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CHAPTER 01

Fluid Properties and Gas Dynamics

1.1 Definition and Classification of Fluids

A fluid may be defined as a substance which is capable of flowing. It has no definite shape of its own and it continuously deforms when shearing forces are applied. It experiences relative motion between their elementary parts as long as shearing forces are present.

A liquid is a fluid which possess a definite volume which varies slightly with pressure and temperature. Liquids are difficult to compress and hence are considered incompressible for practical purposes. It forms a free surface separating it from atmosphere or any other gas present.

A gas is a fluid that possess no definite volume, but always expands until it fills its container. Even a small change in pressure and temperature of the gas has got significant effect on its volume.

A vapour is a gas whose temperature and pressure are such that it is near to the liquid state.

For the purpose of easiness in analysis, fluids are classified as ideal fluids and real fluids. A fluid with no viscosity and surface tension is called an ideal fluid. It is incompressible and encounters no resistance as it moves. Real fluids possess the properties such as viscosity, surface tension and compressibility and therefore resistance is always offered by these fluids when they are set in motion. Only real fluids exist in nature. Ideal fluids are imaginary.

Inviscid (or) Ideal Fluid

An Ideal fluid is one, which has no property other than density. No resistance is encountered when such a fluid flows. Ideal fluids or inviscid fluids are those fluids in which two contacting layers experience no tangential force (shearing stress) but act on each other with normal force (pressure) when the fluids are in motion. In other words inviscid fluids offer no internal resistance. The pressure at every point of an ideal fluid is equal in all directions, whether the fluid is at rest or in motion. Inviscid fluids are also known as perfect fluids or friction less fluids. However, no such fluid exists in nature. The concept of ideal fluids facilitates simplification of the mathematical analysis. Fluids with low viscosities such as water and air can be treated as ideal fluids under certain conditions.

Viscous (or) Real Fluid

Viscous fluids or real fluids are those, which have viscosity, surface tension and compressibility in addition to the density. Viscous or real fluids are those when they are in motion, two contacting layers of the fluids experience tangential as well as normal stresses. The property of exerting tangential or shearing stress and normal stress in a real fluid when it is in motion is known as viscosity of the fluid. Internal friction plays a vital role in viscous fluids during the motion of the fluid. One of the important features of viscous fluid is that it offers internal resistance to the motion of the fluid. Viscous fluids are classified into two categories i.e. Newtonian fluids and Non- Newtonian fluids.

Newtonian Fluid

According to Newton's law of viscosity, for laminar flows, the shear stress is directly proportional to the strain rate or the velocity gradient.

$$t = \mu \frac{du}{dy} \quad ...(i)$$

where μ is the constant of proportionality and is the dynamic viscosity of the fluid. The shear stress is maximum at the surface where the fluid is in contact with it, due to no slip condition. The fluids obeying the Newton's law of viscosity are called as Newtonian fluids. It is clear from Newton's law that equation (i) represents an ideal fluid if $\tau = 0$ then $\mu = 0$. A fluid in which the constant of proportionality μ does not change with shear strain $\frac{du}{dy}$ is said to be Newtonian fluid. Water, air, mercury are some of the examples of Newtonian fluids.

Non-Newtonian Fluids

Non-Newtonian fluids are those fluids which do not obey Newton's law of viscosity and the relation between shear stress and rate of shear strain is non-linear. i.e. the viscosity of non-Newtonian fluid is not constant at a given temperature and pressure but depends on other factors such as the rate of shear in the fluid, the container of the fluid and on the previous history of the fluid. Many important industrial fluids are non-Newtonian in their flow characteristics. These include paints, coal tar, polymers, lubricants, plastics, printer ink and molecular materials etc. The non-Newtonian fluids are further classified into three classes:

i. Time dependent non-Newtonian fluids, for which the rate of shear at any point is a function of the shear stress at that point.

ii. In some fluids the relationship between shear stress and shear rate depends on the time the fluid has been sheared or on its previous history during its motion. These fluids are known as time dependent non-Newtonian fluids.

iii. The third category of fluids contains characteristics of both solids and fluids and exhibit partial elastic recovery after deformation. These are known as viscoelastic fluids. The nature of relation between shear stress and rate of shear strain for Newtonian and non-Newtonian fluids is shown in the figure.

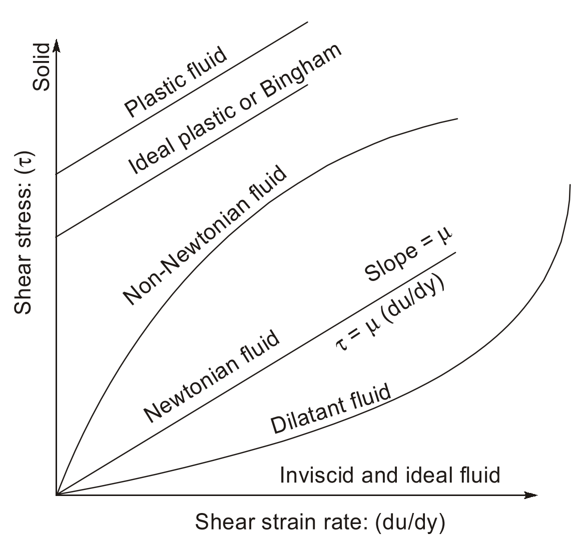


Fig. *Classification of fluids*

Each of the curves in figure can be represented by the equation

$\tau = \tau_y + \mu_p \left(\frac{du}{dy}\right)^n$ where τ_y is yield shear stress and μ_p is apparent viscosity and n is a constant.

Plastics: There mists yield stress τ_y such that the flow takes place only when the shear stress is greater than the yield value.

Bingham Plastics : In addition to yield stress n takes the value I and j. If the coefficient of rigidity which represents the slope of the flow curve.

Pseudo Plastics: Pseudo plastic fluids have no yield stress but the viscosity decreases with rate of shear. ***Examples:*** Colloidal substances like clay,, milk and cement.

Dilatant Fluids: For these fluids yield stress is zero and n > I. Viscosity increases with increasing rate of shear-stress. ***Example:*** Quicksand.

Table 1.1: Classification of Fluids on the Basis of Density and Viscosity

Type of Fluid	Density	Viscosity
1. Ideal fluid	Constant	Zero
2. Incompressible fluid	Constant	Non-zero
3. Inviscid fluid	Constant or Variable	Zero
4. Real fluid	Variable	Non-zero
5. Newtonian fluid	Constant or Variable	$\tau = \mu \frac{du}{dy}$
6. Non-Newtonian fluid	Constant or Variable	$\tau \neq \mu \frac{du}{dy}$
7. Compressible fluid	Variable	Zero
8. Perfect gas	Variable, $p = \rho RT$	Zero

Table 1.2: Classification of flows on the basis of Mach number

Type of Flow	Mach Number, *M*
Incompressible flow	*M* less than 0.3.
Compressible subsonic flow	*M* between 0.3 and 1.
Transonic flow	*M* ranging between values less than 1 and more than 1.
Supersonic flow	*M* greater than 1 but less than 7.
Hypersonic flow	*M* greater than 7.

1.2 Distinction Between a Solid and a Fluid

The molecules of a solid are usually closer together than those of a fluid. The attractive forces between the molecules of a solid are so large that a solid tends to retain its shape. This is not the case for a fluid, where the attractive forces between the molecules are smaller. An ideal elastic solid will deform under load and, once the load is removed, will return to its original state. Some solids are plastic. These deform under the action of a sufficient load and deformation continues as long as a load is applied, providing the material does not rupture. Deformation ceases when the load is removed, but the plastic solid does not return to its original state, The intermolecular cohesive forces in a fluid are not great enough to hold the various elements of the fluid together. Hence a fluid will flow under the action of the slightest stress and flow will continue as long as the stress is present.

1.3 Distinction between gas and liquid

A fluid may be either a gas or a liquid. The molecules of a gas are much farther apart than those of a liquid. Hence a gas is very compressible, and when all external pressure is removed, it tends to expand indefinitely. A gas is therefore in equilibrium only when it is completely enclosed. A liquid is relatively incompressible, and if all pressure, except that of its own vapour pressure, is removed, the cohesion between molecules holds them together, so that the liquid does not expand indefinitely. Therefore a liquid may have a free surface, i.e., a surface from which all pressure is removed, except that of its own vapour. A vapour is a gas whose temperature and pressure are such that it is very near the liquid phase. Thus steam is considered a vapour because its state is normally not far from that of water. A gas may be defined as a highly superheated vapour; that is, its state is far removed from the liquid phase. Thus air is considered a gas because its state is normally very far from that of liquid air. The volume of a gas or vapour is greatly affected by changes in pressure or temperature or both. It is usually necessary, therefore, to take account of changes in volume and temperature in dealing with gases or vapours. Whenever significant temperature or phase changes are involved in dealing with vapours and gases, the subject is largely dependent on heat phenomena (thermodynamics). Thus fluid mechanics and thermodynamics are interrelated.

1.4 Compressible and Incompressible Fluids

Fluid mechanics deals with both incompressible and compressible fluids, that is, with liquids and gases of either constant or variable density. Although there is no such thing in reality as an incompressible fluid, we use this term where the change in density with pressure is so small as to be negligible. This is usually the case with liquids. We may also consider gases to be incompressible when the pressure variation is small compared with the absolute pressure. Ordinarily we consider liquids to be incompressible fluids, yet sound waves, which are really pressure waves, travel through them. This is evidence of the elasticity of liquids. In problems involving water hammer we must consider the compressibility of the liquid. The flow of air in a ventilating system is a case where we may treat a gas as incompressible, for the pressure variation is so small that the change in density is of no importance. But for a gas or steam flowing at high velocity through a long pipeline, the drop in pressure may be so great that we cannot ignore the change in density. For an airplane flying at speeds below 250 mph (100 m/s), we may consider the air to be of constant density. But as an object moving through the air approaches the velocity of sound, which is of the order of 760 mph (1200 km/h) depending on temperature, the pressure and density of the air adjacent to the body become materially different from those of the air at some distance away, and we must then treat the air as a compressible fluid.

1.5 Definitions and Properties of Fluids

1.5.1 Property of a system

Any characteristic of a system is called a property. Some familiar properties are pressure P, temperature T, volume V, and mass m. The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.

1.5.2 System and Control Volume

A system refers to a fixed, identifiable quantity of mass which is separated from its surrounding by its boundaries. The boundary surface may vary with time however no mass crosses the system boundary. In fluid mechanics an infinitesimal lump of fluid is considered as a system and is referred as a fluid element or a particle.

Since a fluid particle has larger dimension than the limiting volume . The continuum concept for the flow analysis is valid.

Control volume is a fixed, identifiable region in space through which fluid flows. The boundary of the control volume is called control surface. The fluid mass in a control volume may vary with time. The shape and size of the control volume may be arbitrary.

1.5.3 Extensive and Intensive Properties

Generally in the study of fluids, the concern is on open systems such as those associated with ducts, pipes or transfer lines, compressor or turbine blades, nozzles or diffuses etc, through which the fluid moves. Such a region is selected for the purpose of analysis and is defined as the control volume. As mentioned in the beginning of this chapter, equations for the governing conservation laws are to be derived for a general control volume. Before deriving the equations, it is useful to define intensive and extensive fluid properties as applied to the control volume.

An extensive property depends on the mass of the fluid contained within the control volume. Examples are, volume, mass and momentum, in the present book, all the extensive properties, except mass, are denoted by capital letter. Examples are U for internal energy, S for entropy and H for enthalpy. A general notation for any extensive property is N.

The property which is independent of the mass of the fluid inside the control volume is called an intensive property. There are two types of intensive properties- The properties like pressure, temperature, etc. M are naturally independent of the mass. The properties like specific internal energy u (internal energy per unit mass), specific entropy s {entropy per unit mass), specific enthalpy h (enthalpy per unit mass), are intensive properties, since they are specific values of the corresponding extensive properties, A general intensive property is denoted by n.

1.5.4 Bulk Modulus of Elasticity:

The bulk modulus of elasticity (K) of a fluid is defined by

$$K = \frac{\text{Increase in pressure}}{\text{Relative change in volume}}$$

Let the pressure changes from p to $p + \Delta p$ in a system consisting of a fluid; the corresponding volume change is $-\frac{\Delta v}{v}$.

Therefore, $$K = \lim_{\Delta P \to 0} -\frac{v\Delta p}{\Delta v}. = -v\frac{dp}{dv}$$

But, $$v = \frac{1}{\rho} \quad \text{i.e. specific volume} = \frac{1}{\text{density}}$$

$$dv = -\frac{\Delta\rho}{\rho^2}$$

Therefore, $$K = \left(-\frac{1}{\rho}\right)\frac{dp}{\left(-\frac{d\rho}{\rho^2}\right)} = \rho\frac{dp}{d\rho}$$

The value of $\frac{dp}{dv}$ or $\frac{dp}{d\rho}$ for gases and vapours depends on the process i.e. isothermal, adiabatic etc.

(i) **Isothermal bulk modulus (K_T):** For an ideal gas in an isothermal process

$$pv = \text{Constant}$$

$$pdv + vdp = 0$$

or
$$-\frac{dp}{dv} = \frac{p}{v}$$

$$K_T = p \quad \text{(for an ideal gas)}$$

(ii) **Adiabatic bulk modulus (K_S):** for and ideal gas in reversible adiabatic process

$$pv^\gamma = \text{Constant}$$

$$p(\gamma v^{\gamma-1})dv + v^\gamma dp = 0$$

$$-\frac{dp}{dv} = \frac{\gamma p}{v}$$

$$K_S = \gamma p$$

K_S and K_T are related as $K_S = \gamma K_T$ (for an ideal gas)

Coefficient of compressibility: Coefficient of compressibility (β) of a fluid is defined as

$$\beta = \frac{\text{Relative change in volume}}{\text{Change in pressure}} = \lim_{\Delta p \to 0} -\frac{\Delta v}{v \Delta p} = -\frac{1}{v}\frac{dv}{dp}$$

As we know,
$$dv = -\frac{d\rho}{dp}, \text{ so}$$

$$\beta = \frac{1}{\rho} - \frac{d\rho}{dp}$$

Comparing equation for K, we get,

$$k = \frac{1}{\beta}$$

Therefore, for ideal gases isothermal coefficient of compressibility is given by,

$$\beta_T = \frac{1}{K_T} = \frac{1}{\rho}$$

The isentropic coefficient of compressibility is given by,

$$\beta_S = \frac{1}{K_S} = \frac{1}{\gamma\rho}$$

$$\beta_T = \gamma\beta_S$$

For liquids, the different between the coefficients of compressibility (or bulk modulus) for isothermal and adiabatic processes is negligibly small.

1.6 Viscosity

The viscosity of a fluid is a measure of its resistance to shear or angular deformation. Motor oil, for example, has high viscosity and resistance to shear, is cohesive, and feels "sticky," whereas gasoline has low viscosity. The friction forces in flowing fluid result from the cohesion and momentum interchange between molecules. Figure indicates how the viscosities of typical fluids depend on temperature. As the temperature increases, the viscosities of all liquids decrease, while the viscosities of all gases increase. This is because the force of cohesion, which diminishes with temperature, predominates with liquids, while with gases the predominating factor is the interchange of molecules between the layers of different velocities. Thus a rapidly-moving gas molecule shifting into a slower moving layer tends to speed up the latter. And a slow-moving molecule entering a faster-moving layer tends to slow down the faster-moving layer. This molecular interchange sets up a shear, or produces a friction force between adjacent layers. At higher temperatures molecular activity increases, so causing the viscosity of gases to increase with temperature.

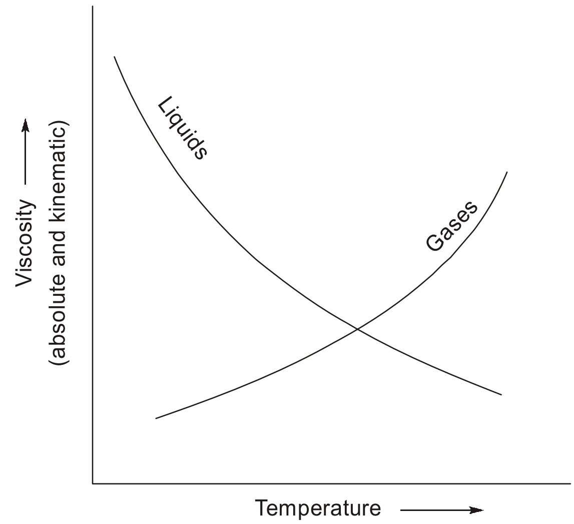


Fig. *Trends in viscosity variation with temperature*

Let us consider the classic case of two parallel plates, sufficiently large that we can neglect edge conditions, a small distance Y apart, with fluid filling the space between. The lower plate is stationary, while the upper are moves parallel to it with a velocity U due to a force F corresponding to some area A of the moving plate.

If the separation distance Y is not too large, if the velocity U is not too high, and if there is no net flow of fluid through the space, the velocity profile will be linear (refer figure (a)). If in addition, there is a small amount of bulk fluid transport between the plates, as could result from pressure fed lubrication for example, the velocity profile becomes the sum of the previous linear profile plus a parabolic profile [refer Figure (b)]; the parabolic additions to the linear profile are zero at the walls (plates) and maximum at the center line. The behaviour of the fluid is much as if it consisted of a series of thin layers, each of which slips a little relative to the next.

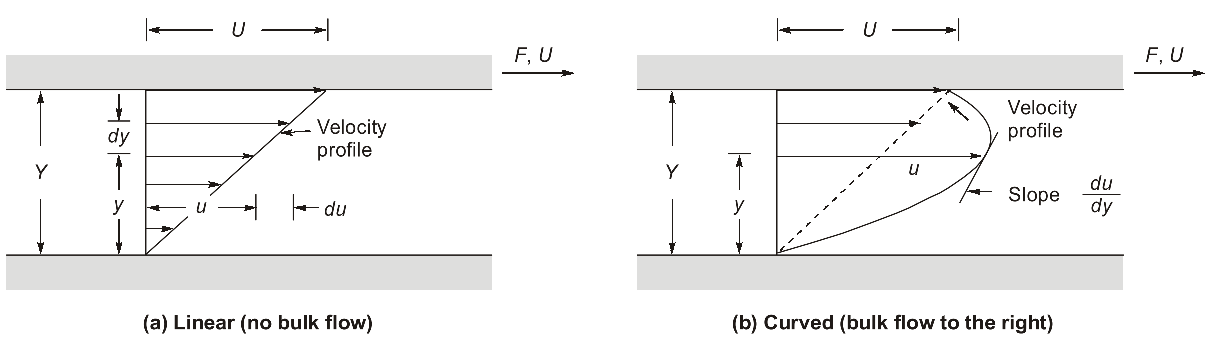


Fig. *Classic case of two parallel plates*

For a large class of fluids as depicted in figure (a), experiments have shown that

$$F \propto \frac{AU}{y}$$

We can see from similar triangles that we can replace $\frac{U}{y}$ by the velocity gradient $\left(\frac{du}{dy}\right)$. If we introduce a constant of proportionality μ, we can express the shearing stress (τ) between any two thin sheets of fluid by

$$\tau = \frac{F}{A} = \mu\frac{U}{y} = \mu\frac{du}{dy} \qquad \text{[Newton's law of viscosity]}$$

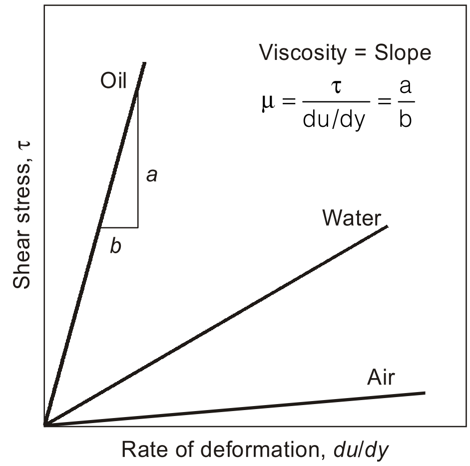


Fig. *Relationship between shear stress and velocity gradient/rate of deformation of Newtonian fluids*

1.6.1 Vapour Pressure of Liquids

All liquids tend to evaporate or vaporize, which they do by projecting molecules into the space above their surfaces. If this is a confined space, the partial pressure exerted by the molecules increases until the rate at which molecules enter the liquid is equal to the rate at which they leave. For this equilibrium condition. We call the vapour pressure the saturation pressure.

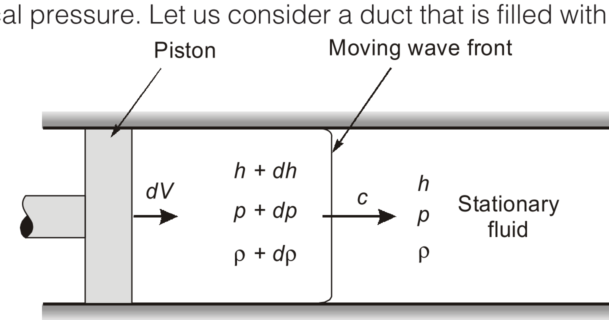
Molecular activity increases with increasing temperature and decreasing pressure, and so the saturation pressure does the same. At any given temperature, if the pressure on the liquid surface falls below the saturation pressure, a rapid rate of evaporation results, known as boiling. Thus we can refer to the saturation pressure as the boiling pressure for a given temperature, and it is of practical importance for liquids.

We call the rapid vaporization and recondensation of liquid as it passes through a region of low absolute pressure cavitation. This phenomenon is often very damaging, and so we must avoid it.

1.7 Compressible Fluid Flow

1.7.1 Velocity of Sound:

A sound wave is an infinitensimal pressure wave and is transmitted rapidly through the fluid medium. The thermodynamic change across such a wave is comparatively small and the speed of the process corresponding to the change is fast. Speed of sound (or the sonic speed) is defined as the speed at which an infinitesimally small pressure wave travels through a medium. The pressure wave may be caused by a small disturbance, which creates a slight rise in local pressure. Let us consider a duct that is filled with a fluid at rest (as shown below).



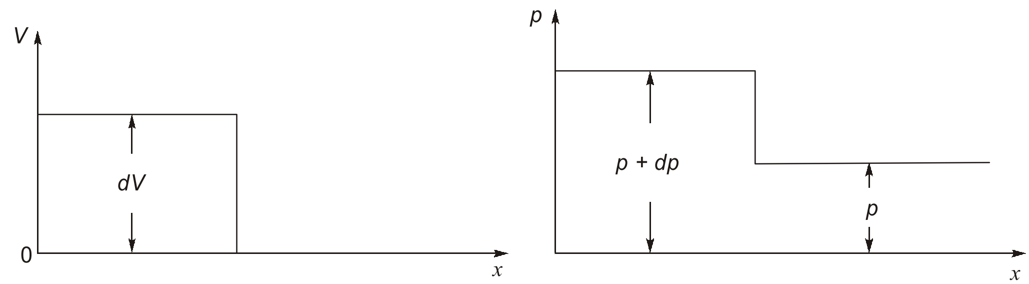


Fig. *Propagation of a small pressure wave along a duct*

A piston flitted in the duct is now moved to the right with a constant incremental velocity *dV*, creating a sonic wave. The wave front moves to the right through the fluid at the speed of sound *C* and separates the moving fluid adjacent to the piston from the fluid still at rest. The fluid to the left of the wave front experiences an incremental change in its thermodynamic properties, while the fluid on the right of the wave front maintains its original thermodynamic properties as shown above.

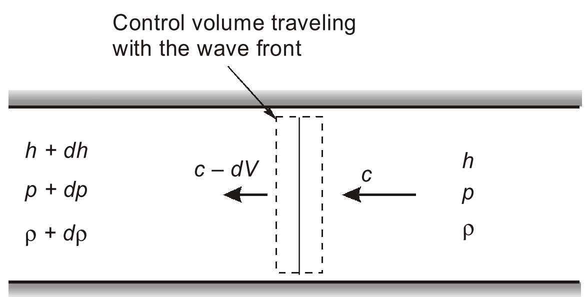


Fig. *Control volume moving with the small pressure wave along a duct*

Now, let us consider a control volume that encloses the wave from and moves with it as shown in figure. To an observer traveling with wave front, the fluid to the right appears to be moving toward the wave front with a speed of *C* and the fluid to the left to be moving away from the wave front with a speed of *C - dV*. Of course, the observer sees the control volume that encloses the wave front (and herself or himself) as stationary, and the observer is witnessing a steady flow process. The mass balance for this single-stream, steady flow process is expressed as

$$\dot{m}_{right} = \dot{m}_{left}$$

$$\rho AC = (\rho + d\rho)A(C - dV) \qquad \text{[Constant area]}$$

By canceling the cross-section area *A* and neglecting higher order terms, this equation reduces to

$$Cd\rho - \rho dV = 0$$

No heat or work crosses the boundaries of the control volume during this steady-flow process, and the potential energy change can be neglected. Then steady-flow energy balance $E_{in} = E_{out}$ becomes

$$h + \frac{C^2}{2} = h + dh + \frac{(C - dV)^2}{2}$$

Which yields, $dh - Cdv = 0$

where we have neglected the second order term dv^2. The amplitude of the ordinary sonic wave is very small and does not cause any appreciable change in the pressure and temperature of the fluid. Therefore, the propagation of a sonic wave is not only adiabatic but also very nearly isentropic.

We know the thermodynamic relation

$$Tds = \text{dh} - vdp$$

$$= dh - \frac{dp}{\rho}$$

This equation reduces to $0 = dh - \frac{dp}{\rho}$ ($dS = 0$ for isentropic process)

$$dh = \frac{dp}{\rho}$$

Considering above equations, we can write the desired expression for the speed of sound as

$$C^2 = \frac{\partial p}{\partial \rho} \text{ at } S = \text{Constant}$$

$$= \left(\frac{\partial p}{\partial \rho}\right)_S \quad \text{for isentropic process}$$

Similarly, we can write, $C^2 = k\left(\frac{\partial p}{\partial \rho}\right)_T$ for isothermal process

when the fluid is an ideal gas ($p = \rho RT$), above equation can be differentiated to yield

$$C^2 = k\left(\frac{\partial p}{\partial \rho}\right)_T = k\left[\frac{\partial(\rho RT)}{\partial \rho}\right]_T = kRT$$

$$C = \sqrt{\gamma RT}$$

(i) **Velocity of sound in incompressible fluids:** As we know, an incompressible fluid cannot experience any change in density.

$$\frac{\partial p}{\partial \rho} = \text{Infinity } (\propto)$$

The velocity of sound in such a fluid is infinity which means that the pressure pulse emitted anywhere in the fluid, are felt simultaneously at all other points.

All real fluids are compressible to some extent, with liquids showing less compressibility. For example, the velocity of sound in water at normal ambient conditions is 1700 m/s which is very high compared to the fluid velocities which can be produced in a liquid medium.

(ii) **Velocity of sound in terms of Bulk Modulus of Elasticity:** Compressibility of a fluid is quantitatively expressed as the inverse of the bulk modulus of elasticity K, of the fluid, which is defined as

$$K = -\frac{dp}{dv / v}$$

where $\frac{dv}{v}$ is the volumetric strain for an infinitesimal pressure change dp.

$$\frac{dv}{v} = -\frac{d\rho}{\rho}$$

The equation for isentropic bulk modulus can be written in terms of the corresponding density change,

$$K_S = \left[\frac{dp}{d\rho / \rho}\right]_S$$

$$C = \sqrt{\frac{K_S}{p}}$$

This equation again confirms that the speed of sound is a direct measure of the compressibility of the medium.

(iii) Velocity of Sound in a Perfect Gas:

For an isentropic process, the relation between pressure and specific volume is,

$$pv^{\gamma} = \text{Constant}, C$$

Since density is the reciprocal of specific volume, the equation becomes,

$$p = C\rho^{\gamma}$$

By taking logarithms and then differentiating the equation, we get

$$\ln p = \ln C + \gamma \ln \rho$$

$$\frac{dp}{p} = \gamma \frac{d\rho}{\rho}$$

$$\left(\frac{dp}{d\rho}\right)_S = \gamma \frac{p}{\rho}$$

$$\text{Velocity of sound, } C = \sqrt{\gamma \frac{p}{\rho}} = \sqrt{\gamma RT} \quad \text{(for perfect gas)}$$

Thus for a perfect gas, the speed of sound is proportional to the square root of the static temperature of the medium. The speed of sound is linked to the average molecular velocity, which from kinetic theory is given by $\sqrt{\frac{8RT}{\pi}}$. It is found that the speed of sound is about three quarter of the average molecular velocity.

(iv) Velocity of sound taking molecular weight into consideration:

$$C = \sqrt{\gamma \left(\frac{\bar{R}}{M_w}\right) T}$$

Since the variation of γ between gases is very small, the molecular weight of the gas (medium) takes a major part in deciding the velocity of sound through the medium. Gases with lower molecular weight have large sound velocities and vice versa.

Table 1.3

Speed of Sound Values for Various Gases

Gas	Molar Mass	γ	Speed of Sound at 0°C (m/s)
Air	28.960	1.404	331
Argon (Ar)	39.940	1.667	308
Carbon dioxide (CO_2)	44.010	1.300	258
Freon 12 (CCl_2F_2)	120.900	1.139	146
Helium (He)	4.003	1.667	970
Hydrogen (H_2)	2.016	1.407	1270
Xenon (Xe)	131.300	1.667	170

1.7.2 Mach Number and Its Significance:

The Mach number at a point in a flow field is defined as the ratio of the local velocity V of flow at the location under consideration to the velocity of sound (C) in the medium at the same location.

$$\text{Mach number, } M = \frac{V}{C}$$

Mach number is a dimensionless parameter named after the Austrian physicist Ernt Mach. The definition of Mach number can also the interpreted as the square root of the ratio between the inertia force due to flow and the elastic force of fluid,

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho A V^2}{k_S A}}$$

Velocity of sound in terms of isentropic bulk modulus can be written as

$$K_S = C^2\rho$$

$$M = \sqrt{\frac{\rho A V^2}{C^2 \rho A}} = \sqrt{\frac{V^2}{C^2}} = \frac{V}{C}$$

Mach number also gives a measure of the relative importance of directed and thermal energies in a flow. The velocity V measures the directed motion of the gas particles and V^2 measures the kinetic energy of the directed flow. The sonic velocity of a given gas is proportional to the random velocity of the gas molecules. Consequently, the Mach number may be regarded as a measure of the ratio of the kinetic energy of the directed flow to the kinetic energy of random molecular motion which is the thermal energy of the system.

Depending upon the magnitude of the flow Mach number, the steady compressible flow can be subdivided as incompressible, subsonic etc as follows:

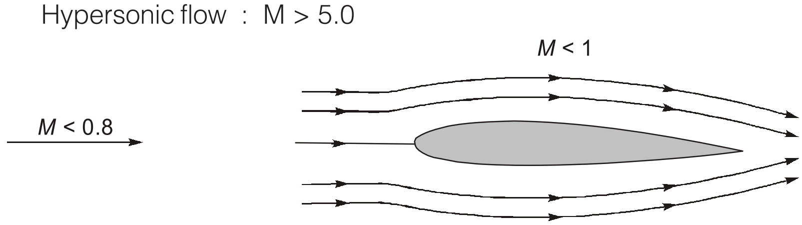
Incompressible flow : $M < 0.3$

Subsonic flow : $0.3 < M < 0.8$

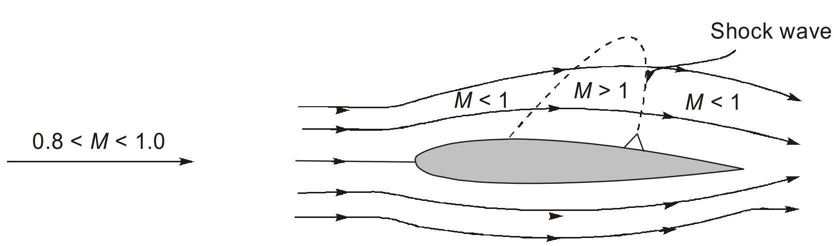
Transonic flow : $0.8 < M < 1.2$

Supersonic flow : $1.2 < M < 5.0$

Hypersonic flow : $M > 5.0$



(a)



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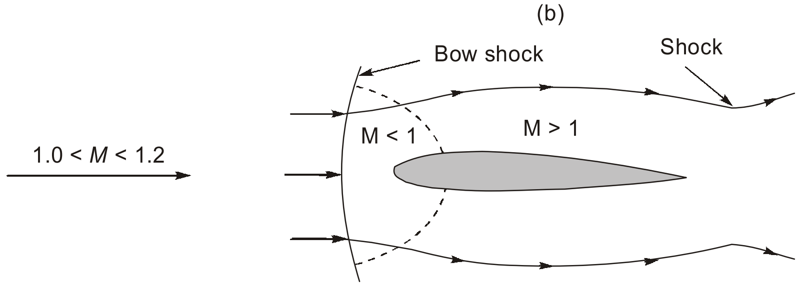

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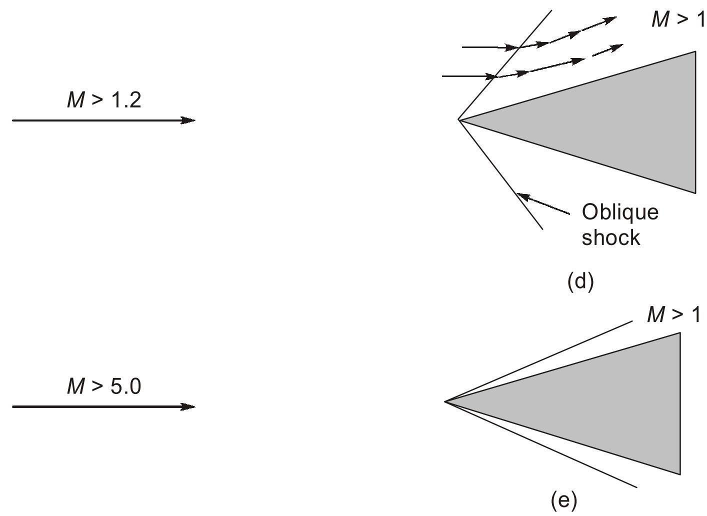


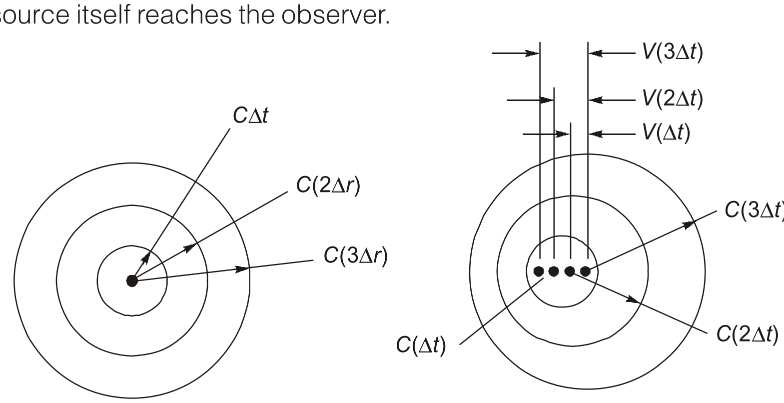
Fig. *Different regimes of compressible flow*

The Mach Cone

Consider a point source of sound that emits a pulse every Δt seconds. Each pulse expands outwards from its origination point at the speed of sound C, so at any instant t the pulse will be a sphere of radius Ct centered at the pulse's origination point. We want to investigate what happens if the point source itself is moving. There are four possibilities, as shown in figure.

(a) $V = 0$. The point source is stationary. Fig. (a) shows conditions after $3\Delta t$ seconds. The first pulse has expanded to a sphere of radius $C(3\Delta t)$, the second to a sphere of radius $c(2\Delta t)$, and the third to a sphere of radius $C(\Delta t)$; a new pulse is about to be emitted. The pulses constitute a set of ever-expanding concentric spheres.

(b) $0 < V < C$. The point source moves to the left at subsonic speed. Fig. (b) shows conditions after 3Δt seconds. The source is shown at times t $=0$, Δt, $2\Delta t$, and $3\Delta t$. The first pulse has expanded to a sphere of radius $C(3\Delta t)$ centered where the source was originally the second to a sphere of radius $C(2\Delta \text{t})$ centered where the source was at time Δt, and the third to a sphere of radius $C(\Delta t)$ centered where the source was at time 2Δt; a new pulse is about to be emitted. The pulses again constitute a set of ever-expanding spheres, except now they are not concentric. The pulses are all expanding at constant speed C. We make two important notes: First, we can see that an observer who is ahead of the source (or whom the source is approaching) will hear the pulses at a higher frequency rate than will an observer who is behind the source (this is the Doppler effect that occurs when a vehicle approaches and passes); second, an observer ahead of the source hears the source before the source itself reaches the observer.



(a) $V = 0$: stationary source (b) $V < C$: Doppler shift

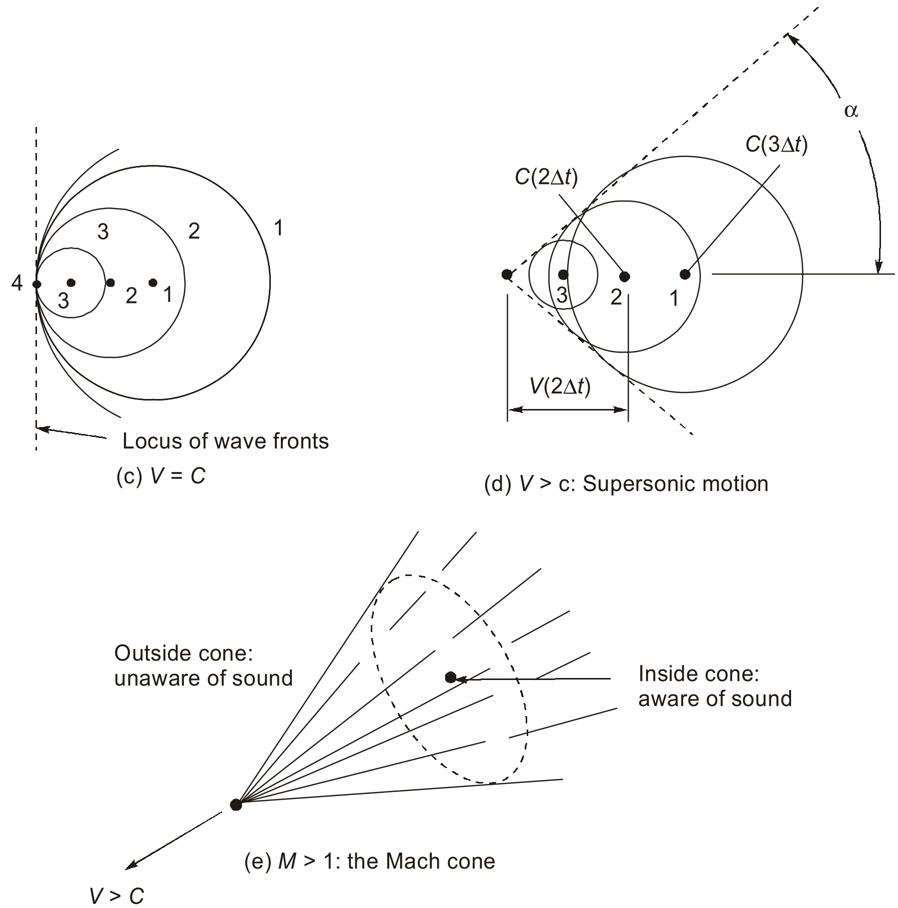


Fig. *Propagation of sound waves from a moving source: The Mach cone*

(c) $V = C$. The point source moves to the left at sonic speed. Fig. (c) shows conditions after $3\Delta t$ seconds. The source is shown at times $t = 0$ (point 1), Δt (point 2), $2\Delta t$ (point 3), and $3\Delta t$ (point 4). The first pulse has expanded to sphere 1 of radius $C(3\Delta t)$ centered at point 1, the second to sphere 2 of radius $C(2\Delta t)$ centered at point 2, and the third to sphere 3 of radius $C(\Delta t)$ centered around the source at point 3. We can see once more that the pulses constitute a set of ever-expanding spheres, except now they are tangent to one another on the left! The pulses are all expanding at constant speed C, but the source is also moving at speed c, with the result that the source and all its pulses are traveling together to the left. We again make two important notes: First, we can see that an observer who is ahead of the source will not hear the pulses before the source reaches the observer second, in theory, over time an unlimited number of pulses will accumulate at the front of the source, leading to a sound wave of unlimited amplitude.

(d) $V > C$. The point source moves to the left at supersonic speed. Fig. (d) shows condition after $3\Delta t$ seconds. By now it is clear how the spherical waves develop. We can see once more that the pulses constitute a set of ever-expanding spheres, except now the source is moving so fast it moves ahead of each sphere that it generates. For supersonic motion, the spheres generate what is called a Mach cone tangent to each sphere. The region inside the cone is called the zone of action and that outside the cone the zone of silence, for obvious reasons, as shown in Fig. (e). From geometry, we see from Fig. (d) that

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$$\sin\alpha = \frac{C}{V} = \frac{1}{M} \quad \Rightarrow \quad \alpha = \sin^{-1}\left(\frac{1}{M}\right)$$

The angle α is termed the Mach angle.

If the body is at rest and the gas is moving over it at a supersonic velocity, all the disturbances generated by the body are swept downstream and lie within the Mach cone shown in figure. There will be essentially jumps in the values of the flow variables when the flow reaches the cone. The cone is therefore termed a conical Mach wave.

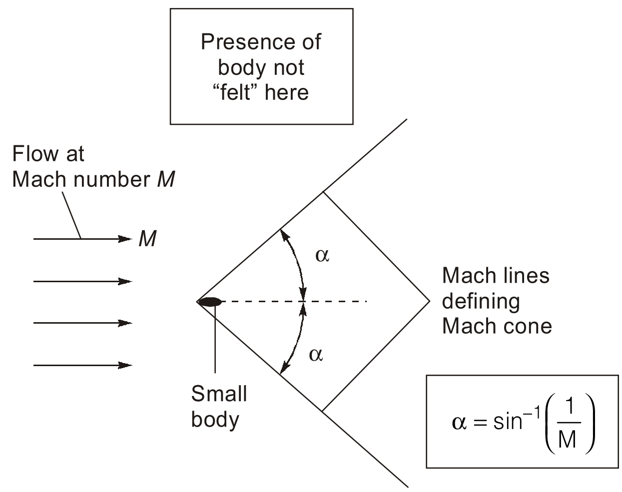

Fig. *Conical Mach Wave*

1.8 Concept of Stagnation Condition

Stagnation conditions are those that would exist if the flow at any point in a fluid stream was isentropically brought to rest. (To define the stagnation temperature, it is actually only necessary to require that the flow be adiabatically brought to rest. To define the stagnation pressure and density, it is necessary, however, to require that the flow be brought to rest isentropically.)

If the entire flow is essentially isentropic and if the velocity is essentially zero at some point in the flow, then the stagnation conditions existing at all points in the flow will be those existing at the zero velocity point as indicated in Fig. (a)

However, even when the flow is non-isentropic, the concept of the stagnation conditions is still useful, the stagnation conditions at a point then being the conditions that would exist if the local flow were brought to rest isentropically as indicated in Fig. (b).

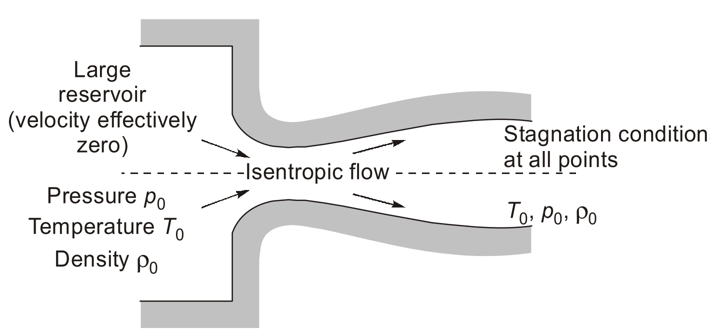

Fig.(a) *Stagnation conditions in an isentropic flow*

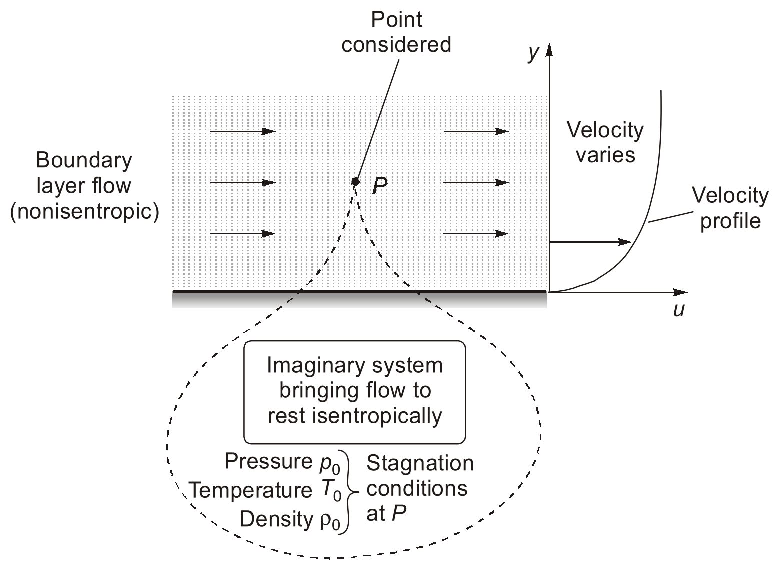

Fig. (b) *Stagnation conditions at a point in a nonisentropic flow*

Let us consider Bernoulli's equation for obtaining information on the reference isentropic stagnation state for incompressible flows:

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{Constant}$$

This equation is valid for a steady, incompressible, frictionless flow along a streamline. It is also valid for an incompressible isentropic process because it is reversible (frictionless and steady) and adiabatic.

For stagnation state:

$$\frac{p_0}{\rho} + 0 = \frac{p}{\rho} + \frac{V^2}{2} \qquad [V = 0 \text{ at stagnation point, } z = \text{constant}]$$

$$p_0 = p + \frac{\rho V^2}{2}$$

For compressible flows, we will focus on ideal gas behaviour. For a compressible flow we can derive the isentropic stagnation relations by applying the mass conservation (or continuity) and momentum equations to a differential control volume and then integrating. Let us imagine the control volume as depicted in figure shown below.

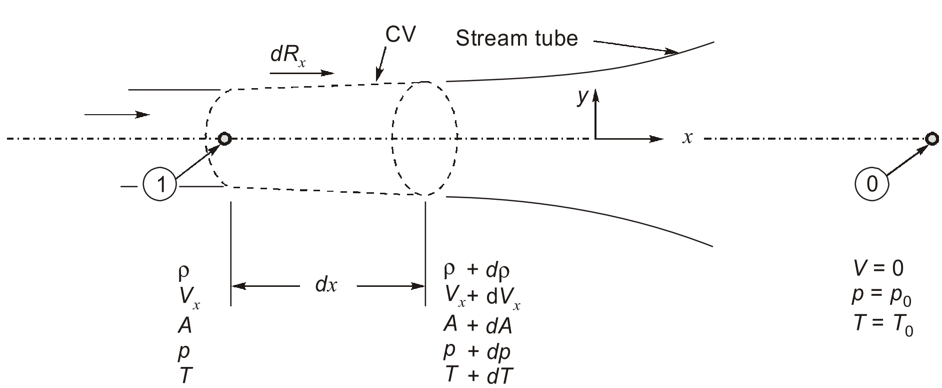

Fig. *Compressible flow in an infinitesimal stream tube*

a. Continuity Equation = 0(1)

Governing equation : $\frac{\partial}{\partial t}\int_{CV}\rho d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0$

Assumptions:

1. Steady flow.
2. Uniform flow at each section.

Then

$$(-\rho V_x A) + \{(\rho + d\rho)(V_x + dV_x)(A + dA)\} = 0$$

or
$$\rho V_x A = (\rho + d\rho)(V_x + dV_x)(A + dA)$$

b. Momentum equation = 0(3) = 0(1)

Governing equation: $F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} V_x \rho dV + \int_{CS} V_x \rho \vec{V}\cdot d\vec{A}$

Assumptions: (3) $F_{B_x} = 0$

(4) Frictionless flow.

The surface forces acting on the infinitesimal control volume are

$$F_{S_x} = dR_x + pA - (p + dp)(A + dA)$$

The force dR_x is applied along the stream tube boundary, figure, where the average pressure is $p + dp/2$, and the area component in the x direction is dA. There is no friction. Thus,

$$F_{S_x} = \left(p + \frac{dp}{2}\right)dA + pA - (p + dp)(a + dA)$$

or
$$F_{S_x} = pdA + \frac{dp\,dA}{2} + pA - pA - dp\,A - pdA - dp\,dA$$

Substituting this result into the momentum equation gives

$$-dp\,A = V_x\{-\rho V_x A\} + (V_x + dV_x)\{(\rho + d\rho)(V_x + dV_x)(A + dA)\}$$

which may be simplified using equation to obtain

$$-dp\,A = (-V_x + V_x + dV_x)(\rho V_x A)$$

Finally,
$$dp = -\rho V_x dV_x = -\rho d\left(\frac{V_x^2}{2}\right)$$

$$\frac{dp}{\rho} + \left(\frac{V_x^2}{2}\right) = 0 \qquad \text{Momentum equation}$$

Above equation is a relation among properties during the deceleration process. In developing this relation, we have specified a frictionless deceleration process. Before we can integrate between the initial and final (stagnation) states, we must specify the relation that exists between pressure, p and density ρ along the process path.

Since the deceleration process is isentropic, then p and ρ for an ideal gas are related by the expression

$$\frac{p}{\rho^k} = \text{Constant}$$

Note:
(i) Notation a and c have been used for velocity of sound.
(ii) Notation γ and k have been used for ratio of specific heats.

Example 1.1 Explain Prandtl velocity ellipse or adiabatic steady flow ellipse with the help of figure.

Solution:

Let us consider one dimensional steady flow through a stream tube. If there is no work and heat transfer and no elevation change, then the SFEE reduces to its simple adiabatic form as follows:

$$h + \frac{V^2}{2} = \text{Constant} = h_0$$

$$c_p T + \frac{V^2}{2} = c_p T_0 \qquad \text{(For perfect gas)}$$

$$\frac{\gamma}{\gamma - 1} RT + \frac{V^2}{2} = \frac{V^2}{2} = \frac{\gamma}{\gamma - 1} RT_0$$

From the definition of velocity of sound, $\gamma RT = a^2$ and $\gamma RT_0 = a_0^2$. The equation thus becomes,

$$\frac{a^2}{\gamma - 1} + \frac{V^2}{2} = \frac{a_0^2}{\gamma - 1} \qquad \text{... (i)}$$

We can note that all terms in above equation are kinetic energy terms, the equation is said to be in kinetic form and is called the kinetic form of adiabatic energy equation. This equation is highly useful since it gives the relation between speed of and flow velocity for a constant stagnation enthalpy flow.

The maximum kinetic energy and thus the maximum velocity, associated with such a flow would occur if all the thermal energy ($C_p T$) of the flow is converted to kinetic energy. When the whole thermal energy is extracted, the temperature of fluid becomes zero and hence the velocity of sound in the medium also becomes zero. Thus substituting $a = 0$ in above equation (i)

$$\frac{a_0^2}{\gamma - 1} + \frac{V^2}{2} = \frac{V_{max}^2}{2} = \text{constant} \qquad \text{... (ii)}$$

Les us consider a section where the flow Mach number is unity. Such a section is critical section.

$$V^* = a^* \text{ for } M = 1.$$

$$\frac{a^2}{\gamma - 1} + \frac{V^2}{2} = \frac{a^{*2}}{\gamma - 1} + \frac{a^{*2}}{2} = \text{constant}$$

$$= \frac{\gamma + 1}{2(\gamma - 1)} a^{*2} = \text{constant} \qquad \text{... (iii)}$$

Using equations (i), (ii) and (iii), we can write

$$\frac{a^2}{\gamma - 1} + \frac{V^2}{2} = \frac{V_{max}^2}{2} = \frac{a_0^2}{\gamma - 1} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2} = \text{constant} \qquad \text{... (iv)}$$

This equation (iv), is full form of adiabatic steady flow energy equation. It shows the maximum velocity condition when the velocity of sound is zero and the maximum sound velocity condition when the fluid velocity is zero. We can observe that equation (iv) is the equation for an ellipse with coordinates a and V. Since only positive values of a and V are practically possible, a first quadrant ellipse is obtained by plotting the equation in a-V coordinates. The ellipse is called Prandtl velocity ellipse or adiabatic steady flow ellipse and is a useful device for examining the relationship between flow velocity and speed of sound. The ellipse plotted for $\gamma = 1.4$ is shown below. Five regimes of flow are indicated on the figure.