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Circuit Theory

Electrical Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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Circuit Theory

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CHAPTER 1

Basics of Circuit Theory

1.1 Introduction

In Electrical and Electronics Engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element*.

An **electric circuit** is an interconnection of electrical elements.

In circuit analysis we need to calculate the voltage across some component or the current through other component or the power absorbed and delivered by different elements. In this chapter we will study about basic electrical variables such as charge, current, voltage, power and energy which will be used throughout the book. We initially focus on the *resistor*, a simple passive component, and a range of idealized active sources of voltage and current. As we move forward, new components will be added to the inventory to allow more complex (and useful) circuits to be considered.

1.2 Basic Quantities

1.2.1 Charge

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also the most basic quantity in an electric circuit is the *electric charge*.

Charge is an electrical property of the atomic particles of which matter consists, measured in Coulombs (C).

The smallest amount of charge that exists is the charge carried by an electron, equal to -1.6×10^{-19} Coulomb. While, a proton carries a charge of $+1.6 \times 10^{-19}$ Coulomb.

The following points should be noted about electric charge:

1. The Coulomb is a large unit for charges. In 1 C of charge, there are $1 / (1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons.
2. According to experimental observations, the only charges that occur in nature are integral multiple of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The law of the conservation of charge states that charge can neither be created nor be destroyed, can be only transferred. Thus the algebraic sum of the electric charges in a system does not change.

1.2.2 Current

Electric current is the time rate of change of charge, measured in amperes (A). Mathematically, the relationship between current i, charge q, and time t is

$$i(t) = \frac{dq(t)}{dt} \quad ...(1.1)$$

The net movement of 1 Coulomb (1C) of charge through a cross section of a conductor in 1 second (1s) produces an electric current of 1 amperes (1A).

The charge transferred between time t_0 and t is obtained by integrating both sides of Equation (1.1). We get

$$q(t) = \int_{t_0}^{t} i(t)\,dt \quad ...(1.2)$$

Reference Direction for Current

The direction of current flow is conventionally taken as the direction of positive charge movement. Figure 1.1 shows the convention that we use to describe a current. The current i_1 is the rate of flow of electric charge from left to right, while the current i_2 is the flow of charge from right to left. Both have same value but opposite direction.

$$i_1 = -i_2$$

A current can be completely described by a value (which can be positive or negative) and a direction (indicated by an arrow).

***Figure*-1.1 :** *Current in a circuit element*

For **example**, a current of 2 A may be represented positively or negatively as shown in Figure 1.2. In other words, a negative current of –2 A flowing in one direction as shown in Figure 1.2 (b) is the same as a current of +2 A flowing in the opposite direction as shown in Figure 1.2 (a).

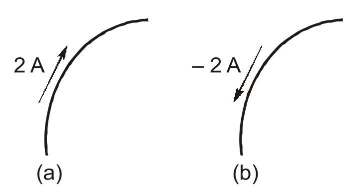


***Figure*-1.2 :** *Conventional current flow (a) Positive current flow (b) Negative current flow*

Types of Current

Different types of current are illustrated in Figure 1.3

- A current that is constant in time is termed as direct current, or simply dc, and is shown by Figure 1.3 (a).
- A current that vary sinusoidally with time [Figure 1.3 (b)]; is often referred as alternating current, or ac.

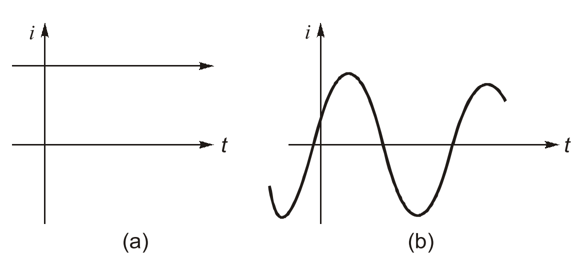


***Figure*-1.3 :** *(a) Direct current (dc) (b) Sinusoidal current (ac)*

1.2.3 Voltage

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b; mathematically,

$$v_{ab}(t) = \frac{dw}{dq} \qquad \text{...(1.3)}$$

where w is energy in joules (J), q is charge in Coulombs (C) and voltage v_{ab} is measured in volts (V). It is evident that

1 volt = 1 Joule/Coulomb = 1 Newton-meter/Coulomb

Thus, **voltage** (or **potential difference**) is the energy required to move a unit charge through an element, measured in volts (V).

Note: Voltage does not exist at a point by itself; it is always determined with respect to some other point. For this reason, voltage is also called potential difference. We often use the terms interchangeably.

Reference Polarity for Voltage

Figure 1.4 shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (–) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways; (1) point a is at a potential of v_{ab} volts higher than point b, or (2) the potential at point a with respect to point b is v_{ab}. It can be represented as

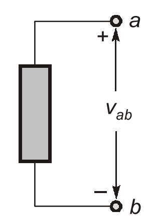


Figure-1.4 : Polarity of Voltage v_{ab}

$$v_{ab} = -v_{ba}$$
$$v_{ab} = v_a - v_b$$
$$v_{ba} = v_b - v_a$$

For **example**, in Figure 1.5, we have two representation of the same voltage. In Figure 1.5 (a), point a is +5 V above point b; in Figure 1.5 (b), point b is –5 V above point a. We may say that in Figure 1.5 (a), there is a 5 V *voltage drop* from a to b or equivalently a 5 V *voltage rise* from b to a. In other words, a voltage drop from a to b is equivalent to a voltage rise from b to a.

Keep in mind that electric current is always through an element and that electric voltage is always across the element or between two points.

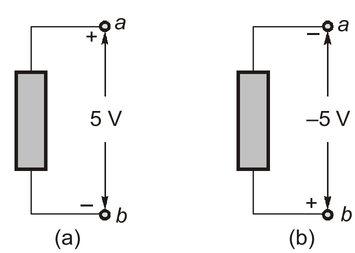


Figure-1.5 : *Two equivalent representations of the same voltage v_{ab}: (a) Point a is 5 V above point b, (b) Point b is –5 V above point a*

Types of Voltage

Types of voltage are:

- A voltage that is constant in time is termed as dc voltage.
- A voltage that vary sinusoidally with time is referred as ac voltage.

1.2.4 Power

Power is the time rate of expending or absorbing energy, measured in watts (W). Thus, in terms of energy, power is defined as

$$p(t) = \frac{dw}{dt} \qquad \text{...(1.4)}$$

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = v(t)i(t)$$

$$p(t) = v(t)\, i(t) \qquad ...(1.5)$$

We see that power is simply the product of the voltage across an element and the current through the element. This is a relation which we shall have frequent use in this book.

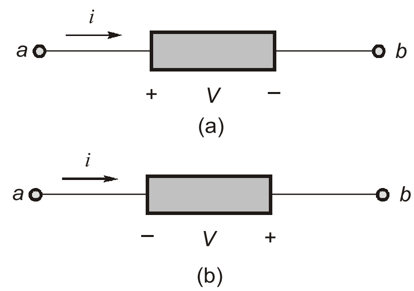


Figure-1.6 : *Passive Sign convention for power (a) Absorbing power (b) Supplying power*

Passive Sign Convention for Power Calculation

Current direction and voltage polarity are important in determining the sign of power. According to passive sign convention, the current enters the circuit element at the positive terminal of the voltage and exit at the negative terminal as shown in Figure 1.6 (a). In this case power absorbed by the circuit element is

$$p = vi$$

This power is also called power dissipated by the element or power received by the element.

If the current enters the circuit element at the negative terminal of the voltage and exit at the positive terminal as shown in Figure 1.6 (b), the power absorbed will be

$$p = -vi$$

If the absorbed power p is negative, then the circuit element actually generates power or , equivalently, delivers power to the rest of the circuit. The power absorbed by an element and the power supplied by the same element are related by

$$\text{Power absorbed} = -\text{Power supplied}$$

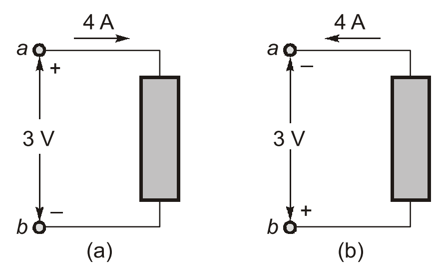


Figure -1.7 : *Two cases of an element with an absorbing power of 12 W (a) $p = 4 \times 3 = 12$ W and (b) $p = 4 \times 3 = 12$ W*

For example, the element in both circuits of Figure 1.7 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Figure 1.8 however, the element is absorbing power of –12 W is equivalent to a supplying power of +12 W.

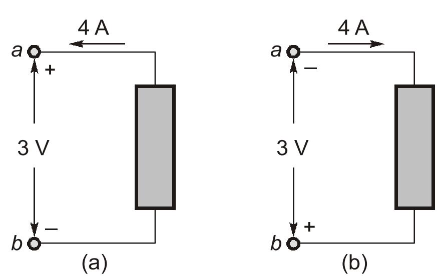


Figure-1.8 : *Two cases of an element with a supplying power of 12 W (a) $p = -4 \times 3 = -12$ W and (b) $p = -4 \times 3 = -12$ W*

Law of Conservation of Energy

Law of conservation of energy must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\Sigma p = 0 \qquad ...(1.6)$$

Thus, sum of the absorbed power is always equal to sum of delivered power in a circuit. Mathematically we can write

$$\Sigma p_{\text{absorbed}} = \Sigma p_{\text{supplied}}$$

1.2.5 Energy

Energy is the capability to perform work. The energy over a time interval is found by integrating the power. The energy absorbed or supplied by an element from time t_0 to t is

$$w(t) = \int_{t_0}^{t} p(\tau)d\tau \qquad ...(1.7)$$

Which is expressed in watt-seconds or Joules (J). The electric power utility companies measure energy in watt-hours (Wh), where

$$1\text{ Wh} = 3{,}600\text{ J}$$

Note: The electric bill that we pay to electric utility companies is paid for electric energy consumed over a certain period of time

1.3 Sources

In this section we introduce a basic circuit element called a source.

Sources are classified as voltage sources and current sources. Further it may be classified as independent and dependent sources.

1.3.1 Independent Voltage Sources

An independent voltage source is characterized by a terminal voltage which is completely independent of the current through it. The circuit symbol is shown in Figure 1.9; the subscript *'s'* merely identifies the voltage as a "source" voltage, and is common but not required.

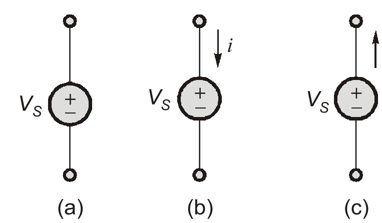
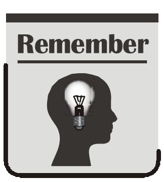


Figure*-1.9 : *Circuit symbol of the independent voltage source



- The presence of the plus sign at the upper end of the symbol of the independent voltage source in Figure 1.9 does not necessarily mean that the upper terminal is numerically positive with respect to the lower terminal. Instead, it means that the upper terminal is V_s volts positive with respect to the lower if V_s is positive.
- If at some instant V_s happens to be negative, then the upper terminal is actually negative with respect to the lower at that instant.

An independent voltage source with a constant terminal voltage is often termed an independent dc voltage source and can be represented by either of the symbols shown in Figure 1.10 (a) and (b). The symbol for an independent ac voltage source is shown in Figure 1.10 (c).

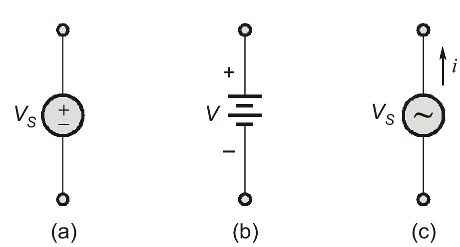


***Figure*-1.10 :** *(a) dc voltage source symbol; (b) battery symbol and (c) ac voltage source symbol*

1.3.2 Independent Current Source

In the independent current source the current through the element is completely independent of the voltage across it.

An independent current source with a constant terminal current is often termed as independent dc current source and can be represented by symbol shown in Figure 1.11 (a). The symbol for an independent ac current source is shown in Figure 1.11 (b).

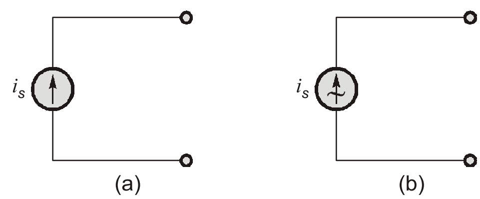


Figure-1.11 : *(a) dc current source symbol (b) ac current source symbol*

Remember

It is a common mistake to view an independent current source as having zero voltage across its terminals while providing a fixed current. In fact, we do not know a priori what the voltage across a current source will be entirely depends on the circuit to which it is connected.

Note: Generally, independent sources supply power to the circuit. However, they can be connected into a circuit in such a way that they absorb power from the circuit.

1.3.3 Dependent Source

Dependent sources are the sources whose output depend on some other voltage or current in a circuit. Both voltage and current types of sources may be dependent, and either may be controlled by a voltage or a current. Thus, there are four types of dependent sources:

1. A voltage-controlled voltage source (VCVS); $v = Av_x$
2. A current-controlled voltage source (CCVS); $v = ri_x$
3. A voltage-controlled current source (VCCS); $i = gv_x$
4. A current-controlled current source (CCCS); $i = Ai_x$

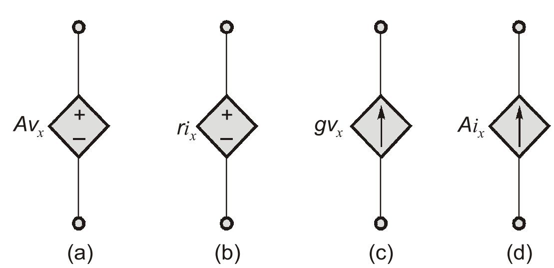


***Figure*-1.12 :** *The four different types of dependent sources*
(a) A voltage-controlled voltage source (b) A current-controlled voltage source
(c) A voltage-controlled current source (d) A current-controlled current source

Note:
- A different symbol, in the shape of a diamond, is used to represent dependent sources.
- Dependent sources are very useful in describing certain types of the electronic circuits.
- A dependent source may absorb or supply power.

1.4 Classification of Network Elements

Active and Passive Elements

- If we have a network element that is absorbing power i.e., energy delivered to the element $\left(\int_{-\infty}^{t} v(t)i(t)\,dt\right)$ is positive then the element is **passive element**. Example of passive elements are resistor, inductor, diodes and capacitor.
- If we have a network element that is delivering power i.e., energy delivered to the element $\left(\int_{-\infty}^{t} v(t)i(t)\,dt\right)$ is negative then the element is **active element**. Op-amps, generators and independent sources are the example of active elements.

Remember: The active element should be able to provide power/power gain to the circuit for infinite duration of time, that is why the charged capacitor or inductor are not active elements.

Bilateral and Unilateral Elements

- For a **Bilateral element**, the voltage-current relationship is the same for current flowing in either direction. Resistors, inductors and capacitors are the examples of bilateral elements.
- For a **Unilateral element**, the voltage-current relationship is different for two directions of current flow. Diode is an Unilateral element.

Lumped and Distributed Elements

- **Lumped elements** are considered as the separate elements which are very small in size. For example, resistor, inductors, capacitors.
- **Distributed elements** are not electrically separable. These are distributed over the entire length of the circuit. For example, transmission lines.

Note: The size of Lumped element is small with respect to signal wavelength. At steady state we can consider distributed element as Lumped element.

Linear and Non-linear Elements

Linearity is the property of an element describing a linear relationship between excitation and response. The property is a combination of both the homogeneity (scaling) property and the additivity property.

- The **homogeneity property** requires that if the input (also called the *excitation*) is multiplied by a constant, then the output (also called the *response*) gets multiplied by the same constant.
  For **example**, if for excitation (voltage or current) $E(t)$ we get response (voltage or current) $R(t)$. Then for excitation $cE(t)$ we will get response $cR(t)$.
- The **additivity property** requires that the response to a sum of inputs is the sum of the responses to each input applied separately.
  For **example**, if for excitation (voltage or current) $E_1(t)$ we get response (voltage or current) $R_1(t)$ and for excitation (voltage or current) $E_2(t)$ we get response (voltage or current) $R_2(t)$ then for excitation $E_1(t) + E_2(t)$ we get response (voltage or current) $R_1(t) + R_2(t)$.

The element that follows **additivity** and **homogeneity property** for relationship between excitation and response is called a **linear element**.

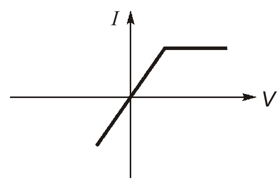
The element that does not follow **additivity** and **homogeneity property** for relationship between excitation and response is called a **non-linear element**.

1.4.1 I-V Characteristic Curves for Different Elements

Following are given some I-V characteristic curves for different elements, looking at these characteristics we can find the type of element.

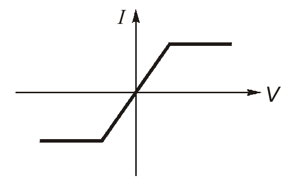
Characteristics

1.



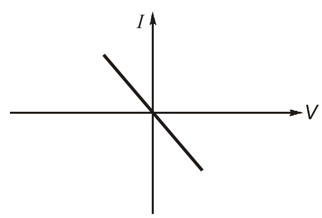
(i) Non-linear
(ii) Unidirectional
(iii) Passive

2.



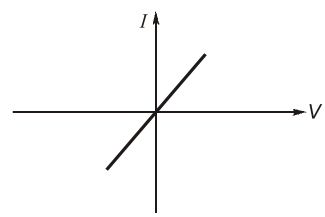
(i) Non-linear
(ii) Bidirectional
(iii) Passive

3.



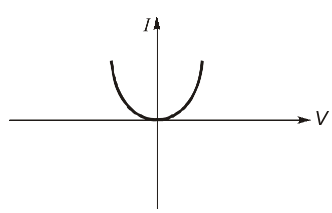
(i) Linear
(ii) Bidirectional
(iii) Active

4.



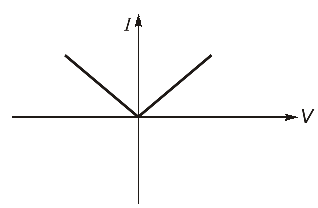
(i) Linear
(ii) Bidirectional
(iii) Passive

5.



(i) Non-linear
(ii) Unidirectional
(iii) Active

6.



(i) Non-linear
(ii) Unidirectional
(iii) Active

Remember

- If the characteristic curve is similar in opposite quadrants then the element is bidirectional otherwise it is unidirectional.
- If ratio of voltage to current at any point on characteristic curve is negative then the element is active otherwise it is passive.
- Every linear element must exhibit bidirectional property.

1.5 Nodes, Paths, Loops and Branches

A branch is a single element or component in a circuit. If several elements in a circuit carry the same current, they can also be referred to as a branch. Figure 1.13 shows branches with different circuit elements.

A node is a connection point between two or more branches which usually means a connection point between two or more elements. For example, Figure 1.14 (a) shows a circuit containing three nodes.

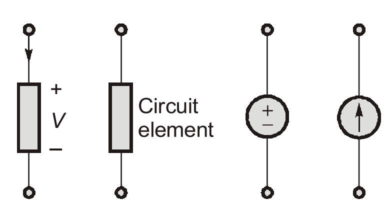


***Figure*-1.13 :** *Definition of a branch*

Sometimes networks are drawn such that two separate junctions are connected by a (zero-resistance) conductor, as in Figure 1.14 (b). However, all that has been done to spread the common point out into a zero-resistance line. Thus, we must necessarily consider all of the perfectly conducting leads or portions of leads attached to the node as part of the node. Note also that every element has a node at each of its ends.

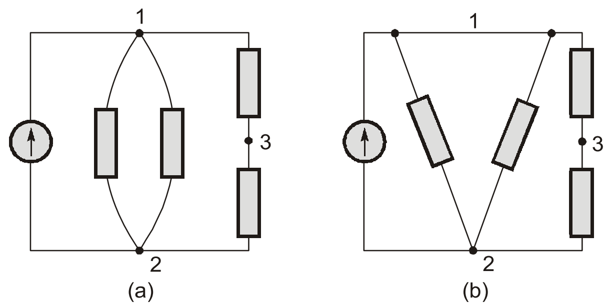


***Figure*-1.14 :** *(a) A circuit containing three nodes and five branches (b) Node 1 is redrawn to look like two nodes; it is still one node*

Figure 1.15 (a) and (b) show the nodes in two different circuit.

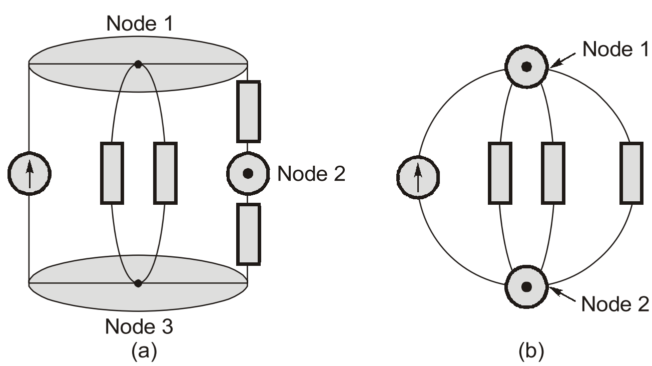


***Figure*-1.15 :** *(a) 3 nodes, 5 branches (b) 2 nodes, 4 branches*

Suppose that we start at one node in a network and move through a simple element to the node at the other end. We then continue from that node through a different element to the next node, and continue this movement until we have gone through as many elements as we wish. If no node was encountered more than once then the set of nodes and elements that we have passed through is defined as a **path**.

If the node at which we started is the same as the node on which we ended, then the path is, by definition, a closed path or a **loop**.

1.6 Circuit Elements

An element is the basic building block of a circuit. By definition, a simple circuit element is the mathematical model of a two-terminal electrical device, and it can be completely characterized by its voltage-current relationship; it cannot be subdivided into other two-terminal devices.

For **example**,

- If the voltage across the element is linearly proportional to the current through it, then element is called as a **resistor**.
- If the terminal voltage is proportional to *derivative of current* with respect to time, then element is called as a **inductor**.
- If the terminal voltage is proportional to *integral of current* with respect to time, then element is called as a **capacitor**.

The capacitor and the inductor, each of which has the ability to both store and deliver finite amounts of energy. They differ from ideal sources in this respect, since they cannot sustain a finite average power flow over an infinite time interval. Although they are classified as linear elements, the current voltage relationships for these new elements are time dependent, leading to many interesting circuits.

1.6.1 Resistors

Resistors are the materials that in general have a characteristic behaviour of resisting the flow of electric charge. This physical property, or ability of materials to resist flow of current, is known as *resistance* and is represented by the symbol R. The resistance of any material with a uniform cross-sectional area depends on A and its length l, as shown in Figure 1.16 (a). We can represent resistance (as measured in the laboratory), in mathematical form,

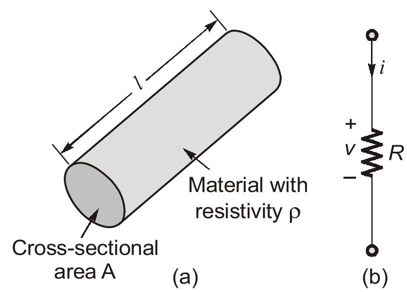


***Figure*-1.16 :** *(a) Resistor (b) Circuit symbol for resistance*

$$R = \rho\frac{l}{A} \quad ...(1.8)$$

where, ρ is known as the *resistivity* of the material in ohm-meters. The inverse of resistivity is called conductivity and is denoted by the symbol σ.

Ohm's Law

Ohm's law states that the voltage across conducting materials is directly proportional to the current flowing through the material, or

$$V = IR \quad ...(1.9)$$

Where the constant of proportionality R is called the resistance. The unit of resistance is the ohm, which is 1 V/A and is denoted by capital omega, Ω.

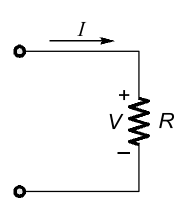


***Figure*-1.17 :** *Circuit symbol for a resistance*

A resistor that obeys Ohm's law is known as a *linear* resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in Figure 1.18 (a) its i-v graph is a straight line passing through the origin. A *non-linear* resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in Figure 1.18 (b). Examples of devices with non-linear resistance are the lightbulb and the diode. Although all practical resistors may exhibit non-linear behaviour under certain conditions, we will assume in this book that all elements actually designated as resistors are linear.

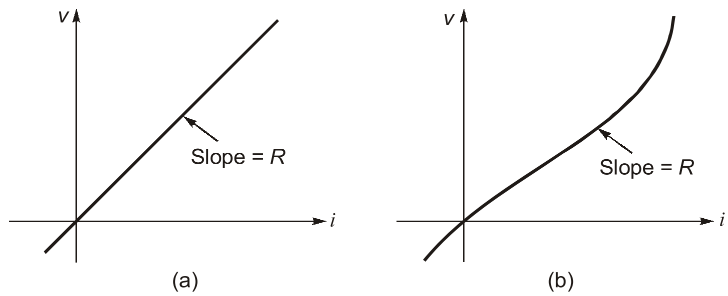


***Figure*-1.18 :** *The i-v characteristic of (a) A linear resistor (b) A non-linear resistor*

The conductance of a circuit element is defined as the inverse of its resistance. The symbol used to denote the conductance of an element is G, where

$$G = \frac{1}{R}$$

The unit of conductance is the Siemens denoted as S. The other notation are mho or inverted omega ℧.

Sign Convention

To apply Ohm's law as stated in equation (1.9), we must pay careful attention to the current direction and voltage polarity. The direction of current i and the polarity of voltage v must conform with the passive sign convention, as shown in Figure 1.16 (b). This implies that current flows from a higher potential to a lower potential in order for $v = iR$. If current flows from a lower potential to a higher potential, $v = -iR$.

Concept of Short Circuit and Open Circuit

Since the value of R can range from zero to infinity, so we consider the two extreme possible values of R. An element with $R = 0$ is called as short circuit, as shown in Figure 1.19 (a). For a short circuit,

$$V = IR = 0$$

Note: The voltage across the terminals of a short circuit is always zero, regardless of the value of the current which could be of any value.

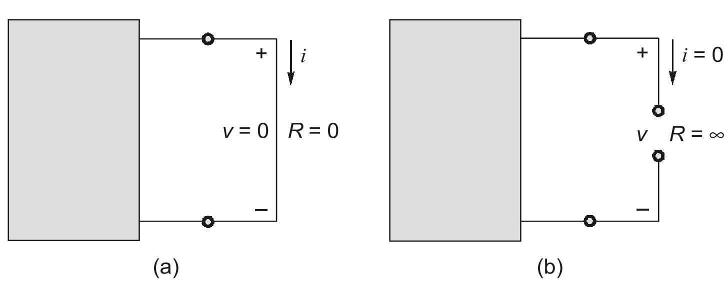


***Figure*-1.19 :** *(a) A Short circuit (R = 0) (b) An open circuit (R = ∞)*

Similarly, an element with resistance $R = \infty$ is known as an open circuit, as shown in Figure 1.19 (b). For an open circuit

$$I = \frac{V}{R} = 0$$

Note: The current through an open circuit is always zero, regardless of the voltage across the terminals which could be of any value.

Power Absorbed by Resistor

The power absorbed by a resistor can be calculated from the expression $P = VI$ together with Ohm's law $V = RI$. If we know the resistance and the voltage, then

$$P = VI = \frac{V^2}{R} = I^2R$$

We should note two things from above equation:

1. The power dissipated in a resistor is a non-linear function of either current or voltage.
2. Since R and G are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy.

1.6.2 Inductors

An inductor is a passive element designed to store energy in its magnetic field. Any conductor carrying electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in Figure 1.20.

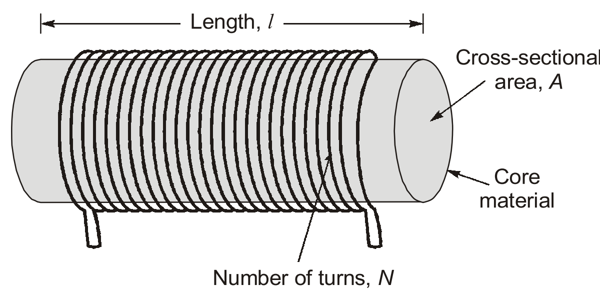


***Figure*-1.20 :** *Typical form of an inductor*

Inductor is a two terminal circuit element consisting of a winding of N turns for introducing inductance into an electrical circuit. Inductance is defined as the property of an electric device by which a time varying current through the device produces a voltage across it.

NOTE

The inductance of an inductor depends on its physical dimension and construction. For the inductor, (solenoid) shown in Figure 1.20.

$$L = \frac{N^2 \mu A}{l} \quad \text{where, } \mu = \mu_o \mu_r$$

where N is the number of turns, l is the length, A is the cross-sectional area, and μ is the permeability of the core.

Voltage, Current and Energy Relationship of an Inductor

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current. Using the passive sign convention (shown in Figure 1.21),

$$v = L\frac{di}{dt} \qquad ...(1.10)$$

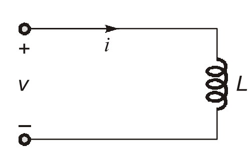


***Figure*-1.21 :** *Circuit symbol for an inductor*

where L is the constant of proportionality called the inductance of the inductor. The unit of inductance is the Henry (H). For the inductor whose inductance is independent of current is known as a linear inductor.

The current voltage relationship from equation (1.10) as

$$di = \frac{1}{L} v\, dt$$

$$i = \frac{1}{L} \int_{-\infty}^{t} v\, dt \quad ...(1.11)$$

or

$$i = \frac{1}{L} \int_{t_0}^{t} v\, dt + i(t_0) \quad ...(1.12)$$

where $i(t_0)$ is the current that accumulates from $t = -\infty$ to t_0. Equation (1.12) shows that inductor current depends on the past history of the voltage. Hence, the inductor is said to have a memory.

The power delivered to the inductor is

$$p = vi = \left(L\frac{di}{dt}\right)i \quad ...(1.13)$$

The energy stored is,

$$w = \int_{-\infty}^{t} p\, dt = \int_{-\infty}^{t} \left(L\frac{di}{dt}\right) i\, dt = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

Since, $i(-\infty) = 0$,

$$w = \frac{1}{2}Li^2 \quad ...(1.14)$$

Note: We see that even though the instantaneous power $p(t)$ may be positive or negative, the total energy stored in the inductor as given by equation (1.14) is always positive (or zero). Therefore, a inductor is a passive network element.

Important Characteristics of an Ideal Inductor:

- There is no voltage across an inductor if the current through it is not changing with time. An inductor therefore act as short-circuit in DC.
- A finite amount of energy can be stored in a inductor even if the voltage through the inductor is zero, such as when the current across it is constant.
- It is impossible to change the current through an inductor by a finite amount in zero time, as this requires an infinite voltage across the inductor. (A inductor resists an abrupt change in the current through it in a manner analogous to the way a mass resists an abrupt change in its velocity).
- An inductor never dissipate energy, but only stores it. Although this is true for the mathematical model, it is not true for a physical inductor due to finite resistances associated with the wire used for designing the inductor.

1.6.3 Capacitors

A capacitor is typically constructed as depicted in Figure 1.22. A capacitor consists of two conducting plates separated by an insulator (or dielectric).

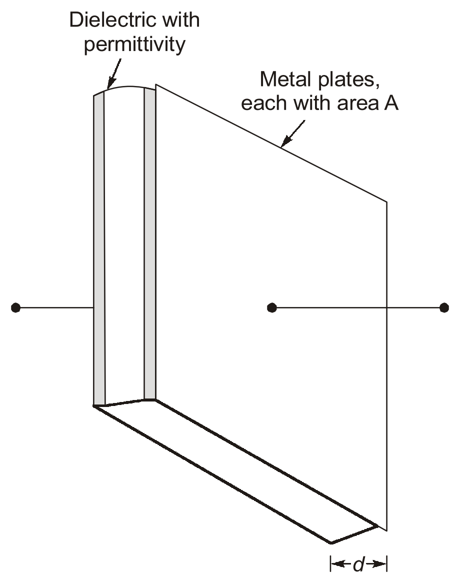


***Figure*-1.22 :** *A typical capacitor*

When a voltage source v is connected to the capacitor, as in Figure 1.23, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that

$$q = Cv \quad \text{...(1.15)}$$

where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the Farad (F). Capacitance is a measure of the ability of a device to store energy in the form of separated charge or an electric field. It depends on the physical dimensions of the capacitor. For example, for the parallel plate capacitor shown in Figure 1.23, the capacitance is given by

$$C = \frac{\varepsilon A}{d} \quad \text{...(1.16)}$$

where ε is the dielectric constant, A is the area of the plates, and d is the space between plates.

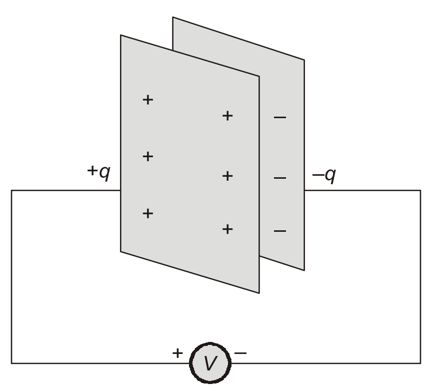


***Figure*-1.23 :** *A Capacitor with applied Voltage V*

Note: In contrast to resistor, which spends or dissipates energy irreversibly, an inductor or capacitor stores or releases energy (i.e., has a memory).

Voltage, Current and Energy Relationship of a Capacitor

To obtain the current voltage relationship of the capacitor, we take the derivative of both sides of equation (1.15).

Since, $$i = \frac{dq}{dt} \quad ...(1.17)$$

differentiating both sides of equation (5.1) gives,

$$i = C\frac{dv}{dt} \quad ...(1.18)$$

where 'v' and 'i' follows the passive sign convention as shown in Figure 1.17. We can see that voltage current relation of capacitor is a linear relation. Capacitors that satisfy equation (1.18) are said to be linear.

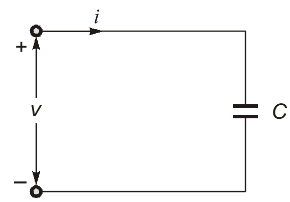


***Figure*-1.24 :** *Circuit symbol of capacitor*

The voltage current relation of the capacitor can be obtained by integrating both sides of equation (1.18). We get

$$v = \frac{1}{C}\int_{-\infty}^{t} i\,dt \quad ...(1.19)$$

$$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0) \quad ...(1.20)$$

where $v(t_0) = \frac{q(t_0)}{C}$ is the voltage across the capacitor at time t_0.

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt} \quad ...(1.21)$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^{t} p dt = C\int_{-\infty}^{t} v dv = \frac{1}{2}Cv^2\Big|_{t=-\infty}^{t}$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$. Thus,

$$w = \frac{1}{2}Cv^2 \quad ...(1.22)$$

Using equation (1.15), we may rewrite equation (1.22) as

$$w = \frac{q^2}{2C} \quad ...(1.23)$$

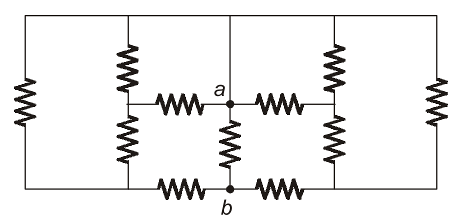
It should be noted that the capacitor is also a non-dissipative element; that is, all the energy supplied to a capacitor is stored in the capacitor. As such, it may be left in storage or it may be reclaimed in the circuit, but it is not dissipated.

Note: We see that even though the instantaneous power $p(t)$ may be positive or negative, the total energy stored in the capacitor as given by equation (1.22) is always positive (or zero). Therefore, a capacitor is a passive network element.

Important Characteristics of an Ideal Capacitor:

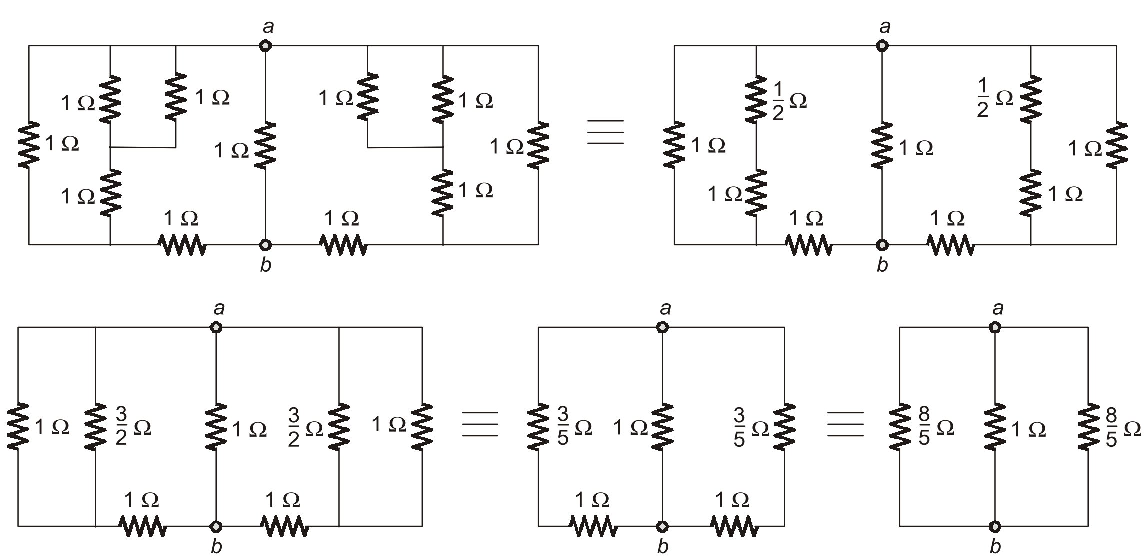
- There is no current through a capacitor if the voltage across it is not changing with time. A capacitor is therefore an open-circuit to DC.
- A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero, such as when the voltage across it is constant.
- It is impossible to change the voltage across a capacitor by a finite amount in zero time, as this requires an infinite current through the capacitor. (A capacitor resists an abrupt change in the voltage across it in a manner analogous to the way a spring resists an abrupt change in its displacement).
- A capacitor never dissipate energy, but only stores it. Although this is true for the mathematical model, it is not true for a physical capacitor due to finite resistance associated with the dielectric as well as packaging.

Example - 1.1 Consider the network shown in the figure below, if all the resistance values in the network are 1 Ω, what is the equivalent resistance that would be measured across the terminal *a* and *b*? How does this result change if the outer two resistors are both replaced by short circuits?



Solution:

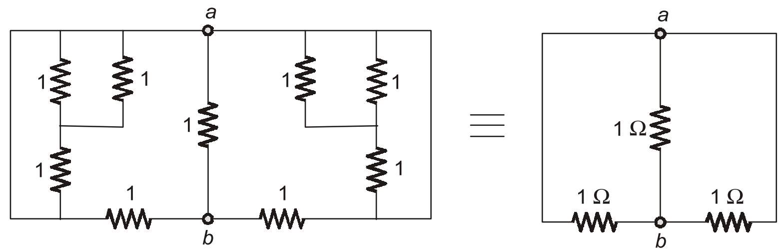
The given network can be simplified as follows



$$\therefore \quad R_{ab} = \frac{8}{5} \parallel 1 \parallel \frac{8}{5} = \frac{\frac{8}{5} \times \frac{8}{5}}{\frac{16}{5}} \parallel 1\,\Omega = \frac{4}{5} \parallel 1$$

$$R_{ab} = \frac{4}{9}\,\Omega$$

Now, if outer two resistors are short circuited then the network can be redrawn as

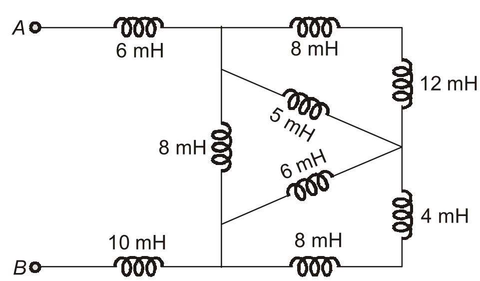


Thus all the three 1 Ω resistors are in parallel having equivalent resistance as

$$R_{ab} = 1\,\Omega \,||\, 1\,\Omega \,||\, 1\,\Omega$$

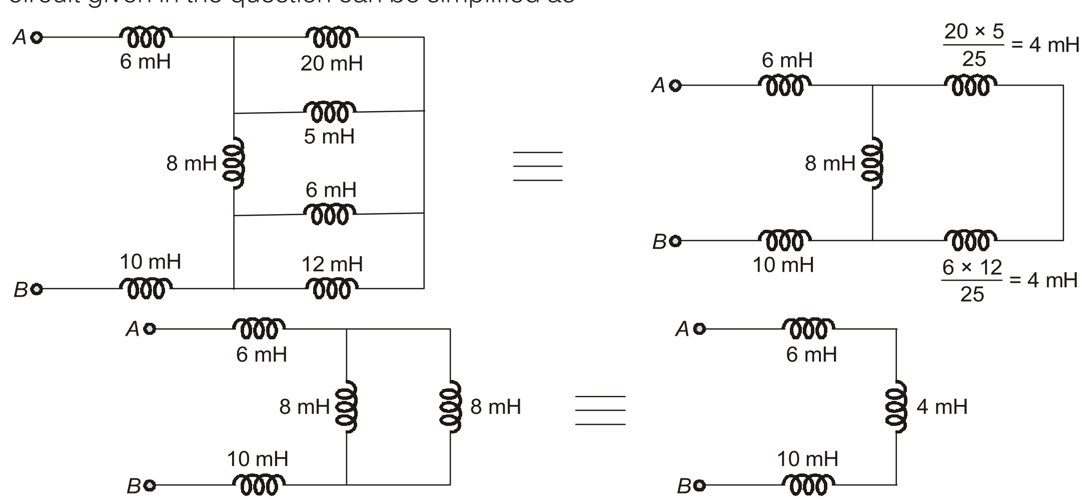
$$R_{ab} = \frac{1}{3}\,\Omega$$

Example - 1.2 Find the equivalent inductance across the terminals *a* and *b* in the circuit shown below.



Solution:

The circuit given in the question can be simplified as



$$L_{ab} = 6 + 4 + 10 = 20\text{ mH}$$

Example - 1.3 Determine the value of L_{eq} that can be used to represent the inductive network of the figure below.