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Fluid Mechanics

Mechanical Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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Fluid Mechanics

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CHAPTER 01

Fluid Properties

1.1 Introduction

- A fluid is a substance which deforms continuously under the influence of shearing forces no matter how small the forces may be.
- Fluids are substance capable of flowing and they conforms to the shape of the containing vessel.
- This property of continuous deformation in technical terms is known as 'flow property', whereas this property is absent in solids.
- If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.
- Fluids are classified as ideal fluids and practical or real fluids.
- Ideal fluids are those fluids which have neither viscosity nor surface tension and they are incompressible. In nature, the ideal fluids do not exist and therefore, they are only imaginary fluids.
- Practical or real fluids are those fluids which possess viscosity, surface tension and compressibility.
- Fluids are considered to be continuum i.e., a continuous distribution of matter with no voids or empty spaces.
- Difference between fluid and solid is that solid can resist a shear stress by static deflection but fluid cannot resist it.

1.2 Fluid Mechanics

- Fluid mechanics is study of fluids either at rest or in motion.
- Total fluid mechanics can be dealt with two different approaches, empirical hydraulics and classical hydrodynamics.
- Hydraulics is mainly concerned with motion of water. It is based on the physical principles and has close correlation with experimental studies which both complement and substantiate the fundamental analysis.
- Hydrodynamics is essentially mathematical science dealing with flow analysis based on concept of an ideal fluid, a fictitious fluid in which both fluid viscosity and fluid compressibility are assumed absent.

1.3 Fluid as Continuum

- Since fluids are aggregations of molecules widely spread for gas and closely spaced for a liquid. The distance between molecules is very large compared to molecular diameter.
- The molecules are not fixed in lattice but move about freely. Thus fluid density or mass per unit volume has no practical meaning because the numbers of molecule occupying a given volume continuously changes.
- But if chosen unit volume is too large there could be noticeable variation in the bulk aggregation of particle. So density can be written as

$$\rho = \lim_{\delta v \to \delta v'} \frac{\delta m}{\delta v}$$

- Since most engineering problems are connected with larger sample volume, so density being a point function and other fluid properties can be thought of as varying continually in space. Such a fluid is called a continuum, which simply means that its variation in properties is so smooth that differential calculus can be used to analyse the substance.

1.4 Fluid Properties

- Any characteristic of a fluid system is called a fluid property.
- Fluid properties are of two types:
  - (i) **Intensive Properties:** Intensive properties are those that are independent of the mass of the fluid system.
    Example: Temperature, pressure, density etc.
  - (ii) **Extensive Properties:** Extensive properties are those whose values depend on the size or extent of the system.
    Example: Total mass, total volume, total momentum etc.
- Following are some of the intensive and extensive properties of a fluid system.
  (i) Viscosity (ii) Surface tension (iii) Vapour pressure (iv) Compressibility and elasticity

1.4.1 Some other Important Properties

1. **Mass Density :** Mass density (or specific mass) of a fluid is the mass which it possesses per unit volume. It is denoted by the Greek symbol ρ. In SI system, the unit of ρ is kg/m^3.
2. **Specific Gravity :** Specific gravity (S) is the ratio of specific weight (or mass density) of a fluid to the specific weight (or mass density) of a standard fluid. The standard fluid chosen for comparison is pure water at 4°C.

$$\text{Specific gravity of liquid} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of water}} = \frac{\text{Specific weight of liquid}}{9810\ \text{N/m}^3}.$$

3. **Relative Density (R.D.) :** It is defined as ratio of density of one substance with respect to other substance.

$$\rho_{1/2} = \frac{\rho_1}{\rho_2}$$

where, $\rho_{1/2}$ = Relative density of substance '1' with respect to substance '2'.

4. **Specific Weight :** Specific weight (also called weight density) of a fluid is the weight it possesses per unit volume. It is denoted by the Greek symbol γ. For water, it is denoted by γ_w. In SI system, the unit of specific weight is N/m^3. The mass density and specific weight γ has following relationship $\gamma = \rho g$; $\rho = \gamma / g$. Both mass density and specific weight depend upon temperature and pressure.
5. **Specific Volume :** Specific volume of a fluid is the volume of the fluid per unit mass. Thus it is the reciprocal of density. It is generally denoted by v. In SI unit specific volume is expressed in cubic meter per kilogram, i.e., m^3/kg.

Example-1.1 Three litres of petrol weigh 23.7 N. Calculate the mass density, specific weight, specific volume and specific gravity of petrol.

Solution:

Mass density of petrol, $\rho_p = \dfrac{M}{V} = \dfrac{W/g}{V} = \dfrac{W}{gV} = \dfrac{23.7}{9.8 \times 3}$ = 0.805 kg/litre = 805 kg/m^3

Mass density of water, ρ_w = 1000 kg/m^3

Specific gravity of petrol = $\dfrac{805}{1000}$ = 0.805

Specific weight of petrol = weight per unit volume

$= \dfrac{23.7}{3.0}$ = 7.9 N/litre = 7.9 kN/m^3

Specific volume = volume per unit mass

$= \dfrac{1}{\rho_p} = \dfrac{1}{805} = 1.242 \times 10^{-3}$ m^3/kg

1.4.2 Viscosity

- Viscosity is a property of the fluids by virtue of which they offer resistance to shear or angular deformation.
- It is primarily due to cohesion and molecular momentum exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers.

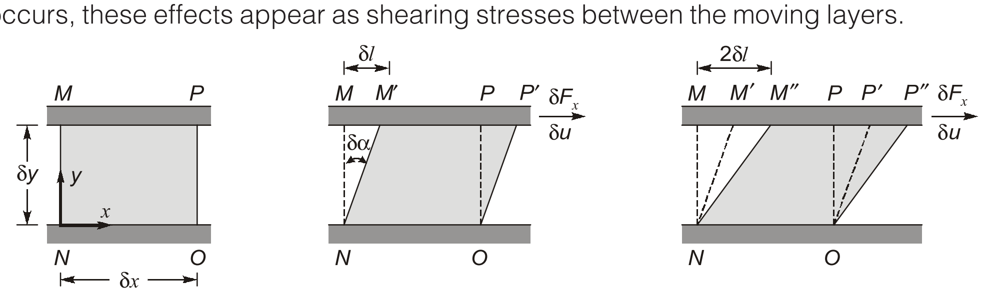


Fig. *(a) Fluid element at time t, (b) deformation of fluid element at time t + δt, and (c) deformation of fluid element at time t + 2δt.*

- Consider the behavior of a fluid element between the two infinite plates as shown in Fig. (a). The rectangular fluid element is initially at rest at time t. Let us now suppose a constant rightward force δF_x is applied to the upper plate so that it is dragged across the fluid at constant velocity δu. The relative shearing action of the plates produces a shear stress, τ_{yx}, which acts on the fluid element and is given by $\tau_{yx} = \lim_{\delta A_y \to 0} \dfrac{\delta F_x}{\delta A_y} = \dfrac{dF_x}{dA_y}$, where δA_y is the area of contact of the fluid element with the plate and δF_x is the force exerted by the plate on that element.

Various positions of the fluid element, shown in Fig. illustrate the deformation of the fluid element from position *MNOP* at time *t*, to *M′NOP′* at time $t + \delta t$, to *M″NOP″* at time $t + 2\delta t$, due to the imposed shear stress. The deformation of the fluid is given by

$$\text{Deformation rate} = \lim_{\delta t \to 0} \frac{\delta\alpha}{\delta t} = \frac{d\alpha}{dt}$$

Distance between the points *M* and *M′* is given by,

$$\delta l = \delta u \, \delta t \qquad \text{...(i)}$$

Alternatively, for small angles, $\delta l = \delta y \delta\alpha$ ...(ii)

Equating Eq. (i) and (ii), we get, $\delta u \delta t = \delta y \delta\alpha$

or

$$\frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y}$$

Taking the limits of both sides

$$\lim_{\delta t \to 0} \frac{\delta\alpha}{\delta t} = \lim_{\delta t \to 0} \frac{\delta u}{\delta y}$$

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$

Thus, the rate of angular deformation is equal to velocity gradient across the flow.

- On the basis of relation between the applied shear stresses and the flow or rate of deformation, fluids can be categorized as Newtonian and non-Newtonian fluids.

1.4.2.1 Newtonian Fluids

- Fluids which obey Newton's law of viscosity are known as Newtonian fluids.
- According to Newton's law of viscosity, shear stress is directly proportional to the rate of deformation or velocity gradient across the flow.

Thus, $\tau \propto \dfrac{du}{dy}$ or $\tau = \mu \dfrac{du}{dy}$

where, μ = absolute or dynamic viscosity

- Water, air, and gasoline are Newtonian fluids under normal conditions.

Dynamic Viscosity (μ)

- Dimension of $\mu = [M\,L^{-1}\,T^{-1}]$
- Unit of μ = N-s/m^2 or Pa.s
- In c. g. s. units, μ is expressed as 'poise', 1 poise = 0.1 N-s/m^2
- (μ) water $\approx 10^{-3}$ N-s/m^2;
  (μ) air $\approx 1.81 \times 10^{-5}$ N-s/m^2 (Both at 20°C and at standard atmospheric pressure)

NOTE: Water is nearly 55 times viscous than air.

Kinematic Viscosity (ν)

- The kinematic viscosity (ν) is defined as the ratio of dynamic viscosity to mass density of the fluid therefore, $\nu = \mu/\rho$
- It is called kinematic because the mass unit cancel, leaving only the dimensions.
- Dimension of $\nu = [L^2\,T^{-1}]$
- Unit of ν = m^2/s or cm^2/s (stoke)
- 1 stoke = 10^{-4} m^2/s
- At 20°C and atmospheric pressure $\nu_{water} = 1.0 \times 10^{-6}$ m^2/s, $\nu_{air} = 15 \times 10^{-6}$ m^2/s

NOTE: Kinematic viscosity of air is about 15 times greater than the corresponding value of water.

Effect of Temperature on Viscosity

- It is necessary to understand the factors contributing to viscosity to analyse temperature effect.
- In liquid, viscosity is caused by intermolecular attraction force which weaken as temperature rises so viscosity decreases.
- In gases, viscosity is caused by the random motion of particle/ molecules. Due to increase in temperature, randomness increases causing increase in viscosity.
- For liquids viscosity decreases with temperature and it is roughly exponential as

$$\mu = ae^{-bT}$$

where a and b are constant for a particular liquid.

For water $a = -1.94, \quad b = -4.80$

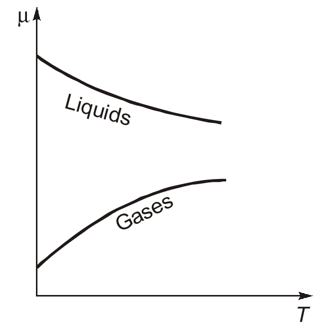


Fig. *Variation of Viscosity with Temperature*

1.4.2.2 Non-Newtonian Fluids

- Fluids for which shear stress is not directly proportional to deformation rate are non-Newtonian. Toothpaste and paint are the examples of non-Newtonian fluid.
- Non-Newtonian fluids commonly are classified as having time-independent or time-dependent behavior.

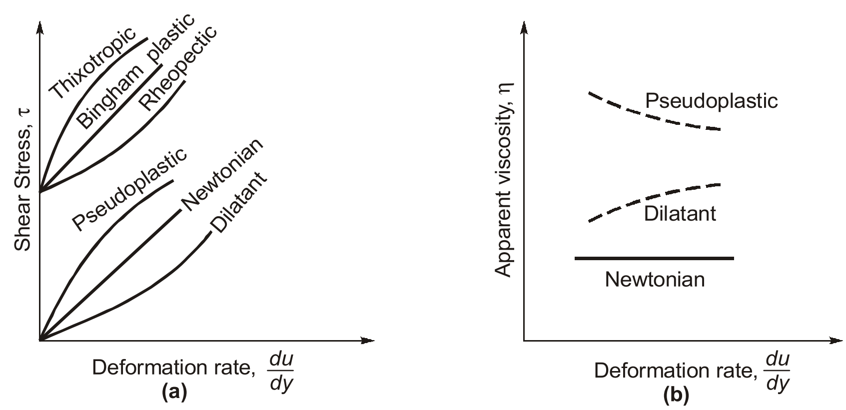


Fig. *(a) Shear stress, τ and (b) Apparent viscosity, η*

- Relation between shear stress and rate of deformation for non-Newtonian fluid can be represented as

$$\tau = k\left(\frac{du}{dy}\right)^n$$

where, n = flow behavior index; k = consistency index

For Newtonian fluid, $n = 1; \quad k = \mu$

above equation can also be represented as

$$\tau = k\left(\frac{du}{dy}\right)^{n-1}\left(\frac{du}{dy}\right) = \eta\frac{du}{dy}$$

where, $\eta = k\left(\frac{du}{dy}\right)^{n-1}$ is referred as the apparent viscosity

NOTE: Dynamic viscosity (μ) is constant (except for temperature effects) while apparent viscosity (η) depends on the shear rate.

- Various types of non-Newtonian fluids are :
  1. **Pseudoplastic** : Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$) are called pseudoplastic fluids (or shear thinning). Most non-Newtonian fluids fall into this group.
     Example: Polymer solutions, colloidal suspensions, milk, blood and paper pulp in water.
  2. **Dilatant :** If the apparent viscosity increases with increasing deformation rate ($n > 1$), the fluid is termed as dilatant (or shear thickening).
     Example: Suspensions of starch, saturated sugar solution.
  3. **Bingham Plastic :** Fluids that behave as a solid until a minimum yield stress, τ_y, and flow after crossing this limit are known as ideal plastic or Bingham plastic. The corresponding shear stress model is $\tau = \tau_y + \mu\dfrac{du}{dy}$.
     Example: Clay suspensions, drilling muds, creams and toothpaste.
  4. **Thixotropic :** Apparent viscosity (η) for thixotropic fluids decreases with time under a constant applied shear stress.
     Example: Paints, printer inks
  5. **Rheopectic :** Apparent viscosity (η) for rheopectic fluids increases with time under constant shear stress.
     Example: Gypsum pastes.

NOTE

(i) There is no relative movement between fluid attached to the solid boundary and solid boundary i.e. the fluid layer just adjacent to the solid surface will have same velocity as of the solid surface.

(ii) Viscoelastic : Fluids which after some deformation partially return to their original shape when the applied stress is released such fluids are called viscoelastic.

(iii) Rheology : Branch of science which deals with the studies of different types of fluid behaviours.

Example-1.2 **Calculate the velocity gradient at distance 0, 100, 150 mm from the boundary if the velocity is a parabola with vortex 150 mm from boundary, where velocity is 1 m/s. Also calculate the shear stress at these points if the fluids has a viscosity of 0.804 Ns/m^2.**

Solution:

Let the equation of velocity profile

$$u = Ay^2 + By + C$$

Now apply boundary condition

(i) $u = 0$ at $y = 0 \Rightarrow c = 0$

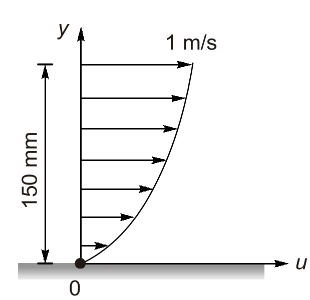
(ii) $u = 1$ m/s at $y = 0.15$ m

$$1 = 0.15^2 \times A + 0.15\,B \qquad \text{...(ii)}$$

(iii) at $y = 0.15$ m at $\dfrac{du}{dy} = 0$

$$\frac{du}{dy} = 2Ay + B$$

$$2A \times 0.15 + B = 0 \qquad \text{...(iii)}$$



From Eq. (ii) and (iii), we get,

$A = -44.4; \quad B = 13.33$

So velocity profile will be given as

$u = -44.4\, y^2 + 13.33\, y$

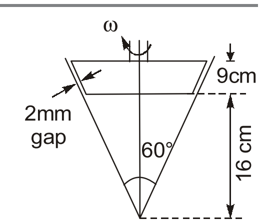
(a) at $y = 0$ mm
$$\frac{du}{dy} = -2 \times 44.4 \times 0 + 13.33 = 13.33 \text{ sec}^{-1}$$

Shear stress,
$$\tau = \mu\frac{du}{dy} = 0.804 \times 13.33 = 10.8 \text{ N/m}^2$$

(b) at $y = 100$ mm
$$\frac{du}{dy} = -2 \times 44.4 \times 0.1 + 13.33 = 4.45 \text{ sec}^{-1}$$
$$\tau = \mu\frac{du}{dy} = 0.804 \times 4.45 = 3.575 \text{ N/m}^2$$

(c) at $y = 150$ mm
$$\frac{du}{dy} = -2 \times 44.4 \times 0.15 + 13.33 = 0$$
$$\tau = 0$$

Example-1.3 For the truncated cone as shown in fig. calculate the torque required if the cone is rotate at 200 rpm. Viscosity of oil in the 2 mm gap between the cone and the housing is 2 poise.

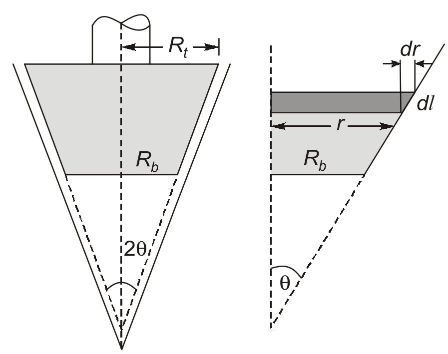


Solution:

Consider the truncated cone as shown in Fig., let R_t and R_b be the radii at top and bottom of the cone of the vertex angle 2θ. Let the cone is rotated at angular speed of ω rad/s and the thickness of the gap is t.

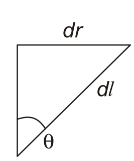
Consider an elementary strip of the cone with radius r.

Shear stress on the sloping wall of the strip



$$\tau = \mu\left(\frac{du}{dy}\right)$$
$$\tau = \mu\left(\frac{u}{t}\right) = \mu\left(\frac{r\omega}{t}\right)$$

Area of sloping wall of strip $= 2\pi r\,(dl) = 2\pi r\left(\frac{dr}{\sin\theta}\right)$



$\therefore$ Shear force on strip, $F = \tau \times \text{Area} = \left(\frac{\mu r\omega}{t}\right) \times \left(\frac{2\pi r dr}{\sin\theta}\right) = \left(\frac{2\pi\mu\omega}{t\sin\theta}\right) r^2 dr$

Torque about central axis due to shear force on the strip, $dT = Fr$

$$dT = \left(\frac{2\pi\mu\omega}{t\sin\theta}\right) r^3 dr$$

$\therefore$ Total torque, $T = \int_{R_b}^{R_t} dT = \frac{2\pi\mu\omega}{t\sin\theta}\int_{R_b}^{R_t} r^3 dr$

$\therefore$
$$T = \frac{2\pi\mu\omega}{t\sin\theta}\left[\frac{R_t^4}{4} - \frac{R_b^4}{4}\right]$$

$$T = \frac{\pi\mu\omega}{2t\sin\theta}\left[R_t^4 - R_b^4\right] \quad \text{...(i)}$$

$$R_t = (16 + 9)\tan 30° = 25 \tan 30° = 14.43 \text{ cm} = 0.144 \text{ m}$$

and $R_b = 16 \tan 30° = 9.24 \text{ cm} = 0.0924 \text{ m}$

∴ $\mu = 2 \text{ poise} = \frac{2}{10}\text{ Ns/m}^2 = 0.2 \text{ Ns/m}^2,\ t = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$$\theta = 30°,\ \omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/sec}$$

∴ Putting these values in Eq. (i), we get,

$$T = \frac{\pi\mu\omega}{2t\sin\theta}\left[R_t^4 - R_b^4\right] = \frac{\pi \times 0.2 \times 20.94}{2 \times 2 \times 10^{-3}\sin 30°}\left[(0.144)^4 - (0.0924)^4\right]$$

$$= 2.35 \text{ N.m}$$

1.4.3 Surface Tension

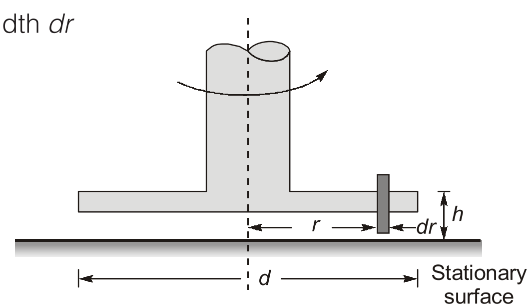
- It is a force which exists on the surface of a liquid when it is in contact with another fluid or a solid boundary. Its magnitude depends upon the relative magnitude of cohesive and adhesive forces.
- Surface tension is a force in the liquid surface and acts normal to a line of unit length drawn imaginarily on the surface. Thus it is a line force.
- It represents surface energy per unit area. It has dimension MT^{-2} and SI unit is N/m.
- Whenever a liquid is in contact with other liquids or gases the interface develops that acts like a stretched elastic membrane, creating surface tension.

Example-1.4 A circular disc of diameter *d* is slowly rotated in a liquid of large viscosity 'μ' at a small distance '*h*' from fixed surface. Derive expression for torque '*T*' necessary to maintain an angular velocity 'ω'.

Solution:

Consider an element of disc at radius *r* and having a width *dr*

Linear velocity at this radius



$$V = r\omega$$

$$\text{Torque} = \text{Shear stress} \times \text{Area} \times r$$

$$\text{Shear stress, } \tau = \frac{\mu du}{dy}$$

Torque required for small ring, dT

∴ $$dT = \frac{\mu du}{dy} \times 2\pi r \cdot dr \cdot r$$

Now assuming that *h* is very small and velocity distribution is linear. So,

$$\frac{du}{dy} = \frac{r\omega}{h}$$

∴ $$dT = \frac{\mu r\omega}{h} \times 2\pi r^2 dr$$

$$= \frac{2\pi\mu\omega}{h} r^3 dr$$

So total torque required,
$$T = \int_0^{d/2} dT = \frac{2\pi\mu\omega}{h}\left[\frac{r^4}{4}\right]_0^{d/2}$$

$$T = \frac{\mu\pi d^4\omega}{32h}$$

Effect of Temperature

- As the surface tension depends directly upon the intermolecular cohession and since this cohession is known to decrease with temperature rise the surface tension decreases with rise of temperature.
- Its value for water-air contact (free-surface of water) reduces from 0.0731 N/m at 17.8°C to 0.0585 N/m at 100°C.

Droplet and Jet

- When a droplet is separated initially from the surface of the main body of liquid, then due to surface tension there is a net inward force exerted over the entire surface of the droplet which causes the surface of the droplet to contract from all the sides and results in increasing the internal pressure within the droplet.
- The contraction of the droplet continues till the inward force due to surface tension is in balance with the internal pressure and the droplet forms into sphere which is the shape for minimum surface area.
- The internal pressure within a jet of liquid is also increased due to surface tension.
- The internal pressure intensity within a droplet and a jet of liquid in excess of the outside pressure intensity may be determined by the expressions derived below.

(i) **Pressure intensity inside a droplet :** Consider a spherical droplet (Fig. (a)) of radius r having internal pressure intensity p in excess of the outside pressure intensity. If the droplet is cut into two halves, then the forces acting on one half (Fig. (b)) will be those due to pressure intensity (p) on the projected area (πr^2) and the tensile force due to surface tension (σ) acting around the circumference ($2\pi r$). These two forces will be equal and opposite for equilibrium and hence we have

$$p(\pi r^2) = \sigma(2\pi r)$$

or
$$p = \frac{2\sigma}{r}$$

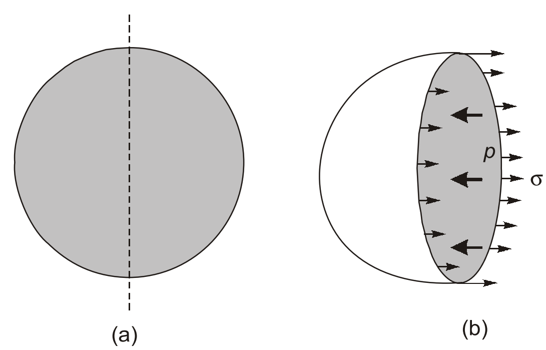


Fig. *Surface Tension (σ) and Internal Pressure (p) in a droplet*

> **NOTE:** Above equation indicates that the internal pressure intensity increase with the decrease in the size of droplet.

(ii) **Pressure intensity inside a soap bubble :** A spherical soap bubble has two surfaces in contact with air, one inside and the other outside, each one of which contributes the same amount of tensile force due to surface tension (Fig.). As such on a hemispherical section of a soap bubble of radius r, the tensile force due to surface tension is equal to $2\sigma(2\pi r)$. However, the pressure force acting on the hemispherical section of the soap bubble is same as in

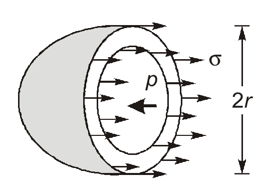


Fig. *Soap Bubble*

the case of a droplet and it is equal to $p(\pi r^2)$. Thus equating these two forces for equilibrium, we have

$$p(\pi r^2) = 2\sigma(2\pi r)$$

or
$$p = \frac{4\sigma}{r}$$

(iii) **Pressure intensity inside a liquid jet :** Consider a jet of liquid of radius r, length l and having internal pressure intensity p in excess of outside pressure intensity. If the jet is cut into two halves, then the forces acting on one half will be those due to pressure intensity (p) on the projected area ($2rl$) and the tensile force due to surface tension (σ) acting along the two sides ($2l$). These two forces will be equal and opposite for equilibrium and hence we have (Fig.),

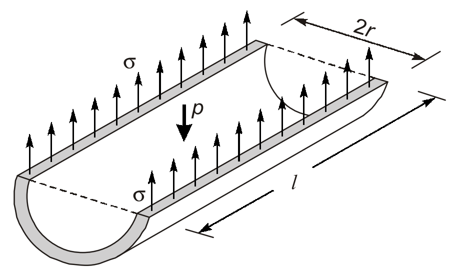


Fig. *Liquid Jet*

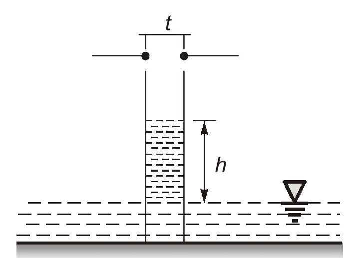
$$p(2rl) = \sigma(2l)$$

or
$$p = \frac{\sigma}{r}$$

Example-1.5 Define capillarity. Derive an equation for capillarity rise between two thin vertical plates spaced 't' distance apart. Calculate the distance between the plates when the capillarity rise is not to exceed 60 mm. Assume surface tension of water at 20°C as 0.075 N/m.

Solution:

Capillarity "It is the phenomenon of rise or fall of liquid surface related to adjacent general level of liquid, due to surface tension, when it is passing through tubes of small thickness."



Let width of plate be L and contact angle be θ so for two vertical plates 't' distance apart. Force due to surface tension = Force due to gravity.

$$\sigma \times (2L)\cos\theta = \rho g(L \times t) \times h$$

So, Height of capacity rise, $h = \dfrac{2\sigma\cos\theta}{\rho g t}$

Now, $\theta = 0.075$ N/m, $\theta = 0°$, $h = 60$ mm

So, $0.06 = \dfrac{2 \times 0.075 \times 1}{9.81 \times 1000 \times t} \times 1000$ mm

$t = 0.255$ mm

Example-1.6 In measuring the unit surface energy of a mineral oil (specific gravity 0.9) by the bubble method. A tube having an internal diameter of 1.5 mm is immersed to a depth of 12.5 mm in the oil. Air is forced through the tube forming a bubble at lower end. What magnitude of unit surface energy will be indicated by a maximum bubble pressure intensity of 150 Pa.

Solution:

Specific weight of mineral oil, $\gamma = 0.9 \times 1000 \times 9.81 = 8829$ N/m^3

Pressure at a depth of 12.5 mm $= \gamma h = 8829.0 \times \dfrac{12.5}{1000} = 110.3625$ Pa

Now pressure difference between inside and outside of bubble,

$$\Delta p = 150 - 110.3625 = 39.64 \text{ Pa}$$

$$\text{Now unit surface energy, } \sigma = \frac{\Delta p d}{4} = \frac{39.64 \times 1.5}{1000 \times 4} = 0.014865 \text{ N/m}$$

Example-1.7 **A system is used to calculate the pressure p_1 in the tank by measuring 15 cm height of liquid in the 1 mm-diameter tube. The fluid is at 20°C. Calculate the true fluid height in the tube and percentage error due to capillarity if fluid is water.**

Solution:

As we know that surface tension of water at 20°C, $\sigma = 0.073$ N/m

So capillary rise 'h' is given as
$$h = \frac{4\sigma\cos\theta}{\rho g D}$$

For water $\theta = 0°$

$$h = \frac{4 \times 0.073 \times 1}{1000 \times 9.81 \times 1 \times 10^{-3}} = 0.02976 \text{ m} = 2.976 \text{ cm}$$

So,
$$\text{True fluid height} = 15 - 2.976 = 12.023 \text{ cm}$$

Now
$$\text{Percentage error} = \frac{15 - 12.023}{12.023} \times 100 = 24.76\%$$

1.4.4 Vapour Pressure

- For phase-change process for any pure liquid, there is a certain relationship which exists between pressure and temperature.
- At a given pressure, the temperature at which any pure liquid changes phase (liquid to vapour) is called the saturation temperature. For water, it is 100°C at 1 atm.
- Similarly, at a given temperature, the pressure at which any liquid changes phase is called the saturation pressure.

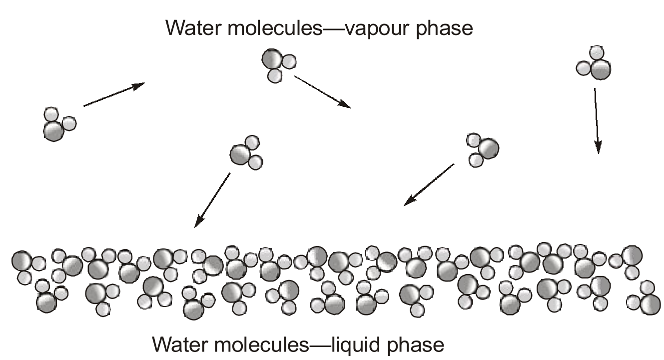


Fig. *Vapour Pressure*

- The vapour pressure of a pure liquid is defined as the pressure exerted by its vapour in phase equilibrium. It is a property of pure liquid vapour.
- Partial pressure is defined as the pressure of a gas or vapour in mixture with other gases.
- Thus, in an open atmosphere the vapour of the liquid exerts a partial pressure on the liquid surface which is known as the vapour pressure of the liquid.
- Thus, for a system of a pure liquid and only vapour of that liquid, the whole pressure on liquid surface will be exerted by the vapour and hence, the term partial pressure is replaced by the vapour pressure.
- The pressure value would be the same whether it is measured in the vapour or liquid phase.
- Vapour pressure of the liquid increases with temperature. Thus, a liquid at higher pressure boils at higher temperature.

- At boiling point of the liquid, vapour pressure becomes equal to the atmospheric pressure.
- Volatile liquids have high vapour pressure (Ex. Benzene).
- Mercury has a very low vapour pressure and hence it is an excellent fluid to be used in barometer.

Cavitation

- When a liquid flows into a region where its pressure is reduced to vapour pressure, it starts vaporizing (or boiling), and vapour pockets or bubbles are formed in the liquid. These vapour bubbles are carried along with flowing liquid until a region of higher pressure is reached, where they suddenly collapse as the vapour condenses to liquid again. Moreover, these bubbles may also collapse even at the point where they are formed, because of the momentary local increase in pressure at this point. When a vapour bubble collapses a cavity is formed and the surrounding liquid rushes to fill it. This process of formation of vapour bubbles and their collapsing is called cavitation.
- Any solid surface in the vicinity of the cavities is also subjected to intense pressure, because, even if the cavities are not actually at the solid surface, the pressure is propagated from the cavities by pressure waves similar to those encountered in water hammer. The formation and collapse of the large number of bubbles on the boundary surface subject the surface to intense local stressing.
- The boundary surface is thus alternatively stressed and relieved of the stresses, thereby ultimately damaged by fatigue, resulting in serious erosion of the surface which is known as pitting.
- Liquids generally contain some dissolved air. This dissolved air is released as the pressure is reduced and thus air pockets or bubbles are formed in liquid. The air pockets are often termed as air locks due to which air cavitation occurs.
- This phenomenon of cavitation is always accompanied by considerable noise and vibrations.
- When the cavitation occurs, the flow is disturbed, flow passage is damaged and the efficiency is lowered.
- Air cavitation is less damaging than vapour cavitation but, both reduces efficiency equally.
- The cavitation number (σ) is used to characterize the susceptibility of the system to cavitatation.

It is defined by
$$\sigma = \frac{p - p_v}{\rho V^2/2}$$

where, p = absolute pressure at point under consideration; p_v = vapour pressure of liquid
ρ = mass density of liquid; V = reference velocity of flow of liquid

- σ is a dimensionless parameter.
- In two geometrically similar systems, if the value of σ is same, they would be equally likely to cavitation or they would have the same degree of cavitation.
- Theoretically, cavitation starts when $\sigma = 0$, i.e., $p = p_v$
- Protection against cavitation:
  (a) To adopt such a hydraulic design of the system which would avoid the development of low pressure.
  (b) If, it is not possible to develop completely cavitation free regions (e.g. in case of turbines and pumps), we can go for special cavitation resistant materials as a coating such as alloys of aluminium, bronze, stainless steel etc.

1.4.5 Compressibility

- Compressibility is the measure of change of volume or density when a substance is subjected to pressure.
- Compressibility effect is more significant in gases. Liquids are considered as non-compressible for general purposes.

- Compressibility becomes important when the flow velocity reaches a significant fraction of speed of sound of fluid.
- Coefficient of compressibility = Percentage change in the volume for a given change in pressure.

or $$\beta = \frac{1}{K}$$

- When applied pressure is withdrawn, fluid expand to their original volume. This property of compressibility of a fluid is expressed by bulk modulus of elasticity.
- Bulk modulus of elasticity is defined as

$$K = \frac{dp}{-dV/V}$$

where, dp = applying pressure, V = original volume

dV = change in volume

(–) ve sign indicate decrease in volume with increase in pressure.

- Bulk modulus of elasticity of liquid is not a constant but increases with increasing pressure.
- In case of water hammer, where change in pressure are either large or very sudden, it is necessary to consider the effect to compressibility of water/liquids.
- Velocity of sound or velocity of pressure waves in fluid system is described as

$$C = \sqrt{\frac{K}{\rho}} \quad \text{where } \rho = \text{mass density of fluid}$$

Example-1.8 The volume of a liquid is reduced by 1.1% by increasing the pressure from 0.5 MPa to 12.5 MPa. Estimate modulus of elasticity of liquid.

Solution:

Modulus of elasticity, $$K = \frac{-dp}{dV/V} = -\left(\frac{12.5 - 0.5}{-\frac{1.1}{100}}\right) = 1090.91 \text{ MPa}$$

Example-1.9 For determining the depth of sea at a place, a charge was exploded at 100 m below the sea water surface. The first reflected wave was recorded after 3 sec at surface. Calculate the depth assuming the sea has a flat bottom. The average value of bulk modulus of elasticity of sea water is 1.96×10^9 N/m^2 and its specific weight = 10×10^3 N/m^3.

Solution:

$$K = 1.96 \times 10^9 \text{ N/m}^2$$

$$\rho = \frac{10^4}{9.81} = 1020 \text{ kg/m}^3$$

$$\text{Velocity of sound} = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{1.96 \times 10^9}{1020}} \simeq 1386 \text{ m/s}$$

Let the depth of sea below water surface be d meter.

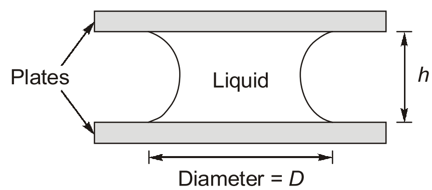
The distance travelled by reflected sound wave

$$= (d - 100) + d = (2d - 100)$$

Time taken by first reflected wave to reach surface in

$$3 = \frac{2d - 100}{1386} \Rightarrow d = 2129 \text{ m}$$

Example-1.10 A very small quantity of liquid having a surface tension σ forms a circular spot of diameter D and between two glasses plates separated by a small distance h, obtain expression for force required to pull the plate apart.



Solution:

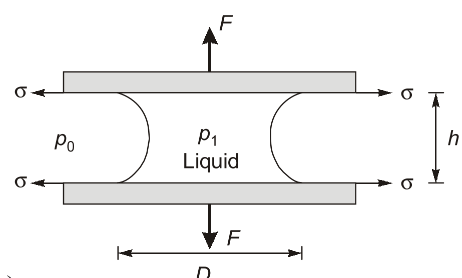
Let p_0 and p_1 be the ambient pressure and pressure inside liquid and

$$\Delta p = p_1 - p_0 \text{ (where } p_0 > p_1)$$

The circular spot is stable if difference in pressure = Total surface tension force

$$\pi Dh \times (\Delta p) = 2\sigma \times \pi D$$

$$\Delta p = \frac{2\sigma}{h}$$



So force required to pull the plate apart

$$F = \left(\frac{\pi D^2}{4}\right)(\Delta p)$$

$$= \frac{2\pi D^2 \sigma}{4h} = \frac{\pi}{2}\left(\frac{D}{h}\right)\sigma D$$

■■■■