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CHAPTER

Water Demand, Water Conveyance and Distribution System

1.1 INTRODUCTION

Environmental engineering is the branch of engineering that deals with the applications of scientific and engineering principles to the improvement and maintenance of the environment with the goals of protecting human health, preserving the valuable ecosystem found in nature and enhancing the quality of human life. In this course of environmental engineering, we will mainly focuses on the water demand, its quality parameter, treatment process and in the similar way for waste water we will discuss about its quality, treatment and disposal.

Whenever an engineer is given the design the duty to design a water supply scheme for a particular section of the community, it becomes imperative upon him, to first of all, evaluate the amount of water available and the amount of water demanded by the public. In this chapter, we are going to study about later part of his duty i.e. to evaluate water demand. We will learn about various types of demands that needs to be fulfilled. We will also study about variations, in demands. In the later part of chapter, we will learn some of the methods that are used for purpose of population forecasting.

1.1.1 Water Demand

Water demand refers to the quantity of water that is required by individuals, communities, industries or any other entities for various purposes. It represnts the amount of water needed to meet the specific needs of a particular area of or population over a given period of time.

Estimtion of demand for water is the key parameter in planning a water supply scheme. In fact, the first study is to consider the demand and then the second requirement is to find sources to fulfil that demand.

To design a water supply, we must first estimate the population for which the scheme should be designed. The scheme once installed must cater for the demand of the projected population upto some predetermined future date.

1.2 VARIOUS TYPES OF WATER DEMAND

The prediction of precise quantity of water demanded by the public is very difficult, because there are so many variable factors affecting water consumption. There are some certain thumb rules and empirical formulae, which are used to assess this quantity, which may give fairly accurate results.

There are different types of water demands:

1.2.1 Domestic Water Demand

Domestic water demand includes the water required in private building for drinking, cooking, bathing, gardening purposes etc. which may vary according to the living conditions of the consumers. The total domestic water consumption is near about 50 to 60% of the total water consumptions. The IS code caps a limit on domestic water consumption between 135 to 225 litre per capita per day (*l*pcd). As per IS code, the minimum domestic water demand under ordinary conditions for a town with full flushing system should be taken as 200 *l*pcd although it can be minimized upto 135 *l*pcd for economically weaker section and LIG colonies (low income group) depending upon prevailing conditions.

NOTE

In developed and an efficient country like USA, this figure usually goes as high as 340 *l*pcd. This is because more water is consumed in rich living, in air-conditioning, air-cooling, bathing in bath tube, dish washing of utensils, car washing, home laundries, garbage grinders, etc.

1.2.2 Industrial Water Demand

The industrial water demand expresses the water required for industries which are either existing or likely to be started in future, in the city for which water supply is being planned. This water requirement will thus vary with the types and number of industries present in the city. In industrial cities, the per capita requirement may finally be computed to be as high as 450 *l*pcd as compared to the normal industrial requirement of 50 *l*pcd.

Table: Water Required by Certain Important Industries

Name of Industry	Unit of Production	Approximate Quantity of Water required per unit of production/raw material in kilo litres
1. Automobiles	Vehicle	40
2. Fertilizers	Tonne	80 - 200
3. Leather	Tonne (or 1000 kg)	40 (or 4)
4. Paper	Tonne	200 - 400
5. Petroleum Refinery	Tonne (Crude)	1 - 2
6. Sugar	Tonne (Crushed cane)	1 - 2
7. Textile	Tonne (goods)	80 - 140
8. Distillery (Alcohol)	kilo litre	122 - 170

1.2.3 Institutional and Commercial Water demand

On an average, a per capita demand of 20 *l*pcd is usually considered to be enough to meet of such commercial and institutional water requirements although of course, this demand may be as high as 50 *l*/h/d for highly commercial cities.

The individual requirements would be as follows:

1. Schools/Colleges : 45 to 135 *l*pcd
2. Offices : 45 *l*pcd
3. Restaurants : 70 *l*pcd
4. Cinema and theatres : 15 *l*pcd
5. Hotels : 180 *l*pcd
6. Hospitals : 340 *l*pcd (when beds is less than 100), and 450 *l*pcd (when beds exceed 100)
7. Airports : 70 *l*pcd
8. Railway : 70 *l*pcd (for junction with bathing facility)

Table: Water Requirements for Commercial Buildings (as per IS code)

S.No.	Type of building	Average consumption in (*l*pcd)
1.	Factories	
	(a) Where bathrooms are required to be provided	45
	(b) Where no bathrooms are required	30
2.	Hospitals (Including laundry, per bed)	
	(a) Number of beds less than 100	340
	(b) Number of beds exceeding 100	450
3.	Nurses homes and medical quarters	150
4.	Hostels	135
5.	Hotels (per bed)	180
6.	Restaurants (per seat)	70
7.	Offices	45
8.	Cinemas, auditoriums and theatres (per seat)	15
9.	Schools	
	(a) Day scholars	45
	(b) Residentials	135

1.2.4 Demand for Public Uses

This includes water requirement for parks, gardening, washing of roads etc. A nominal amount, not exceeding 5% of the total consumption may be provided to meet this demand.

A figure of 10 *l*pcd is usually added on this account while computing total water requirements.

1.2.5 Fire Demand

The quantity of water required for extinguishing fire is not very large. For a total amount of water consumption for a city of 50 lakhs population, it hardly amounts to 1 *l*pcd of fire demand, but this water should be easily available and kept always stored in storage reservoirs, as quantity of water required is in very less duration.

In thickly populated and industrial areas, fire generally breakout and may lead to serious damages, if not controlled effectively. Big cities, therefore, generally maintain full fire fighting squads. Fire fighting personnel require sufficient quantity of water, so as to throw it over the fire at high speed. A provision should, therefore be made in modern public water scheme for fighting fire breakouts.

Following requirements must be met for the fire demand:

- The minimum water pressure available at fire hydrants should be of the order of 100 to 150 kN/m^2 (10 to 15 m of water head) and should be maintained for 4 to 5 hours of constant use of fire hydrant.
- Three jet streams are simultaneously thrown from each hydrants; one on the burning property and one each on the adjacent property on either side of the burning property. The discharge of each stream should be about 1100 *l*/min.

Calculation of Fire Demand:

- For cities having population exceeding 50,000, the water required in kilo litre may be computed using the relation. Kilo litre of water required = $100\sqrt{P}$, where P = Population in thousand
- **Kuchling's Formula :** It states that

$$Q = 3182\sqrt{P}$$

Q = Amount of water required in litre/minute
P = Population in thousands.

- **Freeman's Formula :** It states that

$$Q = 1136\left[\frac{P}{5}+10\right]$$

- **National Board of Fire Underwriters Formula :**

(a) For Central congested high valued city

(i) When population is less than or equal to 2 Lakhs $Q = 4637\sqrt{P}\left[1-0.01\sqrt{P}\right]$ in l/min.

(ii) When population is more than 2 Lakhs, a provision for 54,000 litres/minute may be made with an extra additional provision of 9,100 to 36,400 litres/minute for a second fire.

(b) For a residential city

The required draft for fire-fighting may be as follows:

(i) Small or low building = 2200 litres/minute
(ii) Larger or Higher building = 4,500 litres/minute
(iii) High values residency, apartments, tenements = 7650 to 13500 litres/minute
(iv) Three storeyed buildings in densely built up section = upto 27000 litres/minute

- **Buston's Formula :** It states that, $Q = 5663\sqrt{P}$

NOTE

All the above formulas suffer from the drawback that they are not related to the type of district served and give equal results for industrial and non-industrial areas.

EXAMPLE : 1.1

Compute the 'fire demand' for a city of 2 lakh population by any two formulae (including that of the National Board of Fire Underwriters)

Solution:

(i) The rate of fire demand as per National Board of Fire Underwriters Formula for a central congested city whose population is less than or equal to 2 Lakh is given by

$$Q = 4637\sqrt{P}\left[1-0.01\sqrt{P}\right] = 4637\sqrt{200}\left[1-0.01\sqrt{200}\right]$$

$$= 56303.08\ l/\text{min} = \frac{56303.08\times 60\times 24}{10^6}\text{MLD} = \mathbf{81.08\ MLD}$$

(ii) Kuchling's formula, $Q = 3182\sqrt{P} = 3182\sqrt{200}\ l/\text{min}$; $R = 45000.27\ l/\text{m} = $ **64.8 MLD**

1.2.6 Water Required to Compensate Losses in Thefts and Wastes

This includes the water lost in leakage due to bad plumbing or damaged meters, stolen water due to unauthorised water connections, and other losses and wastes.

These losses should be taken into account while estimating the total requirements.

Even in the best managed water works, this amount may, be as high as 15% of the total consumption, which is nearly 55 $l$pcd.

1.3 THE PER CAPITA DEMAND (q)

It is the annual average amount of daily water required by one person and includes the domestic use, industrial use and commercial use, public use, waste thefts etc.

It may be, therefore expressed as Per Capita Demand (q) in litres per day per head

$$= \frac{\text{Total yearly water requirement of the city in litres (i.e V)}}{365 \times \text{Design Population}}$$

For an average Indian city, as per recommendation of I.S. code, the per capita demand (q) may be taken as (below table).

Table: Break up for per capita demand (q) for an average Indian City

Use	Demand in *l*pcd
(i) Domestic Use	200 (59.7% or 60%)
(ii) Industrial Use	50
(iii) Commercial Use	20
(iv) Civic or Public Use	10
(v) Waste and thefts, etc.	55
(vi) Fire demand	< 1
	Total = 335 = Per Capita Demand (q)

Total Maximum Water Demand:

It is the sum of above six demands and IS code permits for India, a total maximum demand of 335 *l*pcd.

1.3.1 Factors Affecting per Capita Demand

The annual average demand for water (i.e. per capita demand) considerably varies for different towns or cities. This figure generally ranges between 100 to 360 litres/capita/day for Indian conditions. The variations in total water consumption of different cities or towns depend upon various factors, which must be thoroughly studied and analysed before fixing the per capita demand for design purpose. These factors are discussed below:

(a) Size of the City

The total water demand depends on size of population and for the design of water supply scheme for a given population size following guidelines may be adopted (below table).

Table: Variation in Per Capita Demand (q) with population in India

S. No.	Population	Per Capita Demand in Liters/day/Person
1.	Less than 20000	110
2.	20000 - 50000	110 - 150
3.	50000 - 2 Lakhs	150 - 240
4.	2 Lakhs - 5 Lakhs	240 - 275
5.	5 Lakhs - 10 Lakhs	275 - 335
6.	Over 10 Lakhs	335 - 360

(b) Climatic Conditions

At hotter and dry places, the consumption of water is generally more, because more of bathing, clearing, air-coolers, air-conditioning etc. are involved. Similarly, in extremely cold countries, more water may be consumed, because the people may keep their taps open to avoid freezing of pipes and there may be more leakage from pipe joints since metals contract with cold.

(c) Types of Gentry and Habits of People

Rich and upper class communities generally consume more water due to their affluent living standards.

(d) Industrial and Commercial Activities

The pressure of industrial and commercial activities at a particular place increase the water consumption by large amount.

(e) Quality of Water Supplies

If the quality and taste of the supplied water is good, it will be consumed more, because in that case, people will not use other sources such as private wells, hand pumps, etc. Similarly, certain industries such as boiler feeds, etc., which require standard quality waters will not develop their own supplies and will use public supplies, provided the supplied water is upto their required standards.

(f) Pressure in the Distribution Systems

If the pressure in the distribution pipes is high and sufficient to make the water reach at 3^{rd} or even 4^{th} storage, water consumption shall be definitely more.

This water consumption increases because of two reasons:

(i) People living in upper storage will use water freely as compared to the case when water is available scarcely to them.

(ii) The losses and waste due to leakage are considerably increased if their pressure is high. For example, if the pressure increase from 20 m head of water (i.e. 200 kN/m^2) to 30 m head of water (i.e. 300 kN/m^2), the losses may go up by 20 to 30 percent.

(g) Development of Sewerage Facilities

The water consumption will be more, if the city is provided with 'flush system' and shall be less if the old 'conservation system' of latrines is adopted.

(h) System of Supply

Water may be supplied either continuously for all 24 hours of the day, or may be supplied only for peak period during morning and evening. The second system, i.e. intermittent supplies, may lead to some saving in water consumption due to losses occurring for lesser time and a more vigilant use of water by the consumers.

(i) Cost of Water

If the water rates are high, lesser quantity may be consumed by the people. This may not lead to large savings as the affluent and rich people are little affected by such policies.

(j) Policy of Metering and Method of Charging

When the supplies are metered, people use only that much of water as much is required by them. Although metered supplies are preferred because of lesser wastage, they generally lead to lesser water consumption by poor and low income group, leading to unhygienic conditions.

Factors Affecting Losses and Wastes: The various factors on which losses depend and the measure to control them are below:

(i) Water Tight Joints
(ii) Pressure in the Distribution system
(iii) System of supply
(iv) Metering
(v) Unauthorized connections

1.3.2 Variation in demand and effects on the design of various components of a water supply scheme

- The smaller the town, the more variable is the demand
- The shorter the period of draft, the greater is the departure from the mean

(i) Maximum daily Consumption: It is generally taken as 180 percent of the average

Therefore, Maximum daily demand = 1.8 (i.e. 180%) × Average daily demand = $1.8q$

(ii) Maximum hourly Consumption : It is generally taken as 150 percent of its average.

Therefore, Maximum hourly consumption of the maximum day i.e. peak demand

= 1.5 (i.e. 150%) × Average hourly consumption of maximum daily demand

$$= 1.5 \times \left(\frac{\text{Maximum daily demand}}{24}\right) = 1.5 \times \left(\frac{1.8 \times q}{24}\right) = 2.7\left(\frac{q}{24}\right)$$

= 2.7 (Annual average hourly demand)

(iii) Maximum Weekly Demand : Maximum weekly Consumption = 1.48 × Average weekly

(iv) Maximum Monthly Demand : Maximum monthly consumption = 1.28 × Average monthly

NOTE

Goodrich formula is used to compute maximum or peak demand.

$$\boxed{P = 1.8(t)^{-0.1}} \qquad P = \frac{\text{Maximum demand}}{\text{Average demand}}$$

where, t = time in days, t = 1 for maximum daily, $t = \frac{1}{24}$ for maximum hourly

P = Annual average draft for time in t day

The GOI manual on water supply has recommended the following values of the peak factor, depending upon the population.

Table: Peak Factor

S.No.	Population	Peak Factor
1.	Upto 50000 50001 - 200000 Above 2 Lakh	3.0 2.5 2
2.	For Rural water supply scheme, where supply is effected through stand post for only 6 hours	3

Evidently, the peak factor tends to reduce with increasing population.

1.3.3 Coincident draft

The probability of breaking out of fire on day of maximum hourly draft is very meagre. Hence, for general community purposes,

Total draft = Maximum of the two i.e.,

(i) Sum of maximum daily demand and fire demand.

or

(ii) maximum hourly demand

When the maximum daily demand is summed up with fire demand for calculating total draft, it is known as coincident draft.

REMEMBER As far as the design of distribution system is concerned, it is hourly variation in computation that matters.

EXAMPLE : 1.2

A water supply scheme has to be designed for a city having a population of 1,00,000. Estimate the important kinds of drafts which may be required to be recorded for an average water consumption of 250 *l*pcd. Also record the required capacities of the major components of the proposed water works system for the city using a river as the source of supply. Assume suitable data.

Solution:

(i) Average daily draft = (per capita average consumption in lpcd) × population

$= 250 \times 1{,}00{,}000 \text{ litres/day} = 250 \times 10^5 \text{ litres/day} = 25 \text{ MLD}$

(ii) Maximum daily draft may be assumed as 180% of annual average daily draft

∴ Maximum daily draft $= \frac{180}{100}[25 \text{ MLd}] = 45 \text{ MLD}$

(iii) Maximum hourly draft of the maximum day: It may be assumed as 270 percent of annual average hourly draft

∴ Maximum hourly draft of maximum day $= \frac{270}{100}[25 \text{ MLd}] = 67.5 \text{ MLD}$

(iv) Fire flow may be worked out from

$$Q = 4637\sqrt{P}\left[1-0.01\sqrt{P}\right] = 4637\sqrt{100}\left[1-0.01\sqrt{100}\right] = 41733 \; l/\text{min}$$

where P = in thousand population

$$= \frac{41733 \times 60 \times 24}{10^6} \text{ million litres/day} = 61 \text{ MLD}$$

Coincident draft = maximum daily draft + fire draft

= 45 + 61 = **106 MLD**

which is greater than the maximum hourly draft of 67.5 MLD

1.4 DESIGN PERIOD OF WATER SUPPLY UNIT

A water supply scheme includes huge and costly structures (such as dams, reservoirs, treatment works, penstock pipes, etc.) which can not be replaced or increased in their capacities, easily and conveniently. For example, the water mains including the distributing pipes are laid underground, and cannot be replaced or added easily, without digging the roads or disrupting the traffic. In order to avoid these future complications of expansion, the various components of a water supply scheme are purposely made larger, so as to satisfy the community needs for a reasonable number of years to come. This future period or the number of years for which a provision is made in designing the capacities of the various components of the water supply scheme is known as **design period**. The design period should neither be too long nor should it be too short. The design period cannot exceed the useful life of the component structure, and is guided by the following considerations.

1.4.1 Factors Governing the Design Period

- Useful life of component structures and the chances of their becoming old and obsolete. Design period should not exceed there respective values.
- Ease and difficulty that is likely to be faced in expansions, if undertaken at future dates.
- Amount and availability of additional investment likely to be incurred for additional provision.
- The rate of interest on the borrowings and the additional money invested.
- Anticipated rate of population growth, including possible shifts in communities, industries and commercial establishment.

1.4.2 Design Period Values

Water supply projects, under normal circumstances, may be designed for a design period of 30 years excluding completion time of 2 years. The design period recommended by the GOI manual on water supply for designing the various components of a water supply projects are given below table:

Table: Design Period of Various Components of Water Supply Project

S.No.	Units	Design (Parameters) Discharge	Design Period
1.	Water Treatment Unit	Maximum daily demand	15 Years
2.	Main supply pipes (Water mains)	Maximum daily demand	30 Years
3.	Wells and Tube wells	Maximum daily demand	30-50 Years
4.	Demand Reservoir (Overhead or ground level)	Average annual demand	50 Years
5.	Distribution system	Maximum hourly demand/ Coincident draft	30 Years

1.5 POPULATION FORECASTING

Population census enumerations and growth in population etc. are not only used in demographic sphere but also by Engineer and people concerned with economic growth, national planning and policy decision making in the sector of agriculture, growth in industries and infrastructure, drinking water supply schemes and other social welfare activities etc.

1.5.1 Population Growth

Growth of population is of great concern to people engaged in policy planning and decision making at the national level. Population growth means the change (increase) of population size between two dates.

However, a population increasing in size is said to have a positive growth rate and the one declining is to have a negative growth rate.

The number of inhabitants of a country depends on (i) The rate of growth in population and (ii) Migrations

The second factor is of importance only in new countries and the old countries are the sources of migrants.

In order to predict the future population, as correctly as possible, it is necessary to know the factors affecting population growth. These are three main factors responsible for changes in population.

They are: (i) Births (ii) Deaths (iii) Migrations

All these factors are influenced by social and economic factors and conditions prevailing communities.

- The Birth rates may decrease due to excessive family planning practices and legalized abortions. Spread of education and development of extra recreational facilities for the people, also tend to reduce the birth rates.

- The death rates may decrease with the development and advancement of medical facilities, thereby controlling infant mortality rates and adult death rates due to control of infections and other diseases.
- The migrations are dependent upon the industrialization and commercialization of the particular cities or towns. People generally migrate from villages to cities where livelihood are available.
- Besides these three main factors, some other factors like war, natural havocs and disasters may also bring about sharp reduction in the population.
- Considering all these factors, arithmetic balancing is done to arrive at the future population. It can be expressed as $P_t = P_0 + (B - D) - (I - E)$

where, P_0 and P_t refer to the size of population at the beginning and end of a time period, and B, D, I and E refers to the number of births, deaths, immigration and emigration respectively during period under consideration.

1.5.2 Growth Rate Curve

When all the unpredictable factors such as war, or natural disasters do not produce sudden changes, the population would probably follow the growth curve as discussed in the theory of demographic transition. This curve is S-shaped as shown in figure and is known as "the logistic curve". According to this curve, rate of growth of population varies from time to time.

The curve represents early growth AB at an increasing rate

$\left(\text{i.e. geometric or log growth, } \frac{dP}{dt} \propto P\right)$

and late growth DE at a decreasing rate

$\left[\text{i.e. first order } \frac{dP}{dt} \propto (P_S - P)\right]$ as the saturation value (P_S) is approached. The transitional $\left[\text{i.e. } \frac{dP}{dt} = \text{constant}\right]$. What the future holds for a given population, depends upon, as to where the point lie on the growth curve at a given time.

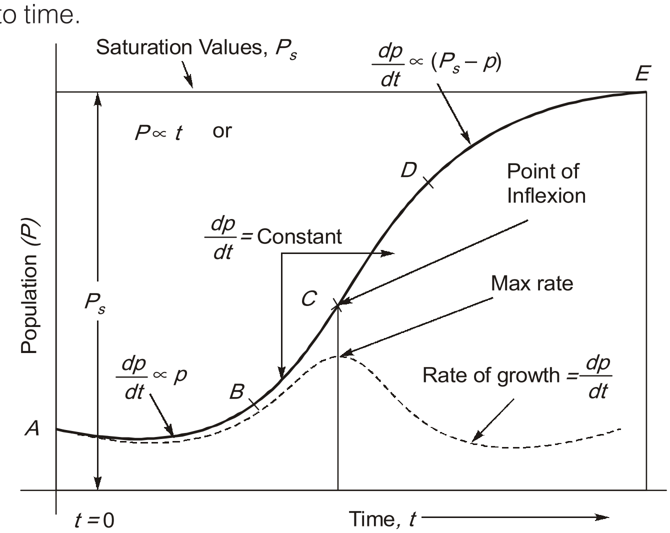

Fig. *Growth Rate Curve*

i.e.

$$\text{in } AB \rightarrow \frac{dP}{dt} \propto P \rightarrow \text{increasing growth rate}$$

$$\text{in } BCD \rightarrow \frac{dP}{dt} = \text{Constant} \rightarrow \text{High growth rate}$$

$$\text{in } DE \rightarrow \frac{dP}{dt} \propto (P_s - P) \rightarrow \text{Decreasing growth rate}, \; P_s = \text{Saturation value}$$

1.6 POPULATION FORECASTING METHODS

The various methods which are generally adopted for estimating future populations by engineers are described below. Some of these methods are used when design period is small, and some are used when the design period is large. The particular method to be adopted for a particular case or for a particular city depends largely upon the factor discussed in these methods and the selection is left to the discretion and intelligence of the designer.

1.6.1 Arithmetic Increase Method

In this method, a constant increment of growth in population is observed periodically. This method is of limited application, mostly used in large and established towns where future growth has been controlled.

This method is based upon the assumption that the population increase at a constant rate, i.e. the rate of change of population with time $\left(\text{i.e. } \frac{dP}{dt}\right)$ is constant.

Thus, $\frac{dP}{dt} = \text{Constant} = k$ or, $dP = k \cdot dt$

or, $\int_{P_{t_1}}^{P_{t_2}} dP = k\int_{t_1}^{t_2} dt$ $P_{t_2} = P_{t_1} + k(t_2 - k_1)$

Here P_{t_2} and P_{t_1} represent the population at time t_2 and t_1 respectively. This time period is usually reckoned in decades. k is the rate of increase of population per unit time (decade), thus $(t_2 - t_1)$ = Number of decades.

The equation can be rewritten as, $P_n = P_0 + n \cdot \bar{x}$

where P_n = perspective or forecasted population after n decades from the present (i.e. last known census)

P_0 = Population at present (i.e. last known census)

n = Number of decades between now and future

$\bar{x}$ = Average (arithmetic mean) of population increase in the known decades.

EXAMPLE : 1.3

The population of 5 decades from 1930 to 1970 are given below in table. Find out the population after one, two and three decades beyond the last known decade, by using arithmetic increase method.

Year	1930	1940	1950	1960	1970
Population	25,000	28,000	34,000	42,000	47,000

Solution: The given data in above table is extended in table below, so as to compute the increase in population (x) for each decade (col. 3), the total increase, and average increase per decade ($\bar{x}$), as shown.

Year (1)	Population (2)	Increase in population (x) (3)
1930	25,000	
1940	28,000	3000
1950	34,000	6000
1960	42,000	8000
1970	47,000	5000
Total		22,000
Average increase per decade ($\bar{x}$)		$\bar{x} = \frac{22000}{4} = 5,500$

∴ (a) Population after 1 decade beyond 1970

$= P_{1980} = P_1 = P_{1970} + 1 \cdot \bar{x} = 47{,}000 + 1 \times 5500 = 52{,}500$

(b) Population after 2 decades beyond 1970

$= P_{1990} = P_2 = P_{1970} + 2 \cdot \bar{x} = 47{,}000 + 2 \times 5500 = 58{,}000$

(c) Population after 3 decades beyond 1970

$= P_{2000} = P_3 = P_{1970} + 3 \cdot \bar{x} = 47{,}000 + 3 \times 5500 =$ **63,500**

1.6.2 Geometric Increase Method

The method of Geometric progression is applicable to the cities with unlimited scope for future expansion and where a constant rate of growth is anticipated.

The basic difference between arithmetic and geometric progression or increase method of population forecasting is that, in Arithmetic method no compounding is done whereas, in Geometric method compounding is done every decade. This method is, therefore, also known as uniform increase method.

In Geometric increase method, a constant value of percentage growth rate per decade (k) analogues to the rate of compounding interest per annual.

Thus, population after one decade can be given by, $P_1 = P_0 + kP_0 = P_0(1 + k)$

Similarly, population after n decades $P_n = P_0(1+k)^n = P_0\left(1+\frac{k}{100}\right)^n$

Where, P_0 refers to initial population i.e. at the end of last known census.

Average percentage growth rate per decade k to be used in the above equation is computed from the percentage growth rate of each decade. The value of k can be calculated as $k = \sqrt[m]{k_1 \cdot k_2 \cdot k_3 \cdots k_m}$

In geometric increase method the growth rate per decade, $k = \sqrt[t]{\frac{P_2}{P_1}} - 1$

EXAMPLE : 1.4

Determine the future population of a satellite town by the Geometric increase method for the year 2011, given the following data.

Year	1951	1961	1971	1981	......	2011
Population in Thousand	93	111	132	161	......	?

Solution: The given data is analysed in table to determine growth rates for each decade.

Year	Population in thousand	Increase in Population in thousand	%age increase in population = growth rate $= \frac{\text{col(3)}}{\text{col(2)}} \times 100$
1951	93		
		18	19.35
1961	111		
		21	18.92
1971	132		
		29	21.97
1981	161		

Constant growth rate, assumed for future

r = geometric mean of past growth rates = $\sqrt[3]{19.35 \times 18.92 \times 21.97}$ = 20.03 % per decade

The population after n decades in now given by equation

$$P_n = P_0\left(1+\frac{r}{100}\right)^n$$

P_{2011} = Population after 3 decades from 1981

$$= P_{1981}\left(1+\frac{20.03}{100}\right)^3 = 1{,}61{,}000(1.2003)^3 = \mathbf{2{,}78{,}417}$$

1.6.3 Incremental Increase Method

This method is another case of arithmetic increase with some modifications. Incremental increase method is adopted for cities which are likely to grow progressively of increasing or decreasing rate rather than a constant rate.

According to this method, population after n decades can be given by

$$P_n = P_0 + n\bar{x} + \frac{n(n+1)}{2}\cdot\bar{y}$$

where, $\bar{x}$ and $\bar{y}$ are the average increase of population per decade and average incremental increase respectively. The other notations carry their usual meaning and $\bar{x}$ and $\bar{y}$ are given by

$$\bar{x} = \text{Average increase of population per decade} = \frac{x_1 + x_2 + \ldots\ldots + x_p}{p} \text{ and}$$

$$\bar{y} = \text{Average of incremental increase} = \frac{y_1 + y_2 + \ldots\ldots + y_p}{p}$$

where, $x_1, x_2, x_3 \ldots x_m$ are increase in each decade, $y_1, y_2, y_3 \ldots y_p$ are incremental increase in each decade.

NOTE

- The "GOI manual on water and water treatment" recommends the use of geometric mean here; and hence, we can safely use that value.
- Geometric increase method gives high results which is suitable for cities growing with fast rate such as new cities whereas arithmetic increase method gives low results which is suitable for cities growing with slow rate such as old cities.

EXAMPLE : 1.5

As per the census records for the years 1911 to 1971, the population of a town is given below in the table. Assuming that the scheme of water supply was to commence in 1996, it is required to estimate the population of 10 years hence i.e. in 2006 and also the intermediate population after 15 year since commencement.

Year	1911	1921	1931	1941	1951	1961	1971
Population	40185	44522	60395	75614	98886	124230	158800

Solution: Let us try to get the solutions using all the three methods to which you have been introduced by now. The incremental population and increase in incremental population are summed up in table below:

Year	Population	Increment	Incremental increase(y)
1911	40185	–	–
1921	44522	4337	–
1931	60395	15873	+ 11536
1941	75614	15219	– 654
1951	98886	23272	+ 8053
1961	124230	25344	+ 2072
1971	158800	37570	+ 9226

From the above table, the following parameters can be worked out as

Total increase in population = 118,615

Total of incremental/decrease = 30,233

Average incremental value decade $(\bar{x}) = \frac{1}{6} \times 118615 = 19769$

Average incremental increase per decade $(\bar{y}) = \frac{1}{5} \times 30233 = 6047$

1. **By Arithmetic Progression Method**

Increase in population from 1911 to 1971, i.e. in 6 decades = 158,800 – 40,185 = 118,615

$$K = \frac{1}{6} \times 118,615 = 19,769$$

Now, using equation $P_n = P_0 + k.n$

∴ Population in 2006 = 158,800 + 19,769 × 3.5 = **227,992**

and population in 2011 = 158,800 + 19,769 × 4 = **237,876**

2. **By Geometrical progression method**

Rate of growth per decade

between 1911 and 1921, $k_1 = \frac{4,337}{40,187} = 0.108$

between 1921 and 1931, $k_2 = \frac{15,873}{44,522} = 0.356$

between 1931 and 1941, $k_3 = \frac{15,219}{60,395} = 0.252$

between 1941 to 1951, $k_4 = \frac{23,272}{75,614} = 0.308$

between 1951 to 1961, $k_5 = \frac{25,344}{98,886} = 0.256$

between 1961 to 1971, $k_6 = \frac{34,570}{124,230} = 0.278$

Geometric mean $= \sqrt[6]{0.108 \times 0.356 \times 0.252 \times 0.308 \times 0.256 \times 0.278}$

or, $k = 0.24426$

Assuming that the future population will grow in geometric progression as in the post during 1911 to 1971.

PRACTICE QUESTIONS

Question : 1

(i) The population of 5 decades from 1930 to 1970 are given in table below. Find out the population after one, two and three decades beyond the last known decade, by using arithmetic increase method.

Year	1930	1940	1950	1960	1970
Population	25000	28000	34000	42000	47000

(ii) Compute the population of the year 2000 and 2006 for a city whose population in the year 1930 was 25000, and in the year 1970 was 47000. Make use of geometric increase method.

[10 Marks]

Solution:

(i) The given data in question table is extended in table below, so as to compute the increase in population (x) for each decade (col. 3), the total increase, and average increase per decade $(\bar{x})$, as shown

Year	Population	Increase in population (x)
(1)	(2)	(3)
1930	25000	
1940	28000	3000
1950	34000	6000
1960	42000	8000
1970	47000	5000
Total		22000
Average increase per decade $(\bar{x})$		$\bar{x} = \frac{22000}{4} = 5500$

The future populations are now computed by using equation as,

$$P_n = P_0 + n \cdot \bar{x}$$

∴ (a) Population after 1 decade beyond 1970

$$P_{1980} = P_1 = P_{1970} + 1 \cdot \bar{x}$$
$$= 47000 + 1 \times 5500$$
$$= 52500$$

(b) Population after 2 decades beyond 1970

$$P_{1990} = P_2 = P_{1970} + 2 \cdot \bar{x}$$
$$= 47000 + 2 \times 5500$$
$$= 58000$$

(c) Population after 3 decades beyond 1970

$$P_{2000} = P_3 = P_{1970} + 3 \cdot \bar{x}$$
$$= 47000 + 3 \times 5500$$
$$= 63500$$

(ii)

In this question, the intermediate census data between 1930 to 1970 is not given, and hence geometric mean method of all known decades is not possible. The growth rate per decade (r) can, however, be computed by using as

$$r = \sqrt[t]{\frac{P_2}{P_1}} - 1 = \sqrt[4]{\frac{47000}{25000}} - 1$$

$$= 0.17095$$

$$= 17.095\% \text{ per decade}$$

Now, $P_n = P_0\left(1+\frac{r}{100}\right)^n$, we have

Hence, $P_{2000} = P_3$ (after 3 decades from 1970 onward)

$$= P_{1970}\left(1+\frac{r}{100}\right)^3$$

$$= 47000\,(1 + 0.17095)^3$$

$$= 47000(1.17095)^3$$

$$= 75459.9 \simeq 75460$$

Population for the year 2006, means that it is after 36 years (3.6 decades) from 1970 onward.

$$P_{2006} = P_{3.6} = P_{1970}\,(1 + 0.17095)^{3.6}$$

$$= 47000\,(1.17095)^{3.6} = 82954$$

■■■■