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Engineering Mechanics

Civil Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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Engineering Mechanics

01
CHAPTER

Composition, Resolution and Equilibrium of Forces

1.1 Units and Dimensions

Types of Units:

1. Fundamental Units
2. Derived Units

1. Fundamental Units : Basically there are three fundamental units in engineering mechanics – Length, Mass and Time.
2. Derived Units: These are derived from above mentioned three basic units (M, L and T) eg. Units of area, pressure etc.

NOTE

1. There are total seven fundamental units – ampere (A), kelvin (K), candela (Cd) and mole (mol) are additional to length, mass and time but we won't require these units.
2. There are two more supplementary unit – radian (rad) and steradian (Sr)

Table 1.1

Quantity	Unit	Dimension
Acceleration	m/s^2	LT^{-2}
Force	kgm s^{-2} or N	MLT^{-2}
Dynamic Viscosity	$N\text{-}s/m^2$	$Ml^{-1}T^{-1}$
Kinematic Viscosity	m^2/s	L^2T^{-1}

1.2 Vectors

Vectors : Those physical quantities which have magnitude, direction and follow vector laws of addition (explained in later stages eg. force, momentum etc.)

> **NOTE**
> 1. Current and Pressure have directions but they don't follow vector laws of addition and hence these are scalar quantities.
> 2. There are two types of product of vectors:
>   (i) Dot product or Scalar product
>
> $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ where θ is angle between $\vec{A}$ and $\vec{B}$.
>
>   (ii) Cross product or Vector product
>
> $|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

1.3 Force

Any pull or push is regarded as force. If a force can change the state of rest or of uniform motion of body then it will be regarded as unbalanced force otherwise balanced force.

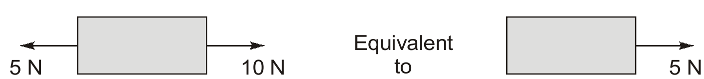


Fig. 1.1 *Unbalanced System of Forces*

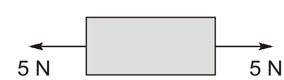


Fig. 1.2 *Balanced System of Forces*

There are different types of forces such as gravitational, frictional, magnetic, inertia or those caused by mass and acceleration.

According to Newton's second law of motion, we can write force as

$$F = ma = \text{mass} \times \frac{\text{length}}{\text{time}^2}$$

One Newton force is defined as the force which gives an acceleration of 1 m/s^2 to a body of mass, 1 kg in the direction of force.

Thus, $1\text{ N} = 1\text{ kg} \times 1\text{ m/s}^2 = 1\text{ kgm/s}^2$

The three requisites for representing the force acting on the body are:

- Magnitude of force
- Its points of application, and
- Direction of its action

> **NOTE**
> Sometimes force is represented in kg-wt. unit.
>
> 1 kg-wt = 9.81 Newton
>
> In general usage, kg-wt. force is called as kg only.

1.4 Effects of a Force

A force may produce the following effects in a body, on which it acts:

1. It may change the motion of a body i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate or retard it or bring it to rest.
2. It may give rise to the internal stresses in the body, on which it acts.

1.5 Characteristics of a Force

To know the effect of force on a body, the following elements of force should be known.

1. Magnitude (i.e. 2 N, 5 kN, 10 kN etc.)
2. Direction or line of action.
3. Sense or nature (push or pull).
4. Point of application.

1.6 Force Systems

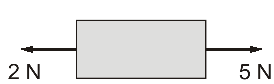
A force system is collection of forces acting on a body in one or more planes. According to the relative position of the line of action of the forces, the forces may be classified as follows:

1. **Collinear:** The forces whose lines of action lie on the same line are known as collinear forces.
2. **Concurrent:** The forces, which meet at one point, are known as concurrent forces. Concurrent forces may or may not be collinear.
3. **Coplanar:** The forces whose line of action lie on the same plane are known as coplanar forces.
4. **Coplanar concurrent:** The forces, which meet at one point and their line of action lie on the same plane, are known as coplanar concurrent forces.
5. **Non-coplanar concurrent:** The forces, which meet at one point but their lines of action do not lie on the same plane, are known as coplanar non-concurrent forces.
6. **Coplanar non-concurrent:** The forces, which do not meet at one point but their line of action lie on the same plane, are known as coplanar non-concurrent forces.
7. **Non-coplanar non-concurrent:** The forces, which do not meet at one point and their line of action do not lie on the same plane, are known as non-coplanar non-concurrent forces.

1.7 Resultant Force

A single force which produces same effect on the body as the system of forces is called as resultant force.

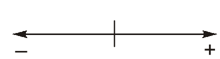
eg. (i)



The resultant of this force system is 3 N (→).

Hence, in vector addition direction also play important role along with magnitudes.

If we assume direction system here as



then resultant here = (+5)N + (–2)N = +3 N

+ indicates resultant force acts in right direction.

Hence, Resultant of $\vec{A}$ and $\vec{B} = \vec{A} + \vec{B}$

Resultant of $\vec{A}$ and (negative of $\vec{B}$) $= \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$

1.8 Triangle Law of Forces

This law states that, if two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle taken in order then the third side will represent the resultant of two forces in the direction and magnitude taken in opposite order.

This triangle law can further be illustrated as:

$$\vec{F_1}+\vec{F_2}+(-\vec{R}) = 0$$

0 because we started from O and finished at O.

$$\Rightarrow \quad \vec{R} = \vec{F_1}+\vec{F_2}$$

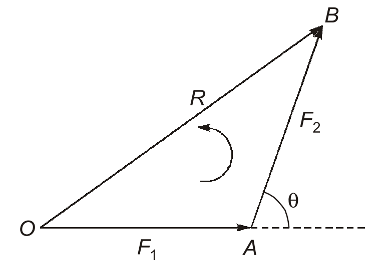


Fig. 1.3

Hence, $\vec{R}$ is the resultant of $\vec{F_1}$ and $\vec{F_2}$

Also, we can further extend it as follow:

$$\vec{F_1}+\vec{F_2}+\vec{R'} = 0$$

$$\Rightarrow \quad \vec{R'} = -(\vec{F_1}+\vec{F_2})$$

Hence $\vec{R'}$ is negative of resultant of $\vec{F_1}$ and $\vec{F_2}$

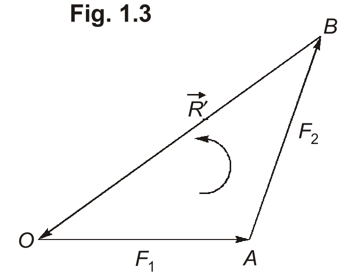


Fig. 1.4

1.9 Parallelogram Law of Forces

This law is used to find resultant of two concurrent forces.

Let system is

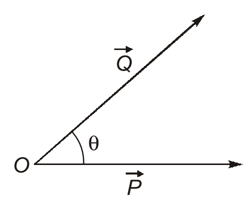


Fig. 1.5

Since any vector can be shifted without changing its direction or magnitude as per principle of transmissibility of forces. So shift $\vec{Q}$

In ΔABC, by triangle law of vector addition,

$\vec{R}$ is resultant of $\vec{P}$ and $\vec{Q}$

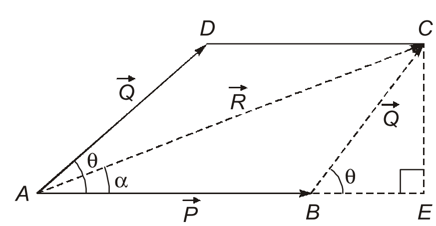


Fig. 1.6

Hence $\quad \vec{R} = \vec{P}+\vec{Q}$

$\quad \theta$ = angle between $\vec{P}$ and $\vec{Q}$

By shifting, $\quad AD = BC = Q$

Draw a right triangle BCE,

$\because \quad BC = Q$

$\Rightarrow \quad BE = Q\cos\theta \qquad CE = Q\sin\theta$

In right triangle AEC, $\quad AC^2 = AE^2 + CE^2$

$$R^2 = (AB + BE)^2 + CE^2$$

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$R^2 = P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

This is magnitude of R.

For direction of $\vec{R}$, let it make an angle of α with horizontal then in right triangle *AEC*,

$$\tan\alpha = \frac{CE}{AE} = \frac{CE}{AB+BE} = \frac{Q\sin\theta}{P+Q\cos\theta}$$

1.10 Polygon Law of Forces

When more than two forces are acting on the body, the triangle law can be extended to polygon law.

If a number of coplanar concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon taken in order, then their resultant can be represented by closing side of the polygon in magnitude and direction in the opposite order.

Let five forces are acting on a body as shown:

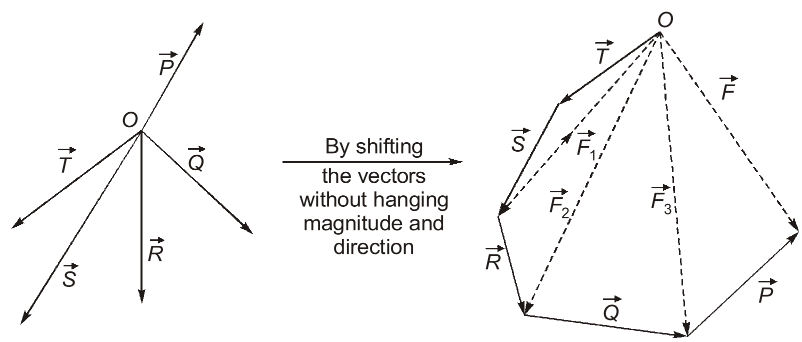


Fig. 1.7

The dotted vector $\vec{F}$ will be the resultant of all vectors taken in opposite to direction of all vectors.

Hence, $\vec{F} = \vec{P}+\vec{Q}+\vec{R}+\vec{S}+\vec{T}$

More analysis

Apply triangle law of vector addition in triangle forming by $\vec{F_1}$, $\vec{T}$ and $\vec{S}$

$$\vec{F_1} = \vec{T}+\vec{S}$$

Similarly,

$$\vec{F_2} = \vec{F_1}+\vec{R}$$

$$\vec{F_2} = (\vec{T}+\vec{S})+\vec{R}$$

also,

$$\vec{F_3} = \vec{F_2}+\vec{Q}$$

$$\vec{F_3} = (\vec{T}+\vec{S}+\vec{R})+\vec{Q}$$

also,

$$\vec{F} = \vec{F_3}+\vec{P}$$

$$\vec{F} = (\vec{T}+\vec{S}+\vec{R})+\vec{P}$$

Hence, triangle law of vector addition is applied to establish polygon law of vector addition.

1.11 Resolution of Forces

Two or more forces can be shown by a single force known as their resultant and the vice-versa is also true. i.e., A force can be resolved into more number of forces.

When A force is resolved into two components only which are at 90° to each other, these are called rectangular components of that force.

For example:

(i) Force F is resolved into

$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

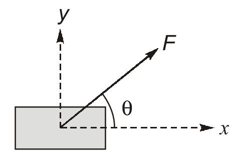


Fig. 1.8

(ii) Weight W is resolved into

$$\text{normal} = W_n = W\cos\theta$$
$$\text{parallel} = W_p = W\sin\theta$$

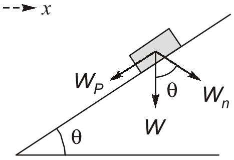


Fig. 1.9

In above 2 examples, F_x and F_y are rectangular components of F and W_P and W_N are rectangular components of W.

1.12 Composition of Forces

Conversion of system of forces into an equivalent single force system is known as the composition of forces. The effect of single equivalent force will be same as the effect produced by number of forces acting on a body.

Let the forces F_1, F_2, F_3 and F_4 are acting on a body in a plane making angle α_1, α_2, α_3 and α_4 with x-axis as shown in figure 1.10. Let R be the resultant force of all the forces acting at the point making an angle θ with horizontal as shown in figure. Resolving the forces along x-axis and y-axis, we get,

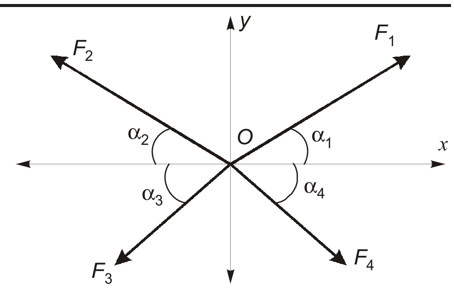


Fig. 1.10

$$\Sigma F_x = F_1\cos\alpha_1 - F_2\cos\alpha_2 - F_3\cos\alpha_3 + F_4\cos\alpha_4$$
$$\Sigma F_y = F_1\sin\alpha_1 + F_2\sin\alpha_2 - F_3\sin\alpha_3 - F_4\sin\alpha_4$$

Component of R along x-axis = $R\cos\theta$

Component of R along y-axis = $R\sin\theta$

$$R\cos\theta = \Sigma F_x$$

and $$R\sin\theta = \Sigma F_y$$

$$R^2(\sin^2\theta + \cos^2\theta) = (\Sigma F_x)^2 + (\Sigma F_y)^2$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

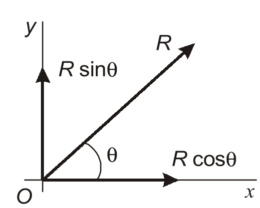


Fig. 1.11

and $$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

A body which is under co-planar system of concurrent forces is in equilibrium if $R = 0$ or

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

EXAMPLE : 1.1

Three forces are acting at a body of magnitudes 7 N, 5 N and 3 N such that the body is in equilibrium. Find the angle between 5 N and 3 N forces.

Solution:

Since all three forces are in equilibrium

then $$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0 \quad \text{....(i)}$$

where $|\vec{F_1}| = 7\text{ N}$

$|\vec{F_2}| = 5\text{ N}$

$|\vec{F_3}| = 3\text{ N}$

From equation (i), $\vec{F_1} = -(\vec{F_2} + \vec{F_3})$

or $(\vec{F_2} + \vec{F_3}) = -\vec{F_1}$

Squaring both sides, we get, $(\vec{F_2} + \vec{F_3})^2 = (-\vec{F_1})^2$

Also, $(\vec{A})^2 = |\vec{A}|^2 = A^2$

$\Rightarrow \quad F_2^2 + F_3^2 + 2F_2F_3\cos\theta = F_1^2$

where θ is angle between F_2 and F_3

$$5^2 + 3^2 + 2 \times 5 \times 3 \times \cos\theta = 7^2$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

Hence angle between 5 N and 3 N forces is 60°.

EXAMPLE : 1.2

If three forces are acting on a body such that $\vec{F_1} \times \vec{F_2} = \vec{F_2} \times \vec{F_3} = \vec{F_3} \times \vec{F_1}$ then prove that the body will be in equilibrium.

Solution:

Given: $\vec{F_1} \times \vec{F_2} = \vec{F_2} \times \vec{F_3}$

or $\vec{F_1} \times \vec{F_2} - \vec{F_2} \times \vec{F_3} = 0$

also $(\vec{F_2} \times \vec{F_3}) = -(\vec{F_3} \times \vec{F_2})$

so, $\vec{F_1} \times \vec{F_2} - (-(\vec{F_3} \times \vec{F_2})) = 0$

$\Rightarrow \quad \vec{F_1} \times \vec{F_2} + \vec{F_3} \times \vec{F_2} = 0$

$\Rightarrow \quad (\vec{F_1} + \vec{F_3}) \times \vec{F_2} = 0$

We know that if $\vec{a} \times \vec{b} = 0$ then $\sin\theta = 0$ or $\theta = 0$ or $\vec{a}$ is parallel to $\vec{b}$.

Here $(\vec{F_1} + \vec{F_3})$ is parallel to $\vec{F_2}$

Hence, $(\vec{F_1} + \vec{F_3}) = \lambda\vec{F_2}$ ...(i)

λ = any arbitrary constant

Similarly, $\vec{F_2} + \vec{F_3} = \mu\vec{F_1}$ ...(ii)

and $\vec{F_2} + \vec{F_1} = \delta\vec{F_3}$ ...(iii)

From equation (i), $\vec{F_1} = \lambda\vec{F_2} - \vec{F_3}$

From equation (iii), $\vec{F_1} = \delta\vec{F_3} - \vec{F_2}$

$$\Rightarrow \quad \lambda\vec{F_2} - \vec{F_3} = \delta\vec{F_3} - \vec{F_2}$$

$$\Rightarrow \quad (\lambda + 1)\vec{F_2} = (\delta + 1)\vec{F_3}$$

$$\Rightarrow \quad \lambda + 1 = 0 \quad \text{or} \quad \delta + 1 = 0$$

$$\Rightarrow \quad \lambda = -1 \quad \text{or} \quad \delta = -1$$

Put in equation, we get, (i)

$$\vec{F_1} + \vec{F_3} = -\vec{F_2}$$

$$\Rightarrow \quad \vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

Since resultant of all three forces is zero.
Hence body is in equilibrium.

EXAMPLE : 1.3

A plane moving with velocity 1000 km/hr turns by 60° angle and its speed remains 1000 km/hr. Find the change in velocity of plane.

Solution:
Here only the magnitude remains same but direction changes and hence velocity changes.

Let initial velocity be $\vec{V_1}$ and final be $\vec{V_2}$

$$|\vec{V_1}| = |\vec{V_2}| = V = 1000 \text{ km/hr}$$

$$\text{Change in velocity} = \vec{V_1} - \vec{V_2}$$

$$= \vec{V_1} + (-\vec{V_2}) = \text{resultant of } \vec{V_1} \text{ and } (-\vec{V_2})$$

$$= \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos(180 - \theta)}$$

$$= \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta} = \sqrt{V^2 + V^2 - 2V^2\cos\theta}$$

$$= \sqrt{2V^2 - 2V^2\cos\theta} = \sqrt{2}V\sqrt{1 - \cos\theta}$$

$$= \sqrt{2}\,V\sqrt{1 - 1 + 2\sin^2\frac{\theta}{2}} = \sqrt{2}\,V\,\sqrt{2}\sin\frac{\theta}{2} = 2V\sin\frac{\theta}{2}$$

$$\text{Change in velocity} = 2 \times 1000 \times \sin\frac{60^\circ}{2} = 1000 \text{ km/hr}$$

EXAMPLE : 1.4

There are two forces P and Q acting on a body. If P is tripled, resultant becomes 4 times and if the direction of P is reversed without changing the magnitude then resultant becomes twice. Calculate the ratio of P, Q and resultant.

Solution:
Let resultant be R
Let angle between P and Q is θ

then $$R^2 = P^2 + Q^2 + 2PQ\cos\theta \qquad \text{...(i)}$$

also when P is tripled, R becomes 4 times,

$$(4R)^2 = (3P)^2 + Q^2 + 2(3P)\,Q\cos\theta$$

$$16\,R^2 = 9P^2 + Q^2 + 6\,PQ\cos\theta \qquad \text{...(ii)}$$

also when P is reversed, R becomes twice

$$(2R)^2 = P^2 + Q^2 + 2PQ\cos(180 - \theta)$$

$$4R^2 = P^2 + Q^2 - 2\,PQ\cos\theta \qquad \text{...(iii)}$$

Now, add equations (i) and (iii)

$$5R^2 = 2P^2 + 2Q^2 \qquad \text{...(iv)}$$

equations (ii) + 3 × equations (iii)

$$28\,R^2 = 12\,P^2 + 4Q^2 \qquad \text{...(v)}$$

Now, 6 × equations (iv) – equations (v)

$$30R^2 - 28R^2 = 12P^2 + 12R^2 - 12P^2 - 4Q^2$$

$$2R^2 = 8Q^2$$

$$Q = \frac{1}{2}R \qquad \text{...(A)}$$

equations (v) – 2 × equations (iv),

$$28R^2 - 10R^2 = 12P^2 + 4Q^2 - 4P^2 - 4Q^2$$

$$18R^2 = 8P^2$$

$$P = \frac{3}{2}R \qquad \text{...(B)}$$

From (A) and (B), we get, $$P : Q : R = \frac{3}{2} : \frac{1}{2} : 1$$

$$P : Q : R = 3 : 1 : 2$$

1.13 Equilibrium of Forces

If a body is moving at a constant velocity or the body is at rest then the body is said to be in equilibrium state. If a number of forces are acting on the body and their resultant comes out to be zero, then the body is said to be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.

In case of coplanar concurrent forces, for a body to be in equilibrium, following conditions must be satisfied.

$$\left.\begin{array}{l}\Sigma F_x = 0 \\ \Sigma F_y = 0\end{array}\right. \quad \left\{\begin{array}{l}\text{Summation of all forces in } x \text{ and } y \text{ direction must be} \\ \text{individually zero for translational equilibrium.}\end{array}\right\}$$

In case of coplanar non-concurrent forces, one additional condition is also required to be satisfied:

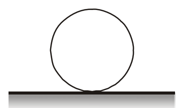
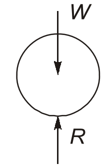
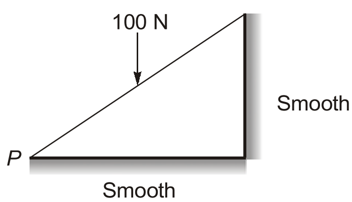
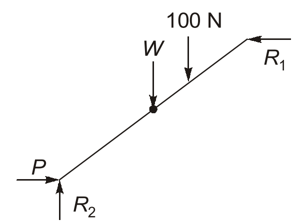
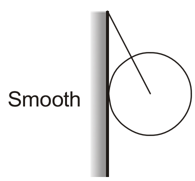
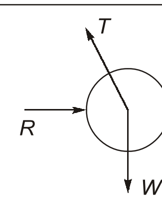
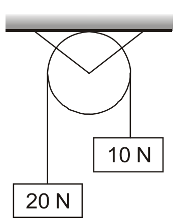
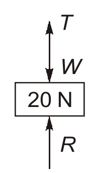
$$\Sigma M = 0 \qquad \{\text{Summation of moment due to all forces must be zero for rotational equilibrium}\}$$

1.14 Free Body Diagram

For analysis of equilibrium condition, the diagram of the body in which the body under consideration is freed from all contact surfaces and is shown with all the forces on it (including self weight, reactions from other contact surfaces) is called the Free Body Diagram (FBD).

Examples:

Table 1.2

Reacting Bodies	FBD required for	FBD
	Ball	
	Ladder	
	Ball	
	Block weighing 20 N	

1.15 Lami's Theorem

If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two.

Mathematically, $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where, P, Q and R are three forces and α, β and γ are the angles as shown in figure 1.12.

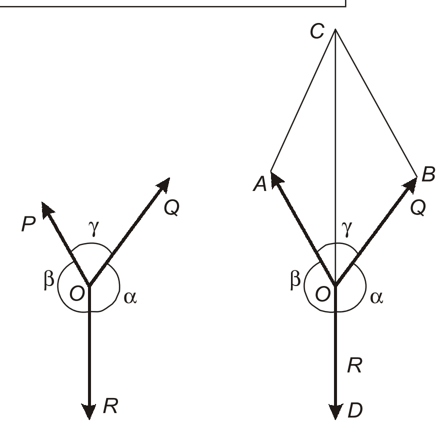


Fig. 1.12

1.15.1 Proof of Lami's Theorem

Consider three coplanar forces P, Q and R acting at a point O as shown in figure 1.12. Now complete the parallelogram $OACB$ with OA and OB as adjacent sides as shown in the figure 1.12. The resultant of two forces P and Q is diagonal OC both in magnitude and direction of the parallelogram $OACB$.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure,

$$BC = P \text{ and } AC = Q$$

$$\angle AOC = (180° - \beta)$$

and $$\angle ACO = \angle BOC = (180° - \alpha)$$

$$\angle CAO = 180° - (\angle AOC + \angle ACO) = 180° - [(180° - \beta) + (180° - \alpha)]$$

$$= 180° - 180° + \beta - 180° + \alpha$$

$$\angle CAO = \alpha + \beta - 180° \quad \text{... (i)}$$

But $$\alpha + \beta + \gamma = 360°$$

or $$\alpha + \beta + \gamma - 180° = 360° - 180° = 180°$$

$$(\alpha + \beta - 180°) + \gamma = 180° \quad \text{... (ii)}$$

From equation (i) and (ii), we get,

$$\angle CAO = 180° - \gamma$$

We know that in triangle *AOC*

$$\frac{OA}{\sin\angle ACO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{\sin\angle CAO}$$

$$\frac{OA}{\sin(180° - \alpha)} = \frac{AC}{\sin(180° - \beta)} = \frac{OC}{\sin(180° - \gamma)}$$

or $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$ Hence Proved

1.16 Sine Law and Cosine Law

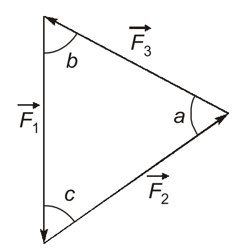


Fig. 1.13

(i) Sine Law: If $\vec{F_1}, \vec{F_2}$ and $\vec{F_3}$ are forces acting on body and body is in equilibrium. i.e, $\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$ then

$$\frac{F_1}{\sin a} = \frac{F_2}{\sin b} = \frac{F_3}{\sin c}$$

(ii) Cosine law

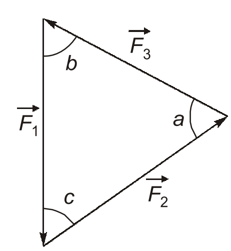


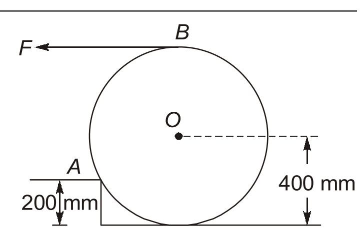
Fig. 1.14

$$\cos a = \frac{F_2^2 + F_3^2 - F_1^2}{2F_2F_3}$$

$$\cos b = \frac{F_1^2 + F_3^2 - F_2^2}{2F_1F_3}$$

$$\cos c = \frac{F_1^2 + F_2^2 - F_3^2}{2F_1F_2}$$

EXAMPLE : 1.5



A roller of radius $r = 400$ mm and weighing 3000 N is to be pulled over a curb of height 200 mm as shown in figure by applying a horizontal force F applied to the end of a string wound around the circumference of roller. Find

(i) the magnitude of force F required to start the roller move over the curb.

(ii) Least pull F through centre of the wheel to just turn the roller over the curb.