

Heat Transfer

Mechanical Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination



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Heat Transfer

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First Edition : 2019

Revised Edition : 2019

Reprint : 2020

Reprint : 2021

Reprint : 2022

Reprint : 2023

Reprint : 2024

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Introduction and Basic Concepts

1.1 Introduction

Heat transfer is the science that seeks to predict the energy transfer that may take place between material bodies as a result of temperature difference.

- The definition of **heat** is provided by classical thermodynamics. It is defined as an energy that flows due to difference in temperature.
- Heat flows in a direction from higher temperature to lower temperature.
- Heat energy can neither be observed nor be measured directly. However, the effects produced by the transfer of this energy are amenable to observations and measurements.

1.1.1 Thermodynamics and Heat Transfer

The science of heat transfer is concerned with the calculation of the rate at which heat flows within a medium, across an interface, or from one surface to another, and the associated temperature distribution. Thermodynamics deals with systems in equilibrium and calculates the energy transferred to change a system from one equilibrium state to another. However, it cannot tell the duration for which heat has to flow to change that state of equilibrium. For example, if 1 kg ingot of iron is quenched from 1000°C to 100°C in an oil bath, thermodynamics tells us that the loss in internal energy of the ingot is

$$mc\Delta T = 1 \text{ kg} \times 0.45 \frac{\text{kJ}}{\text{kg}} \times 900\text{K} = 405 \text{ kJ}$$

But thermodynamics cannot tell us about the time required for the temperature to drop to 100°C. The time depends on various factors such as the temperature of the oil bath, physical properties of the oil, motion of the oil etc. An appropriate heat transfer analysis considers all these factors. Analysis of heat transfer processes requires some concepts of thermodynamics.

The second law of thermodynamics states that if two bodies at temperatures T_1 and T_2 are connected, and if $T_1 > T_2$, then heat will flow spontaneously and irreversibly from body 1 to body 2, causing entropy increase of the universe or entropy generation. Since, all heat transfer processes occur through finite temperature differences overcoming thermal irreversibility, the heat transfer area or operating variables can be optimized in regard to two or more irreversibilities following the principle of minimization of entropy generation or exergy destruction.

1.2 Modes of Heat Transfer

Heat can be transferred in three different modes: conduction, convection and radiation.

1.2.1 Conduction

The mechanism of heat transfer due to a temperature gradient in a stationary medium is called conduction. The medium may be a solid or a fluid. In liquids and gases, conduction is due to the collisions of molecules in course of their random motions. In solids, the conduction of heat is attributed to two effects :

- (i) the flow of free electrons and
- (ii) the lattice vibrational waves caused by the vibrational motions of the molecules at relatively fixed positions called a lattice.

The law which describes the rate of heat transfer in conduction is known as Fourier's law.

According to Fourier's law,

$$q_x = -k \frac{dT}{dx} \quad \dots(1.1)$$

- where q_x is the rate of heat flow per m^2 of heat area normal to the direction of heat flow.
- The minus sign in Eq. (1.1) indicates that heat flows in the direction of decreasing temperature.
- The constant k is known as thermal conductivity.

When the temperature becomes a function of three space coordinates, say, x, y, z in a rectangular Cartesian frame, heat flows along the three coordinate directions. Equation (1.1) under the situation, is written in vector form as

$$q = -k \nabla T \quad \dots(1.2)$$

where,

$$q = iq_x + jq_y + kq_z$$

and,

$$\nabla T = i \frac{\partial T}{\partial x} + j \frac{\partial T}{\partial y} + k \frac{\partial T}{\partial z}$$

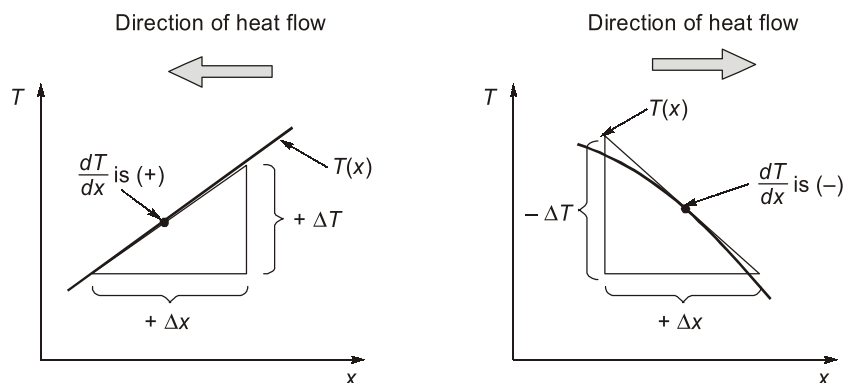
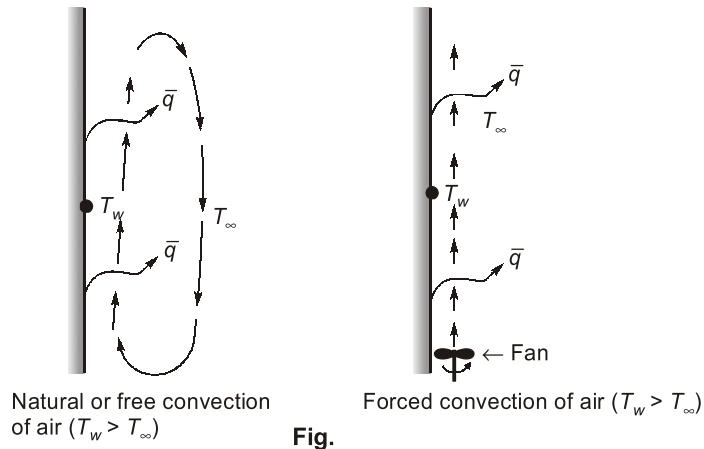


Fig. Sign convention for conduction heat flow

1.2.2 Convection heat transfer

Convection is a process involving mass movement of fluids. When a temperature difference produces a density difference which results in mass movement, the process is called free or natural convection. Here the plate is maintained isothermal at temperature T_w , which is higher than the surrounding fluid temperature T_∞ . The fluid near the wall, on getting heated, moves up due to the effect of buoyancy, and is replaced by the cold fluid moving towards the wall. Thus a circulation current is set up due to the density difference.

When the mass motion of the fluid is caused by an external device like a pump, compressor, blower or fan, the process is called forced convection. Here the fluid is made to flow along the hot surface due to the pressure difference generated by the device and heat is transferred from the wall to the fluid.



Convection process is governed by Newton's law of cooling which states that "The rate of heat transfer by convection between a solid body and the surrounding fluid is directly proportional to the temperature difference between them and is also directly proportional to the area of contact or area of exposure between them.

$$q = hA_s\Delta T$$

where: q = heat flow from surface, a scalar (W)

h = heat transfer coefficient

A_s = Surface area from which convection is occurring

$\Delta T = T_s - T_\infty$ = Temperature difference between surface and coolant.

- h is the heat transfer coefficient, which is not a thermodynamic property of the material, but may depend on geometry of surface, flow characteristics, thermodynamic properties of the fluid etc.
- Typical values of h (W/m²K)

Free convection	Gases : 2 - 25
	Liquid : 50 - 100
Forced convection	Gases : 25 - 250
	Liquid : 50 - 20,000
Boiling/condensation	2500 - 100,000

Whether the convection process is natural or forced, there is always a boundary layer adjacent to the wall where the velocity and temperature vary from the wall to the free stream. Figure shows the velocity and temperature boundary layers for forced flow over a hot horizontal surface. Ludwig Prandtl suggested that the field of flow can be divided into two regions: a thin layer next to the wall, which is called the boundary layer where the shear stress is confined, and the region outside this layer, where the fluid is "ideal" i.e., nonviscous and incompressible.

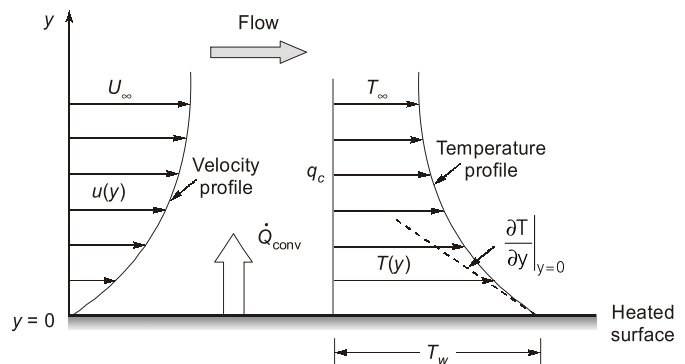


Fig. Velocity and temperature profiles for forced convection heat transfer

Hydrodynamic boundary layer is that region of the flow where viscous forces are present, thermal boundary layer may be defined as the region where temperature gradients are present in the flow.

Hydrodynamic boundary layer thickness (δ) is taken upto where $u = 0.99 v_\infty$, and thermal boundary layer thickness (δ_t) where $T_w = T_\infty$. Generally δ_t is not equal to δ .

The thermal boundary layer is regarded as consisting of a stationary fluid film through which heat is conducted and then it is transported by fluid motion. The rate of convection heat transfer from the wall to the fluid.

$$Q_c = -k_f A \frac{T_\infty - T_w}{\delta_t}$$

where k_f is the thermal conductivity of the fluid film.

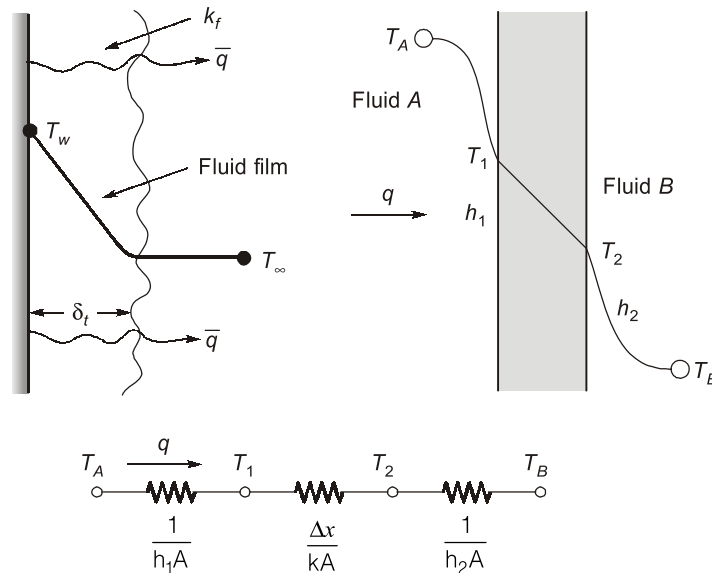


Fig. Heat transfer through a stationary fluid film

The film or surface coefficient of heat transfer h_c may be defined as

$$Q_c = -k_f A \left(\frac{\partial T}{\partial t} \right)_{y=0} = h_c A (T_w - T_\infty)$$

1.2.3 Radiation

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as result of the changes in the electronic configurations of the atoms or molecules.

The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s (in K) is given by the Stefan-Boltzmann law as

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4 \quad \dots(1.4)$$

where $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ is the Stefan-Boltzmann constant.

The idealized surface that emits radiation at this maximum rate is called as black body, and the radiation emitted by a black body is called black body radiation.

The radiation emitted by all real surface is less than the radiation emitted by a black body at the same temperature, and is expressed as

$$\dot{Q}_{\text{emit}} = \epsilon \sigma A_s T_s^4 \quad \dots(1.5)$$

where ϵ is the emissivity of the surface. The property emissivity, whose value is in the range $0 \leq \epsilon \leq 1$, is a measure of how closely a surface approximates a black body.

1.3 Thermal Conductivity

Thermal conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. The thermal conductivity of a material is a measure of the ability of the material to conduct heat. A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator. The thermal conductivities of some common materials at room temperature are given in Table.

The thermal conductivity, k can be defined by Fourier law, equation.

$$k = -\frac{(Q/A)}{(dT/dx)}$$

This equation is used for determination of thermal conductivity of a material. A layer of solid material of thickness L and area A is heated from one side by an electric resistance heater as shown in figure.

If the outer surface of heater is perfectly insulated, then all the heat generated by resistance heater will be transferred through the exposed layer of material. When steady state condition is reached, the temperature of two surface of material T_1 and T_2 are measured and thermal conductivity of material is determined by relation.

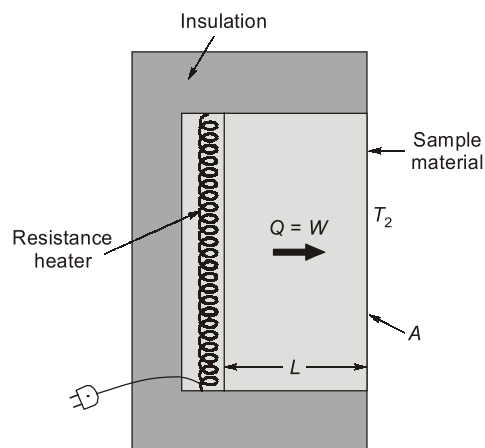


Fig. Experimental set-up for determination of thermal conductivity

Table: Thermal conductivity of some materials at room temperature (300 K)

Material	k(W/(m°C))	Material	k(W/(m°C))
Diamond	2300	Mercury (liquid)	8.54
Silver	429	Glass	0.78
Copper	401	Brick	0.72
Gold	317	Water (liquid)	0.613
Aluminium	237	Human skin	0.37
Iron	80.2	Wood (oak)	0.17
Helium (g)	0.152	Glass fibre	0.043
Soft rubber	0.13	Air (g)	0.026
Refrigerant-12	0.072	Urethane, rigid foam	0.026

1.3.1 Solids

In solids, heat conduction is due to two effects - **flow of free electrons** and **propagation of lattice vibrational waves**. The thermal conductivity is therefore determined In the addition of these two components. In a pure metal, the electronic component is more prominent than the component of lattice vibration and gives rise to a very high value of thermal conductivity. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. Highly ordered crystalline non-metallic solids like diamond, silicon, quartz exhibit very high thermal conductivities (more than that of pure metals) due to lattice vibration only, but are poor conductors of electricity.

Table : The comparison of thermal conductivities of metallic alloys with those of constituting pure metals

Pure metal or alloy	k (W/(m°C))
Copper	401
Nickel	91
Constantan	(55% Cu, 45% Ni)23
Aluminium	237
Commercial bronze	(90% Cu, 10% Al)52

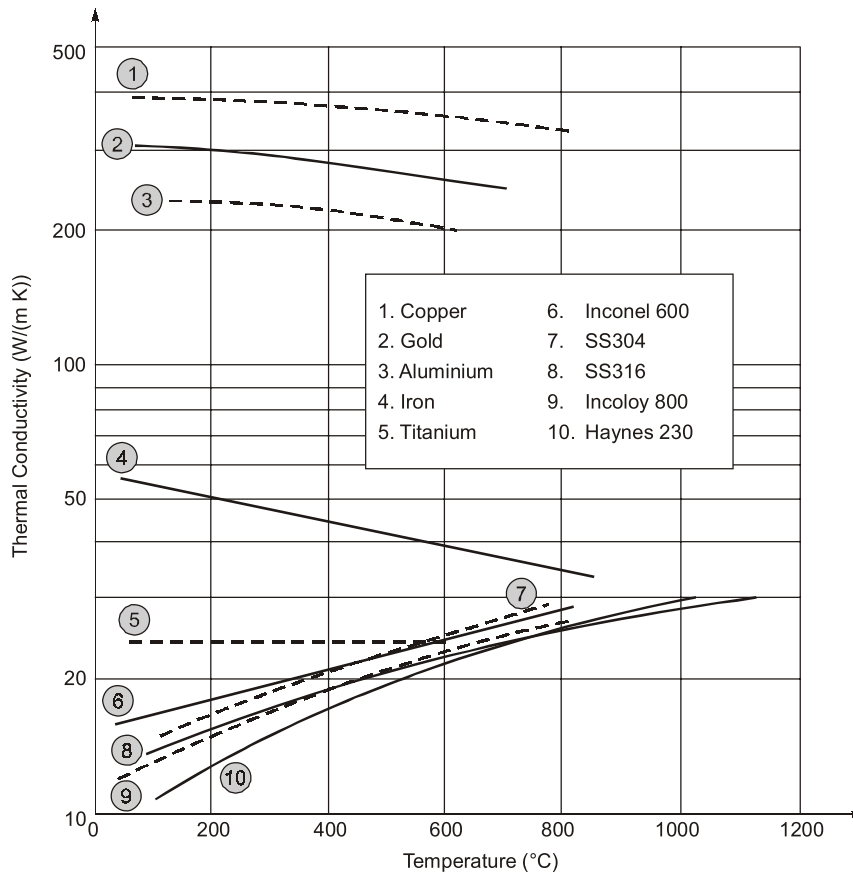


Fig. The variation of thermal conductivity with temperature for typical metals and their alloys

1.3.2 Liquids and Gases

The thermal conductivity for liquids and gases is attributed to the transfer of kinetic energy between the randomly moving molecules due to their collisions. The kinetic theory of gases predicts and the experiments confirm that the thermal conductivity of gases is proportional to the square root of the thermodynamic temperature T , and inversely proportional to the square root of the molar mass M . Therefore, the thermal conductivity of a gas increases with increasing temperature and decreasing molar mass.

Unlike gases, the thermal conductivities of most liquids decrease with increasing temperature, with water being a notable exception. Like gases, the conductivity of liquids decreases with increasing molar mass.

1.4 Simultaneous Heat Transfer

We mentioned that there are three mechanisms of heat transfer, but not all three can exist simultaneously in a medium. For example, heat transfer is only by conduction in opaque solids, but by conduction and radiation in semitransparent solids. Thus, a solid may involve conduction and radiation but not convection. However, a solid may involve heat transfer by convection and/or radiation on its surfaces exposed to a fluid or other surfaces. For example, the outer surface of a cold piece of rock will warm up in a warmer environment as a result of heat gain by convection (from the air) and radiation (from the sun or the warmer surrounding surfaces). But the inner parts of the rock will warm up as this heat is transferred to the conduction and possibly by radiation in a still fluid (no bulk fluid motion) and by convection and radiation in a flowing fluid. In the absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion.

Convection can be viewed as combined conduction and fluid motion, and conduction in a fluid can be viewed as a special case of convection in the absence of any fluid motion.

Thus, when we deal with heat transfer through a fluid, we have either conduction or convection, but not both. Also, gases are practically transparent to radiation, except that some gases are known to absorb radiation strongly absorbs ultraviolet radiation. But in most cases, a gas between two solid surfaces does not interfere with radiation and acts effectively as a vacuum. Liquids, on the other hand, are usually strong absorbers of radiation.

Finally, heat transfer through a vacuum is by radiation only since conduction or convection requires the presence of a material medium.

1.4.1 Combined Convection and Radiation

Heat is transferred from a hot body both by natural convection and radiation. Rate of heat transfer by natural convection,

$$Q_c = h_c A (T_w - T_\infty)$$

and that by radiation

$$Q_r = \sigma A_1 F_{1-2} (T_w^4 - T_\infty^4) = h_r A_1 (T_w - T_\infty)$$

where h_r is known as the radiation heat transfer coefficient (W/m^2K).

$$h_r = \frac{1}{h_r A_1} = \frac{1}{\sigma A_1 F_{1-2} (T_w + T_\infty) (T_w^2 + T_\infty^2)}$$

The total rate of the heat transfer by convection and radiation, which occur in parallel, is

$$Q = Q_c + Q_r = (h_c + h_r) A_1 (T_w - T_\infty)$$

The equivalent physical system and thermal circuit for heat transfer between two bodies 1 and 2 are shown in figure.

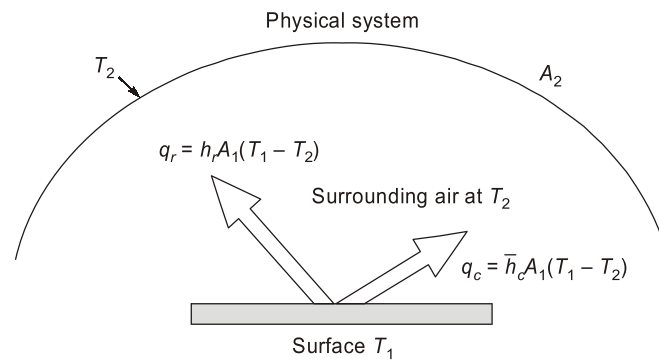


Fig.

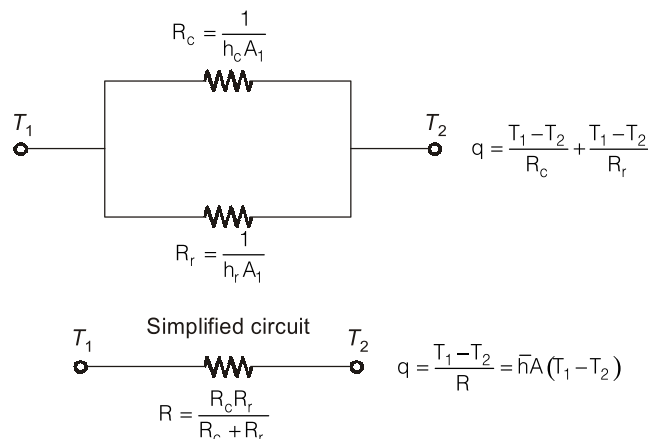


Fig. Thermal circuit with convection and radiation acting in parallel

1.4.2 Combined conduction, convection and radiation

It is easy to envision cases in which all three modes of heat transfer are present, as shown in figure. In this case the heat conducted through the plate is removed from the plate surface by a combination of convection and radiation. An energy balance would give

$$-kA \left. \frac{dT}{dy} \right|_{\text{wall}} = hA(T_w - T_\infty) + \sigma AF_{1-2}(T_w^4 - T_s^4)$$

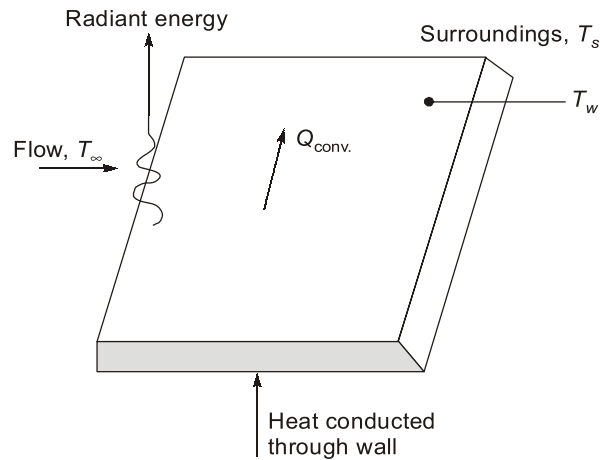


Fig. Combination of conduction, convection and radiation heat transfer

where $Q_{\text{conv.}} = hA(T_w - T_\infty)$, T_w = wall surface temperature, T_∞ = fluid temperature and T_s = temperature of surroundings.

1.5 Electrical analogy of heat transfer rate through a plane wall

The equation for rate of heat conduction through a plane wall can be rearranged as

$$Q = \frac{T_1 - T_2}{\frac{L}{kA}} = \frac{\Delta T}{\frac{L}{kA}}$$

In this equation, the temperature difference, ΔT on two sides of the wall is driving potential, that causes flow of heat. The term L/kA is the quantity, which opposes the heat flow in the material and it is equivalent to a thermal resistance R_{th} of wall. It is also called conduction resistance of wall. Then eq. can be rearranged as

$$Q = \frac{\Delta T}{R_{\text{th}}} = \frac{\Delta T}{R_{\text{wall}}} \text{ W}$$

where,

$$R_{\text{th}} = R_{\text{wall}} = \frac{L}{kA} \text{ } ^\circ\text{C/W or K/W}$$

There is an analogy between a heat flow system and an electric current flow system, where current I is expressed as

$$\text{Current, } I = \frac{V_1 - V_2}{R_e} = \frac{\text{Potential difference}}{\text{Electrical resistance}}$$

where R_e is electric resistance and it is expressed as

$$R_e = \frac{\rho L}{A_c} = \frac{\text{Resistivity of the material} \times \text{Conductor length}}{\text{Cross-section area of conductor}}$$

and the potential difference or voltage difference across the resistances that the rate of heat transfer Q , through a layer analogous to an electric resistance R_e and the temperature difference ΔT analogous to voltage difference ΔV . Such comparison is referred as electrical analogy of rate of heat transfer through a plane wall.

The analogous quantities in the expression are,

$$\Delta V \Rightarrow \Delta T$$

$$I \Rightarrow Q,$$

$$R_e \Rightarrow \frac{L}{kA}$$

Similarly the convection heat transfer given by equation can be rearranged as

$$Q = \frac{T_s - T_\infty}{\frac{1}{hA}} = \frac{\Delta T}{R_{\text{conv}}}$$

where,

$$R_{\text{conv}} = \frac{1}{hA} \text{ } ^\circ\text{C/W or K/W}$$

The R_{conv} is a thermal resistance acts between the surface and its surroundings against the convection heat transfer, thus it is called convection resistance or film resistance or thermal resistance for convection.

Thermal conductance K_c is defined as the reciprocal of thermal resistance and is expressed as

$$K_c = \frac{kA}{L}$$

It is equal to the rate of heat transfer through a solid of area a and thickness l per degree temperature difference.

Example 1.1

An exterior wall of a house may be approximated by a 10 cm-layer of common brick [$K = 0.7 \text{ W/m}\cdot^\circ\text{C}$] followed by a 3.75 cm layer of gypsum plaster [$K = 0.48 \text{ W/m}\cdot^\circ\text{C}$]. What thickness of loosely packed rock wool insulation [$K = 0.065 \text{ W/m}\cdot^\circ\text{C}$] should be added to reduce the heat loss (or gain) through the wall by 80 percent?

Solution:

Assumption; Steady, 1-dimensional heat transfer and all the properties remains constant.

∴ The overall heat loss will be given by

$$q = \frac{\Delta T}{\Sigma R_{th}}$$

Because the heat loss with the rock-wool insulation will be only 20 percent (80 percent reduction) of that before insulation.

$$\frac{q_{\text{with insulation}}}{q_{\text{without insulation}}} = 0.2 = \frac{\Sigma R_{th \text{ without insulation}}}{\Sigma R_{th \text{ with insulation}}}$$

we have for the brick and plaster, for unit area,

$$R_b = \frac{\Delta x}{k} = \frac{0.1}{0.7} = 0.143 \text{ m}^2\cdot^\circ\text{C/W}$$

$$R_p = \frac{\Delta x}{k} = \frac{0.0375}{0.48} = 0.078 \text{ m}^2\cdot^\circ\text{C/W}$$

So that thermal resistance without insulation is

$$R = 0.143 + 0.078 = 0.221 \text{ m}^2\cdot^\circ\text{C/W}$$

Then

$$R_{\text{with insulation}} = \frac{0.221}{0.2} = 1.105 \text{ m}^2\cdot^\circ\text{C/W}$$

and this represents the sum of our previous value and the resistance for the rock wool.

$$1.105 = 0.221 + R_{rw}$$

$$R_{rw} = 0.884 = \frac{\Delta x}{K} = \frac{\Delta x}{0.065}$$

$$\Delta x_{rw} = 0.0575 \text{ m}$$

$$= 5.75 \text{ cm}$$

Consider a plane wall of thickness L , exposed on its both sides to two different environments at temperatures $T_{\infty 1}$, and $T_{\infty 2}$ with heat transfer coefficient h_1 and h_2 , respectively as shown in figure.

The steady state heat transfer rate through the wall, when its two surfaces are maintained at constant temperatures, T_1 and T_2 can be expressed as

When the left face and right face involve convection heat transfer due to temperature difference between surface and surroundings.

The convection heat transfer rate at the left face exposed to environment at $T_{\infty 1}$,

$$Q_2 = h_1 A (T_{\infty 1} - T_1)$$

The convection heat transfer rate at the right face exposed to environment at $T_{\infty 2}$,

$$Q_3 = h_2 A (T_{\infty 2} - T_2)$$

In steady state conditions, the heat transfer rate remains constant;

Thus

$$Q_1 = Q_2 = Q_3 = Q \text{ (say)}$$

Then

$$Q = h_1 A (T_{\infty 1} - T_1)$$

$$= \frac{kA(T_1 - T_2)}{L} = h_2 A (T_2 - T_{\infty 2})$$

It can be modified as

$$Q = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv},2}}$$

where,

$$R_{\text{conv},1} = \frac{1}{h_1 A}, R_{\text{wall}} = \frac{L}{kA}, R_{\text{conv},2} = \frac{1}{h_2 A}$$

The thermal circuit representation provides a useful tool for the analysis of heat transfer problems. The equivalent thermal resistance for a plane wall with convection on both sides is shown in figure. Since these three resistances are in series, therefore, the total thermal resistance ΣR_{th} is sum of the series resistances as in electrical network.

$$\Sigma R_{\text{th}} = R_{\text{conv},1} + R_{\text{wall}} + R_{\text{conv},2}$$

or

$$\Sigma R_{\text{th}} = R_{\text{total}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

and overall temperature difference, $(\Delta T)_{\text{overall}} = T_{\infty 1} - T_{\infty 2}$.

Therefore, heat current,

$$Q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(\Delta T)_{\text{overall}}}{\Sigma R_{\text{th}}}$$

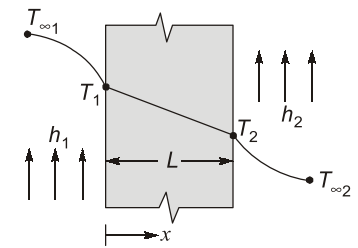


Fig.

1.6 Overall Heat Transfer Coefficient

The problem largely encountered in engineering practice is heat being transferred between two fluids of specified temperatures separated by a wall. In such a situation the surface temperatures are not known, but they can be calculated if the convection heat transfer coefficients on both sides of the wall are known.

For a composite wall with three different layers in series.

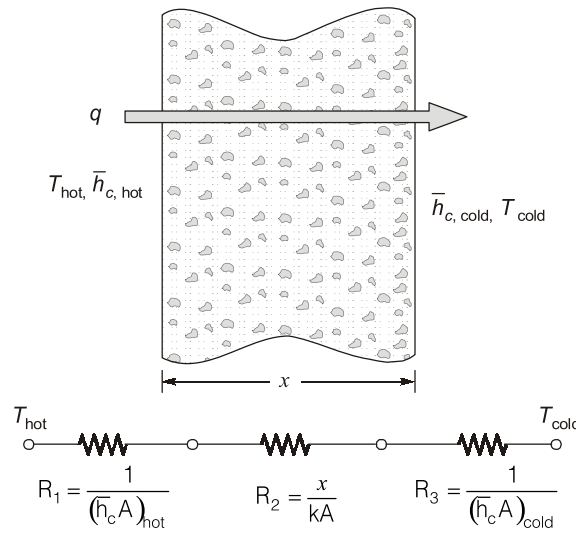


Fig. Thermal circuit with conduction and convection in series

There are three resistances in series:

$$R = R_1 + R_2 + R_3 = \frac{1}{h_{c,1}A} + \frac{x}{kA} + \frac{1}{h_{c,2}A}$$

Now,

$$Q_c = \frac{T_h - T_c}{R} = \frac{T_h - T_c}{\frac{1}{h_{c,1}A} + \frac{x}{kA} + \frac{1}{h_{c,2}A}} = UA(T_h - T_c)$$

where U is known as the overall heat transfer coefficient (W/m^2K) and is given by

$$\frac{1}{UA} = \frac{1}{h_{c,1}A} + \frac{x}{kA} + \frac{1}{h_{c,2}A}$$

or

$$\frac{1}{U} = \frac{1}{h_{c,1}} + \frac{x}{k} + \frac{1}{h_{c,2}}$$

$$\frac{1}{UA} = \frac{1}{h_{c,hot}A} + \frac{L_1}{k_1A} + \frac{L_2}{k_2A} + \frac{L_3}{k_3A} + \frac{1}{k_{c,cold}A}$$

and

$$Q = UA(T_h - T_c)$$

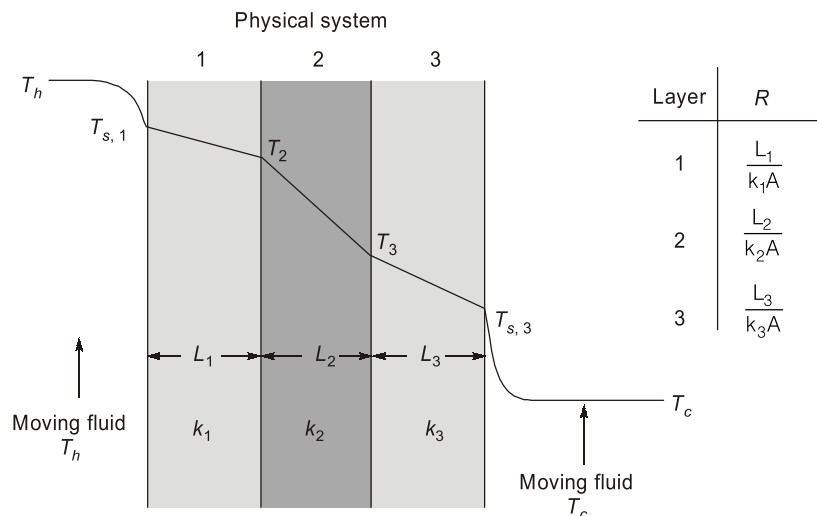


Fig. Thermal circuit

Similarly, for heat transfer from a hot fluid inside a cylinder to the cold fluid outside.

$$Q_c = \frac{T_h - T_c}{R_1 + R_2 + R_3} = \frac{T_h - T_c}{\frac{1}{h_i A_i} + \frac{x_w}{k_w A_{1m}} + \frac{1}{h_o A_o}} = U_o A_o (T_h - T_c)$$

U_o being the overall heat transfer coefficient based on the outside surface area A_o , h_i the inside heat transfer coefficient and h_o the outside heat transfer coefficient.

Now,
$$T_h - T_1 = Q_c R_1 = Q_c \frac{1}{h_i A_i}$$

$$T_1 - T_2 = Q_c R_2 = Q_c \frac{x_w}{k_w A_{1m}}$$

$$T_1 - T_c = Q_c R_3 = Q_c \frac{1}{h_o A_o}$$

From which the interface temperatures T_1 and T_2 can be estimated.

When the wall thickness x_w is small,

$$A_o = A_{1m} = A_i$$

Then
$$\frac{1}{U_o} = \frac{1}{h_i} + \frac{x_w}{k_w} + \frac{1}{h_o} = \frac{1}{U_i}$$

where U_i is the overall heat transfer coefficient based on the inside surface area A_i . It may be noted that

$$U_o A_o = U_i A_i$$

If more resistances are put in series, these are to be added up and the same procedure will follow.

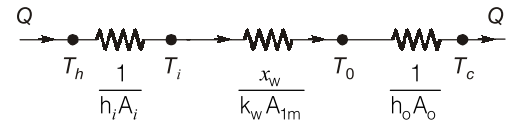
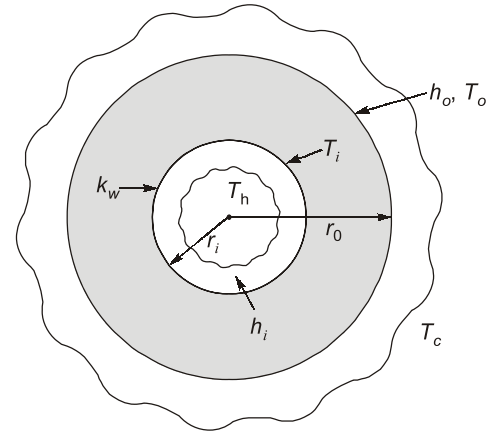


Fig. Radial heat transfer from a hot to a cold fluid through a cylindrical wall

Example 1.2

Water flows at 50°C inside a 2.5 cm inside diameter tube such that $h_i = 3500 \text{ W/m}^2\text{C}$. The tube has a wall thickness of 0.8 mm with a thermal conductivity of $16 \text{ W/m}^2\text{C}$. The outside of the tube loses heat by free convection with $h_o = 7.6 \text{ W/m}^2\text{C}$. Calculate the overall heat transfer coefficient and heat loss per unit length to surrounding air at 20°C .

Solution:

Assumptions: Steady heat transfer, properties remains constant.

This problems involves both conduction and convection.

\therefore There are three resistances.

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3500)\pi(0.025)(1.0)} = 0.00364^\circ\text{C/W}$$

$$R_t = \frac{\ln(d_o/d_i)}{2\pi kL} = \frac{\ln(0.0266/0.025)}{2\pi(16)(1.0)} = 0.00062^\circ\text{C/W}$$

$$[\because d_o = 0.025 + 2(0.0008) = 0.0266 \text{ m}]$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(7.6)\pi(0.0266)(1.0)} = 1.575^\circ\text{C/W}$$

From above three resistance calculations it can be interpreted that controlling resistance for the total heat transfer the other resistances (in series) are negligible in comparison.