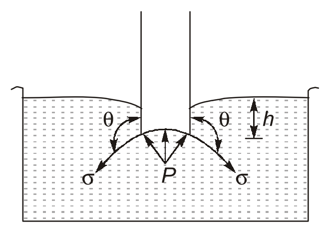
(...here σ = surface tension of liquid)

$$\Rightarrow \quad \frac{\pi}{4}d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

$$\Rightarrow \quad h = \frac{4\sigma\cos\theta}{\rho g d}$$

Expression for capillary fall:

- If glass tube is immersed in mercury, then the level of mercury in the tube is lower than the level of outside liquid.



Under equilibrium,

Surface tension (in downward direction) = Hydrostatic force acting upwards (= pressure intensity at depth 'h' × area)

So, $$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4}d^2$$

$$\Rightarrow \quad h = \frac{4\sigma\cos\theta}{\rho g d}$$

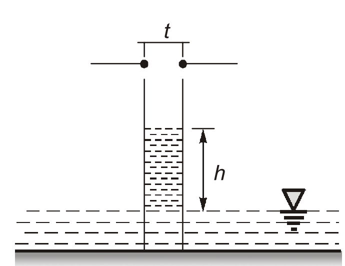
Here, θ = 128° for mercury and glass tube.

EXAMPLE - 1.9

Calculate the distance between the vertical plates spaced 't' apart when the capillarity rise is not to exceed 60 mm. Assume surface tension of water at 20°C as 0.075 N/m.

Solution:

Let width of plate be L and contact angle be θ so for two vertical plates 't' distance apart. Force due to surface tension = Force due to gravity.



$$\sigma \times (2L)\cos\theta = \rho g (L \times t) \times h$$

So, Height of capacity rise, $$h = \frac{2\sigma\cos\theta}{\rho g t}$$

Now, $$\theta = 0.075 \text{ N/m}$$

$$\theta = 0°, \ h = 60 \text{ mm}$$

So, $$0.06 = \frac{2 \times 0.075 \times 1}{9.81 \times 1000 \times t} \times 1000 \text{ mm}$$

$$t = 0.255 \text{ mm}$$