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Energy Conversion

Electrical Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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CHAPTER 1

Basics of Electromechanical Energy Conversion

Introduction

The Main advantage of electric energy over other forms of energy is the relative ease and high efficiency with which it can be transmitted over long distances. Its main use is in the form of a transmitting link for transporting other forms of energy. e.g. mechanical, sound, light, etc. from one physical location to another. Electric energy is seldom available naturally and is rarely directly utilized. Obviously two kinds of energy conversion devices are needed: (1) To convert any available form of energy into electrical energy, (2) To convert electrical energy into desired form of energy.

Electromechanical devices convert mechanical energy into electrical energy or vice-versa. One category of electromechanical devices are transducers, which are used for converting, processing and transporting low energy. Second category of such devices is meant for production of limited force and torque resulting into limited motion, called force producing devices. Third category of these devices are continuous energy conversion devices like motor or generator – which are used for bulk energy conversion and utilization.

Electromechanical energy conversion takes place via the medium of a magnetic or electric field-the magnetic field being most suited for practical conversion devices. Because of the inertia associated with mechanically moving members, the fields must necessarily be slowly varying, i.e. quasistatic in nature. The conversion process is basically a reversible one though practical devices may be designed and constructed to particularly suit one mode of conversion or the other.

1.1 Principle of Energy Conversion

When energy is converted from one form into another, the principle of conservation of energy can be invoked. According to this principle, energy can neither be created nor destroyed, it can merely be converted from one form into another.

In an energy conversion device, out of the total input energy, some energy is converted into the required form, some energy is stored and the rest is dissipated. In view of this, the energy balance equation must include these four energy terms and for a motor, it can be written as

$$\begin{pmatrix}\text{Total electrical}\\ \text{energy input}\end{pmatrix} = \begin{pmatrix}\text{Mechanical}\\ \text{energy output}\end{pmatrix} + \begin{pmatrix}\text{Total energy}\\ \text{stored}\end{pmatrix} + \begin{pmatrix}\text{Total energy}\\ \text{dissipated}\end{pmatrix} \qquad ...(1.1)$$

The principle of energy conversion is based on energy balance equation (1.1). It should be noted that equation (1.1) is written for motor action where electrical energy input and mechanical energy output are treated as positive terms. For generator action,

(Total mechanical energy input) = (Electrical energy output) + (Total energy stored) + (Total energy dissipated)

The various forms of energies involved in equation (1.1) for an electromechanical energy conversion device, are now described below :

(i) Total electrical energy input from the supply mains is W_{ei}.

(ii) The mechanical energy output is W_{mo}.

(iii) Total energy stored in any device = Energy stored in magnetic field, W_{mgs} + Energy stored in mechanical system, W_{ms}.

(iv) Total energy dissipated = Energy dissipated in electric circuit as ohmic losses + Energy dissipated as magnetic core loss (hysteresis and eddy-current losses) + Energy dissipated in mechanical system (friction and windage losses etc.)

Thus the energy balance equation (1.1) can be written in more specific terms as

$$W_{ei} = W_{mo} + (W_{es} + W_{ms}) + (\text{Ohmic energy losses} + \text{Coupling field energy losses}) + (\text{Energy losses in mechanical system})$$

The subscripts *e*, *m*, *mgi*, *s* and *o* stand for electrical, mechanical, magnetic input, stored and output respectively. For example, subscripts *ei* denotes electrical input (energy), subscript *ms* denotes mechanical stored (energy).

If the appropriate terms are grouped together, then the energy balance equation becomes,

$$(W_{ei} - \text{Ohmic energy losses}) = (W_{mo} + W_{ms} + \text{Mechanical energy losses}) + (W_{mgs} + \text{Coupling field energy losses}) \quad \text{...(1.2 a)}$$

or,
$$W_{elec} = W_{mech} + W_f \quad \text{...(1.2 b)}$$

Equation (1.2) leads to the electromechanical energy conversion model of Fig. 1.1. The various losses, i.e. I^2R losses, coupling field losses and the friction and windage losses are irresversible, and these are therefore dissipated as heat.

The energy stored in the mechanical system W_{ms}, is the kinetic energy $\frac{1}{2}$ (mass) u^2 for a linear-motor system or $\frac{1}{2}J\omega_r^2$ for a rotary-motion system. Here, *u* is the linear velocity in m/sec and ω_r is the angular velocity in radians per second. Note that the coupling field is associated with emf *e* and current *i* on the electrical side and torque *T* (or force *F*) and speed ω_r (or *u*) on the mechanical side.

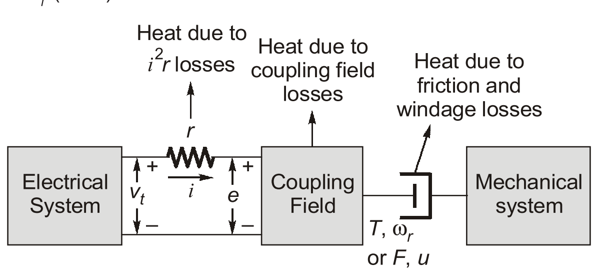


***Fig. 1.1:** General representation of electromechanical energy conversion device*

In equation (1.2b), W_{elec} is the net electrical energy input to the coupling field. W_{mech} is the total energy converted to mechanical form and it is equal to the sum of useful mechanical energy W_{mo}, mechanical energy stored W_{ms} and mechanical energy losses. W_f is the total energy absorbed by the coupling field and it is equal to the sum of both the stored field energy W_{mgs} and the coupling field energy losses.

NOTE

When energy is coupled from one circuit to another, the field through which it is accomplished, known as coupling field. For example, capacitive coupling uses electrical field, thus in capacitive coupling, coupling field is electric field.

It is evident from above that electrical losses (ohmic) and coupling-field losses as well as mechanical losses (friction and windage losses), though always present, play no basic role in the energy conversion process.

For the lossless conversion system of Fig. (1.2 a), equation (1.2) can be written in differential form as

$$dW_{elec} = dW_{mech} + dW_f \qquad \text{...(1.3)}$$

where,
$d\,W_{elec}$ = differential electrical energy input to coupling field
$d\,W_f$ = differential change in energy stored in the coupling field.
$d\,W_{mech}$ = differential change in total mechanical energy (output mechanical energy + stored mechanical energy)

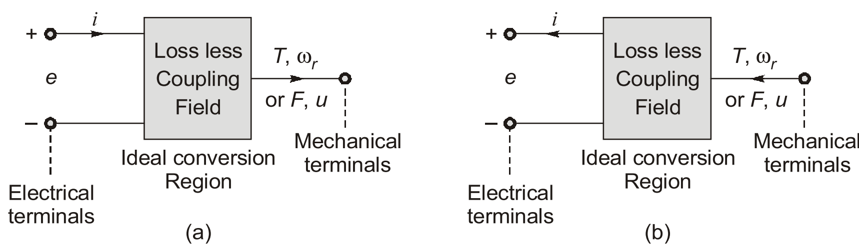


***Fig. 1.2:** Representation of lossless electromechanical energy conversion device (a) motoring mode and (b) generating mode.*

From Fig. 1.1, the differential electrical energy input in time dt is

$$d\,W_{ei} = v_t\, i\, dt$$

Ohmic loss in resistance r in time dt, is $i^2\, r\, dt$.

∴ Differential electrical energy input to the coupling field,

$$d\,W_{elec} = d\,W_{ei} - \text{ohmic loss}$$
$$= (v_t - ir)\, i\, dt = ei\, dt \qquad \text{...(1.4)}$$

Equation (1.3) now becomes,

$$ei\, dt = d\,W_{mech} + d\,W_f \qquad \text{...(1.5)}$$

Energy balance equation (1.5) is obtained by applying the principle of conservation of energy to the motoring mode. This equation along with Faraday's law of induced e.m.f. $e = -\dfrac{d\psi}{dt}$, forms the fundamental basis for the analysis of energy-conversion devices.

1.2 Coupling-field reaction

Coupling field is the link between electrical and mechanical systems. In order that a moving member can rotate, or move, with respect to stationary member, an air gap exist between the stator and rotor. These air gaps have magnetic field set up in them, thus store energy in the form of magnetic field. This energy stored in the coupling field must produce action and reaction on the electrical and mechanical systems for the conversion of energy from electrical to mechanical (motoring mode) or from mechanical to electrical (generating mode).

If the output is mechanical, as in a motor, the coupling field must react with the electrical system in order to absorb electrical energy from it. In a motor, this reaction is back emf of counter emf. If the output is electrical, as in a generator, the coupling field must react with the mechanical system so as to absorb mechanical energy from it. In a generator, this reaction is counter torque, opposite to the applied mechanical torque.

1.3 Energy in Magnetic System

Consider, for example the magnetic system of an attracted armature relay of Fig. 1.3. The resistance of the coil is shown by a series lumping outside the coil which then is regarded as an ideal loss-less coil. The coil current causes magnetic flux to be established in the magnetic circuit. It is assumed that all the flux ϕ is confined to the iron core and therefore links all the N turns creating the coil flux linkages of

$$\lambda = N\phi \qquad ...(1.6)$$

The flux linkage causes a reaction emf of

$$e = \frac{d\lambda}{dt} \qquad ...(1.7)$$

to appear at the coil terminals with polarity (as per Lenz's law) shown in the Fig. 1.3. The associated circuit equation is

$$v = iR + e$$

$$= iR + \frac{d\lambda}{dt} \qquad ...(1.8)$$

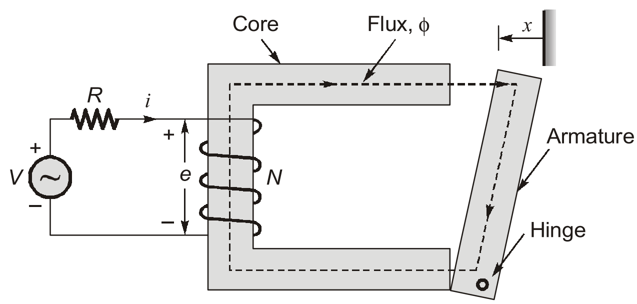


***Fig. 1.3:** Attracted armature relay*

The electric energy input into the ideal coil due to the flow of current i in time dt is

$$dW_e = ei\ dt$$

Assuming for the time being that the armature is held fixed at position x, all the input energy is stored in the magnetic field. Thus,

$$dW_e = ei\ dt = dW_f \qquad ...(1.9)$$

Where dW_f is the change in field energy in time dt. When the expression for e in equation (1.7) is substituted in equation (1.9) we have,

$$dW_e = i d\lambda = F d\phi = dW_f \qquad ...(1.10)$$

Where, $F = Ni$, the magneto motive force (mmf)

The relationship i-λ or F-λ is a functional one corresponding to the magnetic circuit which in general is nonlinear (and is also history-dependent, i.e. it exhibits hysteresis). The energy absorbed by the field for finite change in flux linkages for flux, is obtained from equation (1.10) as

$$\Delta W_f = \int_{\lambda_1}^{\lambda_2} i(\lambda) d\lambda = \int_{\phi_1}^{\phi_2} F(\phi) d\phi \qquad ...(1.11)$$

The energy absorbed by the magnetic system to establish flux ϕ(or flux linkages λ) from initial zero flux is

$$W_f = \int_0^{\lambda} i(\lambda) d\lambda = \int_0^{\phi} F(\phi) d\phi \qquad ...(1.12)$$

Then this is the energy of the magnetic field with given mechanical configuration when its state corresponds to flux ϕ (or flux linkage λ)

The i - λ relationship is indeed the magnetization curve which varies with the configuration variable x (Fig. 1.4). The air-gap between the armature and core varies with position x of the armature. The total reluctance of the magnetic path decreases as x increases) relationship can be expressed as

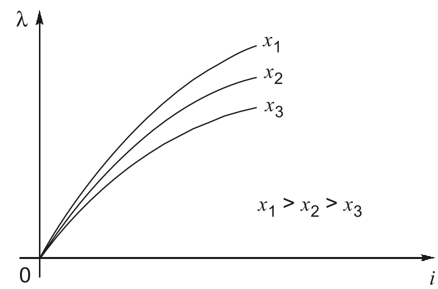


***Fig. 1.4 :** i-λ relationship with variable x*

$$i = i(\lambda, x) \quad ...(1.13)$$

If λ is the independent variable or as

$$\lambda = \lambda(i, x) \quad ...(1.14)$$

If i is the independent variable.

Therefore, the field energy is in general a function of two variables,

i.e. $$W_f = W_f(\lambda, x) \quad ...(1.15)$$

or, $$W_f = W_f(i, x) \quad ...(1.16)$$

As per equation (1.12) the field energy is the area between the λ-axis and i-λ curve as shown in Fig. 1.4. A new term, co-energy is now defined as

$$W_f'(i, x) = i\lambda - W_f(\lambda, x) \quad ...(1.17)$$

wherein by expressing λ as $\lambda(i, x)$, the independent variables of W_f' become i and x. The co-energy of Fig. 1.5 is shown to be complementary area of the i-λ curve. It easily follows from Fig. 1.5 that

$$W' = \int_0^i \lambda \, di$$

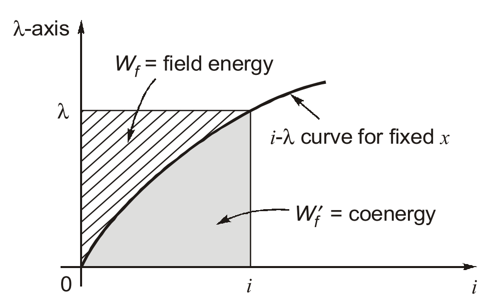


***Fig. 1.5:** Field energy and coenergy*

1.3.1 Linear Case

Electromechanical energy conversion devices are built with air-gaps in the magnetic circuit which separate the stationary and moving members. As a result, the i - λ relationship of the magnetic circuit is almost linear.

Assuming linearity, it follows

$$W_f = \frac{1}{2} i\lambda = \frac{1}{2} F\phi = \frac{1}{2} \Re\phi^2 \quad ...(1.18)$$

where, as it is known, $\Re = F/\phi$ = reluctance of the magnetic circuit. Since the coil inductance,

$$L = \frac{\lambda}{i}, \text{ hence the field energy can be expressed as,}$$

$$W_f = \frac{1}{2}\frac{\lambda^2}{L} \quad ...(1.19)$$

In the linear case the inductance L is independent of i but is a function of configuration x. Thus the field energy is a special function of two independent variables λ and x, i.e.

$$W_f(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L(x)} \quad ...(1.20)$$

The field energy is distributed throughout the space occupied by the field. Assuming no losses and constant permeability, the energy density of the field is

$$W_f = \int_0^B H\,dB = \frac{1}{2}HB = \frac{1}{2}\frac{B^2}{\mu} \text{ J/m}^3 \quad ...(1.21)$$

Where, H = magnetic field intensity (AT/m)

B = magnetic flux density (T)

The energy density expression of equation (1.21) is important from the point of view of design wherein the capability of the material is to be fully utilized in arriving at the gross dimensions of the device.

For the linear case it easily follows from equation (1.21) that coenergy is numerically equal to energy, i.e.

$$W_f' = W_f = \frac{1}{2}\lambda i = \frac{1}{2}F\phi = \frac{1}{2}PF^2 \quad ...(1.22)$$

where, P = ϕ/F = permeance of the magnetic circuit

Also in terms of the coil inductance,

$$W_f' = \int_0^i (\lambda = Li)\,di = \frac{1}{2}Li^2 \quad ...(1.23)$$

or in general, $$W_f'(i, x) = \frac{1}{2}L(x)i^2 \quad ...(1.24)$$

The expression for coenergy density is,

$$W_f' = \int_0^H B\,dH \quad ...(1.25)$$

which for the linear case becomes,

$$W_f' = \frac{1}{2}\mu H^2 = \frac{1}{2}\frac{B^2}{\mu} \text{ J/m}^3 \quad ...(1.26)$$

Example - 1.1 A solenoid of height h and radius r has N turns. For a solenoid current i, calculate

(a) the energy stored inside the solenoid.

(b) the radial magnetic force tending to burst out the solenoid.

(c) the radial pressure on the sides of solenoid.

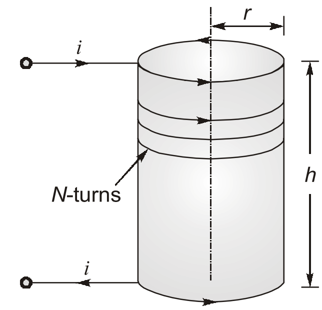
(d) the solenoid inductance.

Assume no flux outside the solenoid.

Solution:

(a) For the direction of current i shown in figure, the magnetic field inside the solenoid is directed axially upward.

∴ Magnetic field intensity inside the solenoid,



$$H = \frac{i \cdot N}{h} \text{ AT/m}$$

$$\text{Flux density, } B = \mu_0 H = \mu_0 \frac{iN}{h} \text{ Wb/m}^2.$$

Field energy density inside the solenoid,

$$w_{fld} = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\mu_0\left(\frac{iN}{h}\right)^2 \text{ J/m}^3$$

Field energy stored inside the solenoid,

$$W_{fld} = w_{fld} \times \text{volume of solenoid}$$

$$= \frac{1}{2}\mu_0\left(\frac{iN}{h}\right)^2 \times \pi r^2 h = \frac{1}{2}\mu_0\frac{(iN)^2}{h} \times \pi r^2 \text{ J}$$

(b) Radial magnetic force,

$$f_e = \frac{\partial W_{fld}(i,r)}{\partial r} = \frac{1}{2}\mu_0\frac{(iN)^2}{h} \times \pi(2r) = \mu_0\frac{(iN)^2}{h}\pi r \text{ (Newton)}$$

(c) Radial pressure on the sides of solenoid

$$= \frac{f_e}{\text{Surface area of solenoid}}$$

$$= \mu_0\frac{(iN)^2}{h}\pi r \times \frac{1}{2\pi r h} = \frac{1}{2}\mu_0\left(\frac{iN}{h}\right)^2 \text{ N/m}^2$$

(d) Solenoid inductance, $$L = \frac{N^2}{\Re l} = \frac{N^2 \cdot \mu_0 \cdot \pi r^2}{h} \text{ H}$$

1.4 Field Energy and Mechanical Force

Consider once again the attracted armature relay excited by an electric source as in Fig. 1.6. The field produces a mechanical force F_f in the direction indicated which drives the mechanical system (which may be composed of passive and active mechanical elements). The mechanical work done by the field when the armature moves a distance dx in positive direction is

$$dW_m = F_f\, dx$$

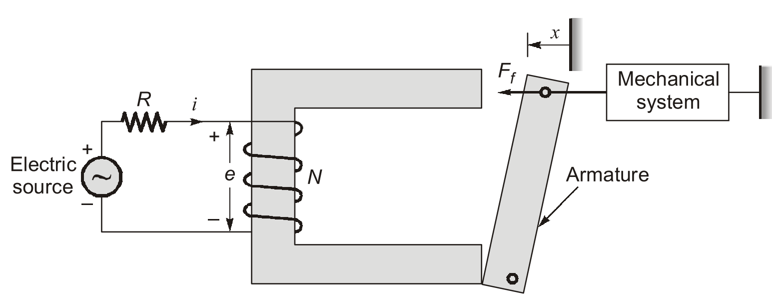


Fig. 1.6: *Production of mechanical force*

This energy is drawn from the field by virtue of change dx in field configuration. As per the principle of energy conservation.

Mechanical energy output = electrical energy input – increase in field energy

or in symbolic form $F_f dx = i d\lambda - dW_f$ ...(1.27)

It may be seen that $F_f dx$ is the gross mechanical output, a part of which will be lost in mechanical friction.

From equation (1.17),

$$W_f = i\lambda - W_f'(i,x) \quad ...(1.28)$$

Then,

$$dW_f = d(i\lambda) - dW_f'(i,x)$$

$$= id\lambda + \lambda di - \left(\frac{\partial W_f'}{\partial i} \cdot di + \frac{\partial W_f'}{\partial x} dx\right) \quad ...(1.29)$$

Substituting for dW_f from equation 1.29 in equation 1.27, we have

$$F_f\, dx = idl - \left[id\lambda + \lambda di - \left(\frac{\partial W_f'}{\partial i} di + \frac{\partial W_f'}{\partial x} dx\right)\right] \quad ...(1.30)$$

or

$$F_f dx = \left[\frac{\partial W_f'}{\partial i} - \lambda\right] di + \frac{\partial W_f'}{\partial x} dx \quad ...(1.31)$$

Because the incremental changes di are not present on the left-hand side of equation (1.31), di's coefficients should be zero on the left side i.e.,

$$\frac{\partial W_f'}{\partial i} - \lambda = 0$$

$$\lambda = \frac{\partial W_f'}{\partial i} \quad ...(1.32)$$

It then follows from equation 1.31,

$$F_f = \frac{\partial W_f'(i,x)}{\partial x} \quad ...(1.33)$$

The expression for mechanical force developed above applies when i is an independent variable, i.e. it is a current excited system.

If (λ, x) are taken as independent variables,

$$W_f = W_f(\lambda, x)$$

$$dW_f = \frac{\partial W_f}{\partial \lambda} d\lambda + \frac{\partial W_f}{\partial x} dx \quad ...(1.34)$$

Substituting equations 1.34 in equation 1.27

$$F_f dx = id\lambda - \frac{\partial W_f}{\partial \lambda} d\lambda - \frac{\partial W_f}{\partial x} dx$$

or

$$F_f dx = -\frac{\partial W_f}{\partial x} dx + \left(i - \frac{\partial W_f}{\partial \lambda}\right) d\lambda \quad ...(1.35)$$

Since $d\lambda$, the independent differential, is not present on the left hand side of this equation,

$$i - \frac{\partial W_f}{\partial \lambda} = 0$$

or

$$i = \frac{\partial W_f(\lambda, x)}{\partial \lambda} \quad ...(1.36)$$

Hence,

$$F_f = -\frac{\partial W_f(\lambda, x)}{\partial x} \quad ...(1.37)$$

In this form of expression for the mechanical force of field origin, λ is the independent variable, i.e it is a voltage-controlled system as voltage is derivative of λ.

The electromagnetic torque T_e can be obtained as,

$$T_e = -\frac{\partial W_f(\lambda,\theta)}{\partial\theta} = -\frac{\partial W(\phi,\theta)}{\partial\theta} \quad ...(1.38)$$

1.4.1 Determination of Mechanical Force

Nonlinear Case

It was seen above that the mechanical force is given by the partial derivatives of coenergy or energy as per equations 1.33 and equation 1.37. In the general nonlinear case, the derivative must be determined numerically or graphically by assuming a small increment Δx. Thus,

$$F_f \approx \left.\frac{\Delta W_f'}{\Delta x}\right|_{i=\text{constant}} \quad ...(1.39\text{ a})$$

or

$$F_f \approx -\left.\frac{\Delta W_f}{\Delta x}\right|_{\lambda=\text{constant}} \quad ...(1.39\text{ b})$$

These two expressions will give slightly different numerical values of F_f because of finite Δx. Obviously F_f is the same in each case as $\Delta x \to 0$. Calculation of F_f by equation is illustrated.

Linear Case

From equation (1.24),

$$W_f'(i,x) = \frac{1}{2}L(x)i^2$$

$$\therefore \quad F_f = \frac{\partial W_f'}{\partial x} = \frac{1}{2}i^2\frac{\partial L(x)}{\partial x} \quad ...(1.40)$$

From equation, it is obvious that the force acts in a direction to increase the inductance of the exciting coil, a statement already made.

Alternatively, from equation (1.20),

$$W_f(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L(x)}$$

$$\therefore \quad F_f = -\frac{\partial W_f}{\partial x} = -\frac{1}{2}\left(\frac{\lambda}{L(x)}\right)^2\frac{\partial L(x)}{\partial x} \quad ...(1.41)$$

It may be seen that equation 1.41 and equation 1.40 are equivalent as $i = \lambda/L$.

Also from equation 1.18, $W_f(f, x) = \frac{1}{2}\Re(x)\phi^2$

$$\therefore \quad F_f = -\frac{\partial W_f}{\partial x} = -\frac{1}{2}\phi^2\frac{\partial \Re(x)}{\partial x} \quad ...(1.42)$$

It must be remembered here that there is no difference between λ as independent variable or ϕ as independent variable as these are related by a constant ($\lambda = N\phi$). It follows from equation that the force acts in a direction to reduce reluctance of the magnetic system, a statement that has been made already. Another expression for F_f can be derived as below:

From equation (1.18), $W_f(\lambda, x) = \frac{1}{2}\lambda i(x)$

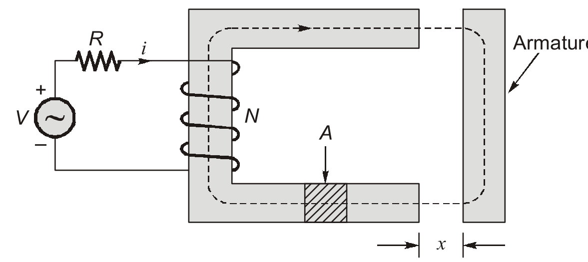
$$F_f = -\frac{\partial W_f}{\partial x} = -\frac{1}{2}\frac{\lambda\partial i(x)}{\partial x} \quad ...(1.43)$$

Example - 1.2 The electromagnetic relay of figure below is excited from a voltage source

$$v = \sqrt{2}V \sin \omega t$$

Assuming the reluctance of the iron path of the magnetic circuit to be constant, find the expression for the average force on the armature, when the armature is held fixed at distance x.

Solution:



Reluctance of the iron path $= a$ (say)

Reluctance of air gap, $\Re = \dfrac{1}{\mu A}$

Thus, Reluctance of the air path $= \dfrac{2x}{\mu_0 A} = bx$

Total reluctance of the magnetic path,

$$R = a + bx$$

$$W_f(\phi, x) = \frac{1}{2}\Re(x)\phi^2 \qquad \text{(Note that } \lambda = N\phi)$$

$$F_f = -\frac{\partial W_f(\phi, x)}{\partial x} = -\frac{1}{2}\phi^2\frac{\partial \Re}{\partial x} = -\frac{1}{2}b\phi^2$$

$$= -\frac{1}{2}\phi^2\frac{\partial \Re}{\partial x} = -\frac{1}{2}b\phi^2 \qquad \text{...(i)}$$

Notice that, $\dfrac{\partial \Re}{\partial x} = b$ is positive, so that F_f is negative, i.e. it acts in a direction to reduce x (which means in a direction to reduce reluctance $\Re$).

Now i and v are related by the circuit equation,

$$V = iR + L\frac{di}{dt}$$

whose steady-state solution is,

$$\bar{I} = \frac{\sqrt{2}V}{\sqrt{R^2 + \omega^2 L^2}} \angle -\tan^{-1}\frac{\omega L}{R}$$

Then, $$i = \frac{\sqrt{2}V}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$L = \frac{N^2}{\Re} \qquad \left\{\because \lambda = Li \Rightarrow N\phi = Li \Rightarrow \frac{NF}{\Re} = Li \Rightarrow \frac{N^2 i}{\Re} = Li\right.$$

Then, $$\phi = \frac{Ni}{\Re}$$

$$= \frac{\sqrt{2}NV}{\sqrt{(\Re R)^2 + (N^2\omega)^2}} \sin\left(\omega t - \tan^{-1}\frac{\omega N^2}{R\Re}\right)$$

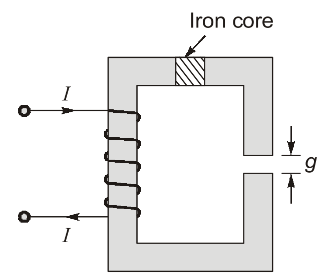
Substituting ϕ in equation (i),

$$F_f = -\frac{bN^2V^2}{(R\Re)^2 + (N^2\omega)^2} \sin^2\left(\omega t - \tan^{-1}\frac{N^2\omega}{R\Re}\right)$$

$$F_f(av) = \frac{1}{T}\int_0^T F_f dt;\ T = \frac{2\pi}{\omega} \qquad \left\{\because \frac{1}{T}\int_0^T \sin^2\left(\omega t - \tan^{-1}\frac{N^2\omega}{R\Re}\right) d(t) = \frac{1}{2}\right.$$

$$= -\frac{1}{2}\frac{bN^2V^2}{(R\Re)^2 + (N^2\omega)^2}$$

Example - 1.3 For the magnetic circuit of figure below, length of iron path = 120 cms., g = 0.5 cm, area of cross-section of iron = 5 × 5 cm², μ_r = 1500, I = 2 A, N = 1000 turns.



Calculate and compare the field-energy stored and field-energy density in iron as well as in airgap. Neglect fringing and leakage flux.

Solution:

$$\text{Total reluctance} = \frac{\text{Length of iron path}}{\mu_0\mu_r \times \text{Area}} + \frac{\text{Gap length}}{\mu_0 \times \text{Area}}$$

$$= \frac{120\times 10^{-2}}{4\pi\times 10^{-7}\times 1500\times 25\times 10^{-4}} + \frac{0.5\times 10^{-2}}{4\pi\times 10^{-7}\times 25\times 10^{-4}}$$

$$= \frac{10^9}{4\pi\times 25}\left[\frac{120}{1500} + \frac{0.5}{1}\right] = 1.8462\times 10^6 \text{ A/Wb}$$

Flux,
$$\phi = \frac{NI}{\Re} = \frac{1000\times 2}{1.8462\times 10^6}\text{ mWb} = 1.0833\text{ mWb}$$

$$\text{Field energy stored in iron} = \frac{1}{2}\phi^2 \times \text{reluctance offered by iron path}$$

$$= \frac{1}{2}[1.0833\times 10^{-3}]^2 \times \frac{120\times 10^{-2}}{4\pi\times 10^{-7}\times 1500\times 25\times 10^{-4}} = 0.14942\text{ J}$$

$$\text{Field energy stored in Air gap} = \frac{1}{2}\phi^2(\Re_{airgap})$$

$$= \frac{1}{2}(1.0833\times 10^{-3})^2 \times \frac{0.5\times 10^{-2}}{4\pi\times 10^{-7}\times 25\times 10^{-4}} = 0.93387\text{ J}$$

$$\text{Entry density in iron core} = \frac{\text{Energy stored in iron}}{\text{Volume of iron}} = \frac{0.14942}{120\times 10^{-2}\times 25\times 10^{-4}} = 49.81\text{ J/m}^3$$

$$\text{Energy density of air gap} = \frac{\text{Energy stored in air gap}}{\text{Volume of air gap}} = \frac{0.93387}{0.5\times 10^{-2}\times 25\times 10^{-4}} = 74709.6\text{ J/m}^3$$

$$\frac{\text{Energy stored in air gap}}{\text{Energy stored in iron}} = \frac{0.93387}{0.14942} = 6.25$$

$$\frac{\text{Energy density in air gap}}{\text{Energy density in iron}} = \frac{74709.6}{49.807} = 1499.98 \approx 1500$$

This example demonstrates that most of the field energy is stored in the air gap.

1.5 Multiple-Excited Magnetic Field Systems

Electromechanical transducers have the special requirement of producing an electrical signal proportional to forces or velocities or producing force proportional to electrical signal (current or voltage). Such transducers require two excitations - one excitation establishes a magnetic field of specified strength while the other excitation produces the desired signal (electrical or mechanical).

Fig. 1.7 shows a magnetic field system with two electrical excitation - one on stator and the other on rotor. The system can be described in either of the two sets of three independent variables; $(\lambda_1, \lambda_2, \theta)$ or (i_1, i_2, θ).

In terms of the first set,

$$T_f = \left.\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta}\right|_{\theta_1, \theta_2} \quad ...(1.44)$$

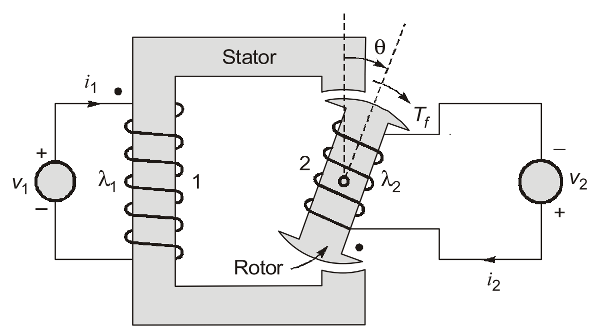


Fig. 1.7

where the field energy is given by,

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 d\lambda_1 + \int_0^{\lambda_2} i_2 d\lambda_2 \quad ...(1.45)$$

Analogous to equation 1.36,

$$i_1 = \left.\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1}\right|_{\lambda_2, \theta}$$

$$i_2 = \left.\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2}\right|_{\lambda_1, \theta}$$

Assuming linearity,

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \quad ...(1.46\text{ a})$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2; \quad (L_{12} = L_{21}) \quad ...(1.46\text{ b})$$

Solving for i_1 and i_2 in terms of λ_1, λ_2 and substituting in equation 1.45 gives upon integration*

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\beta_{11}\lambda_1^2 + \beta_{12}\lambda_1\lambda_2 + \frac{1}{2}\beta_{22}\lambda_2^2 \quad ...(1.47)$$

where,

$$\beta_{11} = \frac{L_{22}}{(L_{11}L_{22} - L_{12}^2)}$$

$$\beta_{22} = \frac{L_{11}}{(L_{11}L_{22} - L_{12}^2)}$$

$$\beta_{12} = \beta_{21} = \frac{-L_{12}}{(L_{11}L_{22} - L_{12}^2)}$$

The self and mutual - inductance of the two exciting coils are functions of angle θ.

If currents are used to describe the system state

$$T_f = \left.\frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}\right|_{i_1, i_2} \quad ...(1.48)$$

Where the coenergy is given by

$$W_f'(i_1, i_2, \theta) = \int_0^{i_1} \lambda_1 di_1 + \int_0^{i_2} \lambda_2 di_2 \quad \text{...(1.49 a)}$$

In the linear case,

$$W_f'(i_1, i_2, \theta) = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1 i_2 + \frac{1}{2}L_{22}i_2^2 \quad \text{...(1.49 b)}$$

Where inductances are function of angle θ.

Example - 1.4 Two coupled coils have self and mutual-inductance of

$$L_{11} = 2 + \frac{1}{2x};\ L_{22} = 1 + \frac{1}{2x};\ L_{12} = L_{21} = \frac{1}{2x}$$

over a certain range of linear displacement x. The first coil is excited by a constant current of 20 A and the second by a constant current of –10 A. Find:

(a) Mechanical work done if x changes from 0.5 to 1 m.

(b) Energy supplied by each electrical source in part (a).

(c) Change in field energy in part (a).

Hence verify that the energy supplied by the sources is equal to the increase in the field energy plus the mechanical work done.

Solution:

Since it is the case of current excitations, the expression of co-energy will be used.

Here, $i_1 = 20$ A, $i_2 = -10$ A

$$W_f'(i_1, i_2, x) = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1 i_2 + \frac{1}{2}L_{22}i_2^2$$

$$= \left(2 + \frac{1}{2x}\right) \times 200 + \frac{1}{2x} \times (-200) + \left(1 + \frac{1}{2x}\right) \times 50 = 450 + \frac{25}{x}$$

(a)

$$F_f = \frac{\partial W_f'}{\partial x} = -\frac{25}{x^2}$$

$$\Delta W_m = \int_{0.5}^{1} F_f dx = \int_{0.5}^{1} -\frac{25}{x^2} dx = -25 \text{ J}$$

(b)

$$\Delta W_{e1} = \int_{\lambda_1(x=0.5)}^{\lambda_1(x=1)} i_1 d\lambda_1 = i_1[\lambda_1(x = 1) - \lambda_1(x = 0.5)]$$

$$\lambda_1 = L_{11}i_1 + L_{12}i_2$$

$$= \left(2 + \frac{1}{2x}\right) \times 20 + \frac{1}{2x} \times (-10) = 40 + \frac{5}{x}$$

$$\lambda_1(x = 0.5) = 50,\ \lambda_1(x = 1) = 45$$

∴ $\Delta W_{e1} = 20(45 - 50) = -100$ J

Similarly, $\Delta W_{e2} = i_2[\lambda_2(x = 1) - \lambda_2(x = 0.5)]$

$$\lambda_2 = L_{12}i_1 + L_{22}i_2$$

$$\lambda_2 = \frac{1}{2x} \times 20 + \left(1 + \frac{1}{2x}\right) \times (-10) = -10 + \frac{5}{x}$$

$$\lambda_2(x = 0.5) = 0,\ \lambda_2(x = 1) = -5$$

$$\Delta W_{e2} = -10(-5) = 50 \text{ J}$$

Net electrical energy input,

$$\Delta W_e = \Delta W_{e1} + \Delta W_{e2}$$
$$= -100 + 50 = -50 \text{ J}$$

(c) For calculating the change in the field energy, β's have to be obtained.

$$\beta_{11} = \frac{L_{22}}{D}; \; D = L_{11}L_{22} - L_{12}^2$$
$$= \frac{2x+1}{4x+3}$$

Similarly, $$\beta_{22} = \frac{4x+1}{4x+3}$$
$$\beta_{12} = -\frac{1}{4x+3}$$

At $x = 0.5$; $$\beta_{11} = \frac{2}{5},\ \beta_{22} = \frac{3}{5},\ \beta_{12} = -\frac{1}{5}$$

At $x = 1$; $$\beta_{11} = \frac{3}{7},\ \beta_{22} = \frac{5}{7},\ \beta_{12} = -\frac{1}{7}$$

The values of λ have already been calculated at x = 0.5, 1 m

As per equation, the field energy is given by

$$W_f = \frac{1}{2}\beta_{11}\lambda_1^2 + \beta_{12}\lambda_1\lambda_2 + \frac{1}{2}\beta_{22}\lambda_2^2$$

The field energy at x = 0.5 m and x = 1 m is then calculated as

$$W_f(x = 0.5) = \frac{1}{2}\times\frac{2}{5}\times(50)^2 = 500 \text{ J} \qquad \{\because \lambda_2(x=0) = 0\}$$

$$W_f(x = 1) = \frac{1}{2}\times\frac{3}{7}\times(45)^2 - \frac{1}{7}\times 45\times(-5) + \frac{1}{2}\times\frac{5}{7}\times(-5)^2 = 475 \text{ J}$$

Hence, $$\Delta W_f = W_f(x=1) - W_f(x=0.5)$$
$$= 475 - 500 = -25 \text{ J}$$
$$\Delta W_f + \Delta W_m = -25 - 25 = -50 = \Delta W_e \text{ (verified)}$$

Note: In the linear case with constant current excitation

$$\Delta W_f = \Delta W_f'$$

ΔW_f can be easily calculated from part (a) without the need of calculating β's. Thus

$$W_f' = 450 + \frac{25}{x}$$
$$\Delta W_f' = W_f'(x=1) - W_f'(x=0.5)$$
$$= 475 - 500 = -25 \text{ J}$$

So, $$\Delta W_f = -25 \text{ J}$$

***Example* - 1.5** A doubly-excited magnetic field system has coil self and mutual inductance of

$$L_{11} = L_{22} = 2$$
$$L_{12} = L_{21} = \cos\theta$$

where θ is the angle between the axes of the coils.

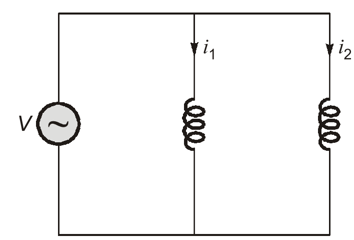
(a) The coils are connected in parallel to a voltage source $v = V_m \sin \omega t$. Derive an expression for the instantaneous torque as a function of the angular position θ. Find therefrom the time-average torque. Evaluate for θ = 30°, $v = 100 \sin 314t$.

(b) If coil 2 is shorted while coil 1 carries a current of $i_1 = I_m \sin \omega t$, derive expression for the instantaneous and time-average torques. Compute the value of the time-average torque when $\theta = 45°$ and $i_1 = \sqrt{2}\sin 314t$.

(c) In part (b) if the rotor is allowed to move, at what value of angle will it come to rest?

Solution:

(a)
$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$
$$= \frac{1}{2}\left(\frac{\partial L_{11}}{\partial \theta}\right)i_1^2 + \left(\frac{\partial L_{12}}{\partial \theta}\right)i_1 i_2 + \frac{1}{2}\left(\frac{\partial L_{22}}{\partial \theta}\right)i_2^2$$



Substituting the value of inductances,
$$T_f = -(\sin\theta)i_1 i_2$$
Using KVL in given circuit,
$$V_m \sin \omega t = 2\frac{di_1}{dt} + (\cos\theta)\frac{di_2}{dt}$$
$$V_m \sin \omega t = (\cos\theta)\frac{di_1}{dt} + 2\frac{di_2}{dt}$$
Solving these we get, $\quad \dfrac{di_1}{dt} = \dfrac{di_2}{dt} = \dfrac{V_m \sin\omega t}{(2+\cos\theta)}$

Integrating, $\quad i_1 = i_2 = \dfrac{-V_m \cos\omega t}{\omega(2+\cos\theta)}$

Substituting in T_f, $\quad T_f = -\dfrac{V_m^2 \cos^2 \omega t}{\omega^2(2+\cos\theta)^2}\sin\theta$

$$T_f(\text{av}) = -\frac{V_m^2 \sin\theta}{2(2+\cos\theta)^2\omega^2}$$

Given: $\quad \theta = 30°$,
$\quad v = 100 \sin 314t$

$\therefore \quad T_f(av) = \dfrac{-(100)^2 \sin 30°}{2(2+\cos 30°)^2 \times (314)^2} = -3.087 \times 10^{-3}$ Nm

(b) From circuit equations, $\quad 0 = (\cos\theta)\dfrac{di_1}{dt} + 2\dfrac{di_2}{dt}$

or $\quad \dfrac{di_2}{dt} = -\dfrac{1}{2}(\cos\theta)\dfrac{di_1}{dt}$

or $\quad i_2 = -\dfrac{1}{2}(\cos\theta)i_1$

Given, $\quad i_1 = I_m \sin \omega t$

$\therefore \quad i_2 = -\dfrac{1}{2}I_m(\cos\theta)\sin\omega t$

Substituting in T_f, $\quad T_f = (\sin\theta) \times \dfrac{1}{2}I_m^2(\cos\theta)\sin^2 \omega t = \dfrac{1}{2}I_m^2(\sin\theta)(\cos\theta)\sin^2 \omega t$