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Measurements & Instrumentation

Electrical Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016
E-mail: infomep@madeeasy.in
Contact: 9021300500
Visit us at: www.madeeasypublications.org

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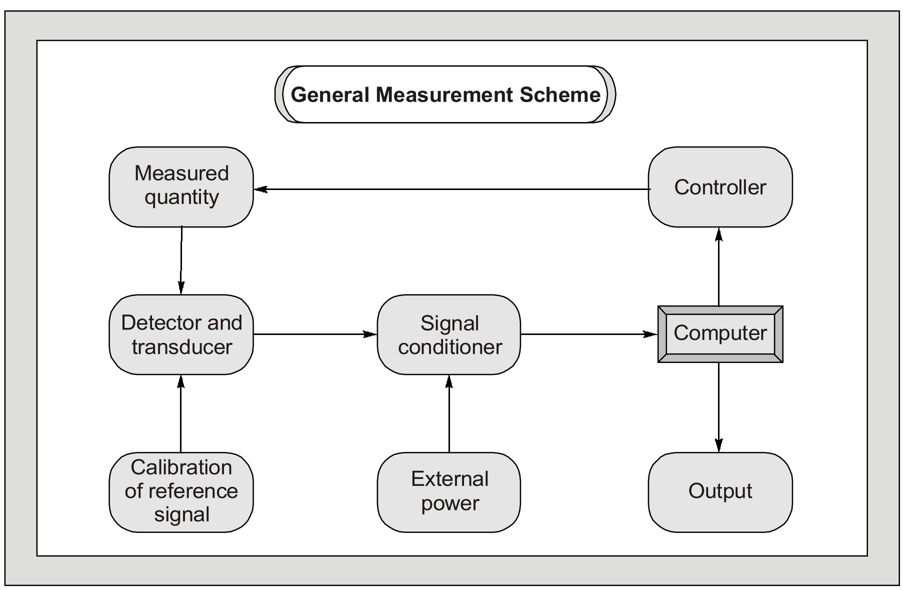
Introduction to Measurements & Instrumentation

Measurement and instrumentation systems have wide applications such as measurement of electrical and physical quantities like current, voltage, power, temperature, pressure, displacement etc.

The need for measurement arises when one wants to generate data for design or when one wants to propose a theory based on a set of measurement and instrumentation for commerce.

The measurement and instrumentation systems can also be used to locate things or events. Like employees present in a building, the epicenter of an earthquake. Sometimes, measurement systems are made a part of control system. One can observe the change in the field of measurement and instrumentation due to the introduction of new standards, and sensors.



This course on instrumentation and measurement is intended to make the engineers familiar about the art of modern instrumentation and measurement systems. It is well suited for classroom courses of engineering as well as for various competitive examinations.

Equal importance has been provided to both theory as well as problems with illustrative examples after every topic. It has been tried to cover every topic so that even a beginner understands it easily to excel in the subject of measurement and instrumentation.

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01
CHAPTER

Introduction

1.1 MEASUREMENTS AND IT'S SIGNIFICANCE

The measurement of a given quantity is essentially an act or the result of comparison between the quantity (whose magnitude is unknown) and a predefined standard. Measurement is the process by which one can convert physical parameters to meaningful numbers. The measuring process is one in which the property of an object or system under consideration is compared to an accepted standard unit, a standard defined for that particular property. For the result of the measurement to be meaningful, the standard used for comparison purposes must be accurately defined and should be commonly accepted. Also, the apparatus used and the method adopted must be provable. The importance of measurement is simply expressed in the following statement of the famous physicist "Lord Kelvin":

"I often say that when you can measure what you are speaking about and can express it in numbers, you know something about it; when you can't express it in numbers your knowledge is of a meager and unsatisfactory kind."

1.1.1 Methods of Measurement

Direct Measurement

- In this method, the measured or the unknown quantity is directly compared against a standard.
- This method of measurement sometimes produces human errors and hence gives inaccurate results.

Indirect Measurement

- In this method, direct measurement of some other quantity is done which leads to measurement of the main quantity. ex; measuring wire elongation for measuring heating effect of current.
- This method of measurement is more accurate and more sensitive.
- These are more preferred over direct measurement.

1.1.2 Nature of Measuring Instruments

Mechanical

- These instruments are used for stable and static conditions:
- They are unable to respond rapidly to measurements of dynamic and transient conditions because of having moving parts that are bulky, heavy and rigid possessing high inertia.

Electrical

Electrical methods of indicating the output of detectors are more rapid than mechanical methods, but they are having limited time response characteristics.

Electronic

These instruments require use of semiconductor devices. The response time of these instruments are extremely small as a very small inertia of electron is only involved. The sensitivity of these instruments is also very high. Faster response, lower weight, lower power consumption are some of the advantages of electronic instruments.

1.2 TYPES OF MEASURING INSTRUMENTS

1.2.1 Absolute Instruments

These instruments give the magnitude of the quantity under measurement in terms of physical constants of the instruments e.g. Tangent Galvanometer, Rayleigh's current balance. It is not necessary to calibrate them and there is no need to compare them with the other standard instruments.

1.2.2 Secondary Instruments

In these type of instruments, the quantity being measured can only be measured by observing the output indicated by the instrument. These instruments are calibrated by comparing with an absolute instrument. They are utilized in day to day life, and are standardized using absolute instruments.

1.3 DEFLECTION AND NULL TYPE INSTRUMENTS

1.3.1 Deflection Type

The deflection of the instrument provides a basis for determining the quantity under measurement i.e. PMMC Ammeter, Electrodynamometer and moving iron instruments. They are less accurate, less sensitive, but have faster response.

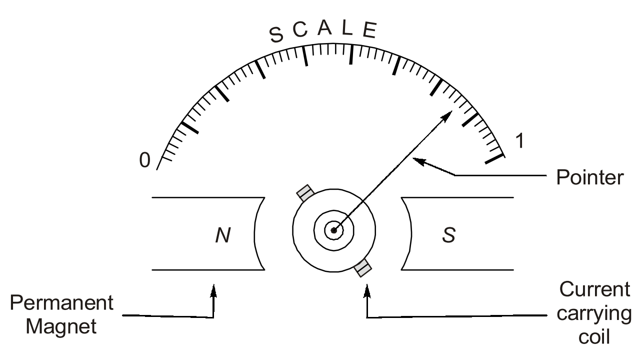


***Fig. 1.1** PMMC (Deflection Type Instrument)*

1.3.2 Null Type Instruments

In null type instruments, a zero or null indication leads to determination of the magnitude of measured quantity. Null type instruments are more accurate, highly sensitive and are less suited for measurements under dynamic conditions than deflection type instruments. Sometimes due to various factors; it is difficult to obtain null deflection. Due to this problem, they are less preferred than deflecting type or electronic meters.

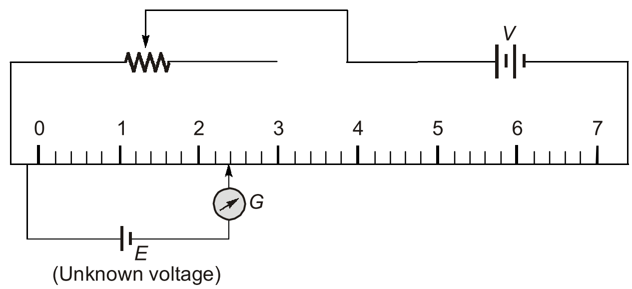


***Fig. 1.2 :** Null Type Instrument*

1.3.3 Calibration

The calibration of all instruments is important since it affords the opportunity to check the instrument against a known standard and subsequently to find errors and accuracy. Calibration procedures involve a comparison of the particular instrument with a primary standard or, a secondary standard or, an instrument of known accuracy.

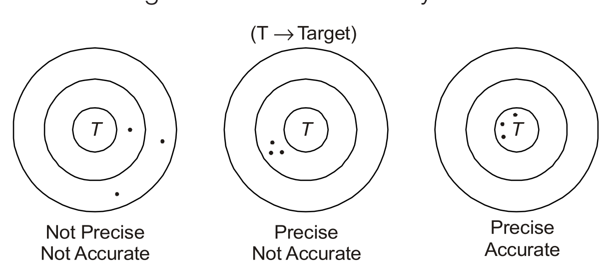
1.4 CHARACTERISTICS OF INSTRUMENT AND MEASUREMENT SYSTEMS

1.4.1 Accuracy

- It is the closeness with which an instrument reading approaches the true value of the quantity being measured.
- The accuracy can be specified in terms of inaccuracy or limits of error.
- The best way to conceive the idea of accuracy is to specify it in terms of the true value of the quantity being measured.
- The accuracy of a measurement means conformity to truth.

1.4.2 Precision

- It is a measure of the reproducibility of the measurements i.e. given a fixed value of a variable, precision is a measure of the degree to which successive measurements differ from one another.
- The term "Precise" means clearly or sharply defined.
- Precision is used in measurements to describe the consistency or the reproducibility of results.
- Precision instruments are not guaranteed for accuracy.



- Precision depends upon number of significant figures.
- The more are significant figures the more is precision.
- Significant figures convey actual information regarding the magnitude and the measurement precision of a quantity.

Example: 302 A (Number of significant figures = 3)
302.10 V (Number of significant figures = 5)
0.000030 Ω (Number of significant figures = 2)

EXAMPLE - 1.1

In calculating voltage drop, a current of 4.37 A is recorded in a resistance of 31.27 Ω. Calculate the voltage drop across the resistor to the appropriate number of significant figures.

Solution:

Current, $I = 4.37A$ (3 significant figures)

Resistance, $R = 31.27\Omega$ (4 significant figures)

Voltage drop, $V = IR = 4.37 \times 31.27 = 136.6499$ volt

Since number of significant figures used in multiplication is 3.

So answer can be written only to a maximum of three significant figures i.e. $V = 137$

REMEMBER

- 248 volt ; 0.000248 MV
- $\Rightarrow$ 248.0 volt $\Rightarrow$ More precise than other two.

1.4.3 Linearity

- If the output is proportional to input then, the instrument is called linear.
- Non-linear behaviour of an instrument doesn't essentially lead to inaccuracy.
- Most of the time it is necessary that measurement system component should have linear characteristics. For example, the resistance used in a potentiometer should vary linearly with displacement of the sliding contact in order that the displacement is directly proportional to the sliding contact voltage. Any departure from linearity will result in error in the reading of system.

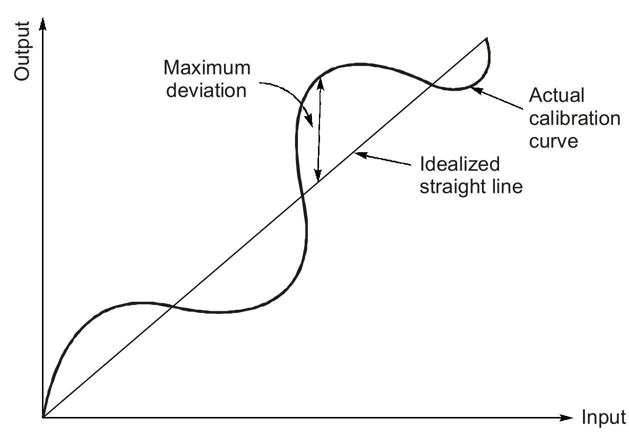


***Fig. 1.3 :** Linearity w.r.t. actual calibration curve and idealized straight line*

1.4.4 Reproducibility

It is the degree of closeness with which a given value may be repeatedly measured. It may be specified in terms of units for a given period of time. The measure of reproducibility leads to measure of precision of an instrument.

1.4.5 Static Sensitivity

- The "static sensitivity" of an instrument is the ratio of the magnitude of the output signal or response to the magnitude of input signal or the quantity being measured.
  It's units are mm/μA; per volts etc. depending upon type of input and output.
- Sometimes the static sensitivity is expressed as the ratio of the magnitude of the measured quantity to the magnitude of the response.

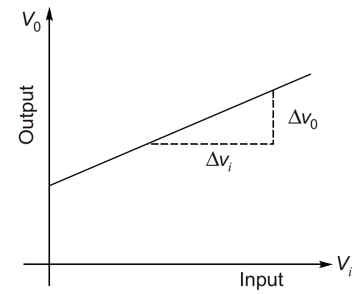


Fig. 1.4 : *Sensitivity*

$$\text{Static Sensitivity} = \frac{\text{Small change in output}}{\text{Small change in input}} = \frac{\Delta V_0}{\Delta V_i}$$

- The sensitivity of an instrument should be high and therefore, instrument should not have a range greatly exceeding the value to be measured.

$$\text{Deflection Factor} = \frac{1}{(\text{Static Sensitivity})}$$

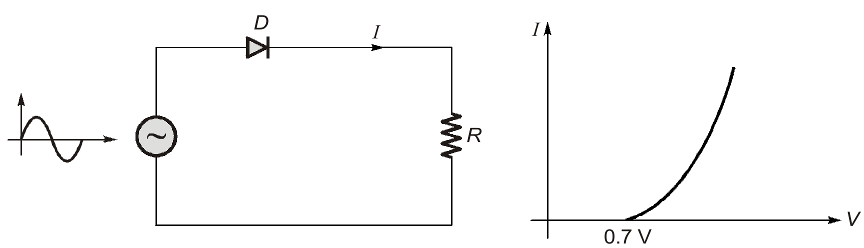
1.4.6 Resolution or Discrimination

- The small measurable input change that can be measured by the instrument is called resolution or discrimination.
- If the input is slowly increased from some arbitrary (non-zero) input value, it will again be found that output doesn't change at all until a certain increment is exceeded. This increment is called resolution.

1.4.7 Dead Time & Dead Zone

Dead Time: The time required for the measurement to begin to respond to the changes in the measurand is known as dead time. It is the time after which the instrument begins to respond after the measured quantity has been changed.

Dead Zone: Dead zone is the largest change of input quantity for which there is no output of the instrument.



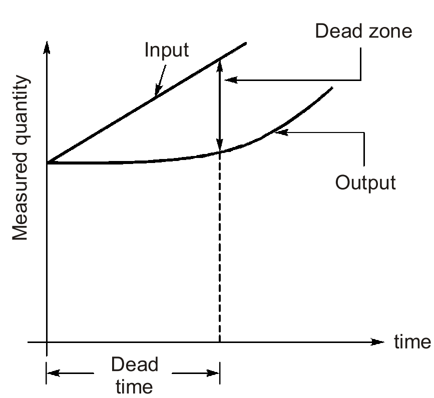


Fig. 1.5 : *Dead Zone and Dead Time*

1.4.8 Signal to Noise Ratio (S/N)

- Noise is an unwanted signal superimposed upon the signal of interest thereby causing a deviation of the output from it's expected value.
- The ratio of desired signal to the unwanted noise is called signal to noise ratio and is expressed as

$$\frac{S}{N} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

- In any measurement system, it is desired to have a large signal-to-noise ratio. This can be achieved by increasing the signal level without increasing the noise level or decreasing the noise level with some suitable technique.

1.5 ERRORS IN MEASUREMENTS AND THEIR ANALYSIS

Measurements done in a laboratory or at some other place always involve errors. No measurement is free from errors. If the precision of the equipment is adequate, no matter what it's accuracy is, a discrepancy will always be observed between two measured results.

1.5.1 True Value

The true value of quantity to be measured may be defined as the average of an infinite number of measured values when the average deviation due to various contributing factors tends to zero.

1.5.2 Guarantee Errors

The accuracy and precision of an instrument depends upon it's design, the material used and the workmanship that goes into making the instrument. Components are guaranteed to be within a certain percentage of the rated value. Thus, the manufacturer has to specify the deviations from the "nominal value" of a particular quantity. The limits of these deviations from the specified value are defined as ***"Limiting Errors"*** or ***"Guarantee Errors"***.

For example, the magnitude of resistance of a resistor is 200 Ω with a limiting error of ±10 Ω. The magnitude of the resistance will be between the limits

$$R = 200 \pm 10\ \Omega$$

or $$R \geq 190\ \Omega$$

and $$R \leq 210\ \Omega$$

Hence, the manufacturer guarantees that the value of resistance of the resistor lies between 190 Ω and 210 Ω.

1.5.3 Absolute (Relative) Limiting Error

The relative (fractional) error is defined as the ratio of the error to the specified (nominal) magnitude of a quantity.

$$\text{Relative limiting error, } \varepsilon_r = \left(\frac{\text{Measured value} - \text{True value}}{\text{True value}}\right) \times 100$$

or, $$\%\ \varepsilon_r = \left(\frac{\text{Actual value} - \text{Nominal value}}{\text{Nominal Value}}\right) \times 100$$

or, $$\%\ \varepsilon_r = \left(\frac{A_m - A_T}{A_T}\right) \times 100 \qquad \begin{cases} A_m = \text{Measured value} \\ A_T = \text{True value} \end{cases}$$

Now, $\% \varepsilon_r = \frac{A_m - A_T}{A_T}$ or $\frac{A_m}{A_T} = 1 + \varepsilon_r$ or $\boxed{\frac{A_T}{A_m} = \frac{1}{1+\varepsilon_r}}$

$$\boxed{A_T = \left(\frac{1}{1+\varepsilon_r}\right) A_m}$$

Here, $\boxed{\frac{1}{1+\varepsilon_r} = \text{Correction factor}}$

REMEMBER Nominal value = True value and Actual value = Measured value

EXAMPLE - 1.2

A resistance has nominal value of 50 Ω. When it is measured it's actual value is 60 Ω. Find the % error.

Solution:

$$\% \text{ error}, \varepsilon_r = \left(\frac{A_m - A_T}{A_T}\right) \times 100 = \left(\frac{60-50}{50}\right) \times 100 = 20\%$$

$$\boxed{\% \text{ error} = 20\%}$$

EXAMPLE - 1.3

The measured value of a resistor is 100 Ω and it's relative error is ±10% then, it's true value and the range is?

Solution:

$$\varepsilon_r = \pm 10\% \text{ of } 100 = \pm 10\ \Omega$$

Range, $A_T = (100 - 10) \text{ to } (100 + 10)$

$= 90\ \Omega \text{ to } 110\ \Omega$

1.5.4 Error at Desired Scale (or) Reading Value

Error at any desired scale is given by:

$$\boxed{\% \varepsilon_r = \frac{\% \text{ full scale error} \times \text{Full scale value}}{\text{Reading (desired) value}}}$$

REMEMBER As the desired instrument reading approaches the full scale value of measurement of the unknown quantity, error is reduced in the measurement.

1.6 COMBINATION OF QUANTITIES WITH LIMITING ERRORS

When two or more quantities, each having a limiting error, are combined, it is advantageous to be able to compute the limiting error of the combination.

1.6.1 Sum or Difference of Two or more quantities

Let,
$$x_1 = a \pm \varepsilon_{r1}$$
$$x_2 = b \pm \varepsilon_{r2}$$
$$x_3 = c \pm \varepsilon_{r3}$$
∴ $$x = x_1 + x_2 + x_3$$
or, $$x = -x_1 - x_2 - x_3$$
So, $$x = \pm (x_1 + x_2 + x_3)$$
Relative limiting error in x is given by
$$\delta R = \pm [\delta R_1 + \delta R_2 + \delta R_3]$$
$$\in_x \times R = \pm(\in_{r_1} \times a + \in_{r_2} \times b + \in_{r_3} \times c)$$
Dividing LHS and RHS by R, we have
$$\in_x = \pm\left(\frac{a}{R}\in_{r_1} + \frac{b}{R}\in_{r_2} + \frac{c}{R}\in_{r_3}\right)$$
$$\in_x = \pm\left(\frac{a}{a+b+c}\in_{r_1} + \frac{b}{a+b+c}\in_{r_2} + \frac{c}{a+b+c}\in_{r_3}\right)$$
$$\boxed{\varepsilon_x = \pm\left(\frac{a}{a+b+c}\cdot\varepsilon_{r1} + \frac{b}{a+b+c}\cdot\varepsilon_{r2} + \frac{c}{a+b+c}\varepsilon_{r3}\right)}$$
(ε_x = worst possible error)

EXAMPLE - 1.4

Three resistances R_1 = 10 ± 2%, R_2 = 20 ± 5%, R_3 = 50 ± 3% are connected in series. Find the % limiting error for the series combination.

Solution:
$$\varepsilon_R = \pm\left(\frac{10}{10+20+50}\times 2 + \frac{20}{10+20+50}\times 5 + \frac{50}{10+20+50}\times 3\right)$$
or, $$\varepsilon_R = \pm 3.375\%$$
$$\boxed{\%\ \text{Limiting error} = \pm 3.375\%}$$
Given, $$R_T = 10 + 20 + 50 = 80\ \text{W}$$
$$\boxed{R_{\text{measured}} = (80 \pm 3.375\%)\,\Omega}$$

1.6.2 Multiplication or Division Terms

Let, $$x = \frac{x_1\,x_2}{x_3} \text{ or } \frac{x_2\,x_3}{x_1} \text{ or } x_1 x_2 x_3 \text{ or } \frac{x_1}{x_1 x_3}$$
Then, relative limiting error is $$x = x_1\,x_2\,x_3$$
Taking natural log both sides
$$\ln x = \ln x_1 + \ln x_2 + \ln x_3$$
Differentiating both sides, we have
$$\frac{\partial x}{x} = \pm \pm\left[\frac{\partial x_1}{x_1} + \frac{\partial x_2}{x_2} + \frac{\partial x_3}{x_3}\right]$$
$$\boxed{\varepsilon_x = \pm(\varepsilon_{r1} + \varepsilon_{r1} + \varepsilon_{r3})}$$

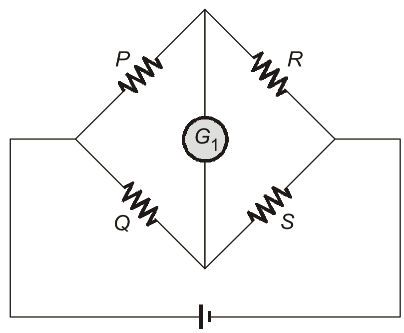
NOTE

When, $x = \dfrac{x_1 x_2}{x_2 + x_3}$ or $\dfrac{x_1}{x_2 + x_3}$ or $\dfrac{x_1 x_2}{x_2 - x_1}$

Then, multiplication or division form is not applicable for finding relative limiting error.

EXAMPLE - 1.5

In the measurement of unknown resistance by using a Wheatstone bridge if $P = 20 \pm 5\%$, $Q = 50 \pm 3\%$ and $S = 30 \pm 2\%$. Find the value of the unknown resistance R and it's limiting error.



Solution:

Limiting error, $\varepsilon_R = \pm(5 + 3 + 2) = \pm 10\%$

$$R = \frac{P}{Q} \cdot S = \frac{20}{50} \times 30 = 12\,\Omega$$

So, unknown resistance, $R = 12 \pm 10\%$

1.6.3 Power of a Factor

Let, $x = x_1^m \cdot x_2^n \cdot x_3^p$ or $\dfrac{x_1^m x_2^n}{x_3^p}$ or $\dfrac{x_1^m}{x_1^n x_3^p}$

$$x = x_1^m \, x_2^n \, x_3^p$$

Taking natural log both sides

$$\ln x = m \cdot \ln x_1 + n \cdot \ln x_2 + p \cdot \ln x_3$$

Differentiating both sides, we have

$$\frac{\partial x}{x} = \pm \left[m \cdot \frac{\partial x_1}{x_1} + n \cdot \frac{\partial x_2}{x_2} + p \cdot \frac{\partial x_3}{x_3} \right]$$

Then, Relative limiting error is $\varepsilon_x = \pm(m\,\varepsilon_{r1} + n\,\varepsilon_{r2} + p\,\varepsilon_{r3})$

EXAMPLE - 1.6

Three resistors having resistances of 250 Ω, 500 Ω and 375 Ω are connected in parallel. The 250 Ω resistor has a +0.025 fractional error, the 500 Ω resistor has a –0.036 fractional error, and the 375 Ω resistor has a +0.014 fractional error. Determine (a) the total resistance neglecting errors, (b) total resistance considering the error of each resistor and (c) the fractional error of the total resistance based upon rated values.

Solution:

(a) Total resistance of resistors connected in parallel and neglecting their errors is,

$$R = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{250}+\frac{1}{500}+\frac{1}{375}} = 115.38\ \Omega$$

(b) The fractional error in $R_1 = 250\ \Omega$ is +0.025

$\therefore$ $\delta R_1 = (0.025 \times 250) = +6.25\ \Omega$

Hence, $R_1 = 250 - 6.25 = 243.75\ \Omega$

Similarly, $\delta R_1 = (-0.036 \times 500) = -18\ \Omega$

and $R_2 = 500 + 18 = 518\ \Omega$

$\therefore$ $\delta R_3 = (+0.014 \times 375) = 5.25\ \Omega$

and $R_3 = 375 - 5.25 = 369.75\ \Omega$

Therefore the resultant resistance of three resistances in parallel,

$$R = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{243.75}+\frac{1}{518}+\frac{1}{369.75}} = 114.45\ \Omega$$

(c) The fractional error of the parallel resistances based on the rated values is

$$\frac{115.38 - 114.45}{114.45} = 0.00814 = +0.814\%$$

NOTE

When x is of the form $\frac{x_1^m}{x_2^n + x_3^p}$ or $\frac{x_1^m + x_2^n}{x_3^p}$

then, above method is not applicable for finding relative limiting error.

EXAMPLE - 1.7

The power is measured in a resistor by passing current through the ammeter and ammeter measures $I = (5 \pm 4\%)$ A across the resistance of $R = (10 \pm 2\%)\ \Omega$. Find the power consumed by the resistor and it's limiting error.

Solution:

Power consumed, $P = I^2R = 5^2 \times 10 = 250$ watts

and limiting error, $\varepsilon_p = \pm(2\varepsilon_I + \varepsilon_R) = \pm(2 \times 4 + 2) = 10\%$

$\therefore$ $P = (250 \pm 10\%)$ watt

1.6.4 Special Case

Resistance in parallel:

Let, $R_1 = 10 \pm 10\%$ (Range = 9 Ω to 11 Ω)

and $R_2 = 20 \pm 5\%$ (Range = 19 Ω to 21 Ω)

Equivalent resistance of parallel combination is $R = \frac{R_1 R_2}{R_1 + R_2}$

True value; $R = \frac{10 \times 20}{10 + 20} = 6.66\ \Omega = R_T$

Resistance in lower range; $R_L = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{9 \times 19}{9 + 19} = 6.107\ \Omega$ = Measured value in low range = L_m

Resistance in higher range; $R_H = \dfrac{11 \times 21}{11 + 21} = 7.21875\ \Omega$ = Measured value for high range = H_m

Error in low range (for low value) = $\%\ \varepsilon_r = \left(\dfrac{L_m - R_T}{R_T}\right) \times 100$
Error in high range (for high value) = $\%\ \varepsilon_r = \left(\dfrac{H_m - R_T}{R_T}\right) \times 100$

For present case:

Error in low range, $\%\varepsilon_r = \left(\dfrac{6.10 - 6.66}{6.66}\right) \times 100 = -8.39\ \%$

Error in high range, $\%\varepsilon_r = \left(\dfrac{7.2187 - 6.66}{6.66}\right) \times 100 = 8.28\ \%$

EXAMPLE - 1.8

A 4-dial decade box has

Decade *a* of 10 × 1000 Ω ± 0.1%,

Decade *b* of 10 × 100 Ω ± 0.1%

Decade *c* of 10 × 10 Ω ± 0.5%

Decade *d* of 10 × 1 Ω ± 1.0%

It is set at 4639 Ω. Find the percentage limiting error and the range of resistance value.

Solution:

$$\text{Error for decade } a = \pm 4000 \times \frac{0.1}{100} = \pm 4\ \Omega$$

$$\text{Error for decade } b = \pm 600 \times \frac{0.1}{100} = \pm 0.6\ \Omega$$

$$\text{Error for decade } c = \pm 30 \times \frac{0.5}{100} = \pm 0.15\ \Omega$$

$$\text{Error for decade } d = \pm 9 \times \frac{1}{100} = \pm 0.09\ \Omega$$

∴ Total error = ±(4 + 0.6 + 0.15 + 0.09) = 4.84 Ω

Relative limiting error, $\varepsilon_r = \pm \dfrac{4.84}{4639} = \pm 0.00104$

% limiting error, $\%\ \varepsilon_r = \pm 0.00104 \times 100 = \pm 0.104\%$

$$\text{Limiting value of resistance} = 4639\,(1 \pm 0.00104)$$
$$= (4639 \pm 4.84)\ \Omega$$

PRACTICE QUESTIONS

Question: 1

Two sets of large number of 20 kΩ and 30 kΩ resistors are used to make a large number of 12 kΩ and 50 kΩ resistors choosing one from each group. If the standard deviations of the two sets of resistors of 20 kΩ and 30 kΩ are respectively 5% and 10%. Find the standard deviations of the combined resistor-sets of 12 kΩ and 50 kΩ?

Question: 2

What do you mean by dimensions of a quantity? The energy stored in a parallel-plate capacitor per unit volume is given by

$$W = K \in^a V^b d^c$$

where, K = Constant ; $\in$ = Permittivity of medium ; V = Voltage between plates ;

d = Distance between plates

Find the values of a, b and c.

Question: 3

The limiting errors for a four dial resistance box are :

Units : ±0.2%

Tens : ±0.1%

Hundreds : ±0.05%

Thousands : ±0.02%

If the resistance value is set at 4325 Ω calculate the limiting error for this value.

Question: 4

A power transformer was tested to determine losses and efficiency. The input power was measured as 3650 W and the delivered output power was 3385 W, with each reading in doubt by ±10 W. Calculate the percentage uncertainty in the losses of the transformer and the percentage uncertainty in the efficiency of the transformer, as determined by the difference in input and output power readings.

Practice Questions Explanations

1. ***Solution*:**

Let, R_1 = 20 kΩ and R_2 = 30 kΩ

$$\sigma_{R_1} = \text{standard deviation of 20 k}\Omega\text{ resistor} = 20 \times \frac{5}{100}\text{k}\Omega = 1\text{k}\Omega$$

$$\sigma_{R_2} = \text{standard deviation of 30 k}\Omega\text{ resistor} = 30 \times \frac{10}{100}\text{k}\Omega = 3\text{ k}\Omega$$

R_3 = 50 kΩ resistor is made by series combination of R_1 and R_2, so

$$R_3 = R_1 + R_2 = 20 + 30 \text{ k}\Omega = 50 \text{ k}\Omega$$

Similarly, R_4 = 12 kΩ resistor is made by parallel combination of R_1 and R_2

So,

$$R_4 = \frac{R_1 R_2}{R_1 + R_2} = \frac{20 \times 30}{20 + 30} = 12 \text{ k}\Omega$$

σ_{R_3} = standard deviation of R_3 and

σ_{R_4} = standard deviation of R_4

Standard deviation of R_3,

$$\sigma_{R_3} = \sqrt{\left(\frac{\partial R_3}{\partial R_1}\right)^2 \sigma_{R_1}^2 + \left(\frac{\partial R_3}{\partial R_2}\right)^2 \sigma_{R_2}^2}$$

where,

$$\frac{\partial R_3}{\partial R_1} = \frac{\partial (R_1 + R_2)}{\partial R_1} = 1$$

and

$$\frac{\partial R_3}{\partial R_2} = \frac{\partial (R_1 + R_2)}{\partial R_2} = 1$$

So,

$$\sigma_{R_3} = \sqrt{1^2 \times 1^2 + 1^2 \times 3^2} \text{ k}\Omega = \sqrt{10} \text{ k}\Omega = 3.16 \text{ k}\Omega$$

$$= \frac{3.16}{50} \times 100 = 6.32\%$$

Standard deviation of R_4,

$$\sigma_{R_4} = \sqrt{\left(\frac{\partial R_4}{\partial R_1}\right)^2 \sigma_{R_1}^2 + \left(\frac{\partial R_4}{\partial R_2}\right)^2 \sigma_{R_2}^2}$$

where,

$$\frac{\partial R_4}{\partial R_1} = \frac{\partial}{\partial R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \frac{R_2}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)}$$

$$= \frac{R_2^2}{(R_1 + R_2)^2} = \left(\frac{R_2}{R_1 + R_2}\right)^2 = \left(\frac{30}{20 + 30}\right)^2 = 0.36$$

Similarly,

$$\frac{\partial R_4}{\partial R_2} = \frac{\partial}{\partial R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \left(\frac{R_1}{R_1 + R_2}\right)^2 = \left(\frac{20}{20 + 30}\right)^2 = 0.16$$

$$\sigma_{R_4} = \sqrt{\left(\frac{\partial R_4}{\partial R_1}\right)^2 \sigma_{R_1}^2 + \left(\frac{\partial R_4}{\partial R_2}\right)^2 \sigma_{R_2}^2} = \sqrt{(0.36)^2 \times 1^2 + (0.16)^2} \; k\Omega$$

$$= \sqrt{0.36} = 0.6 \text{ k}\Omega = \frac{0.6}{10} \times 100 = 6\%$$

2. *Solution*:

Dimensions: Every quantity has a quality which distinguishes it from all other quantities. This unique quality is called Dimensions. The dimensions written in a characteristics notation.

For example: $[L]$ for length, $[T]$ for time etc.

Writing the dimensions of various quantities in LMTI system.

Energy density, $$W = \frac{\text{Energy}}{\text{Volume}}$$

$$[W] = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$$

Now, Force, $$F = \frac{Q_1Q_2}{\in d^2}$$

∴ Permittivity, $$\in\ = \frac{Q_1Q_2}{Fd^2}$$

or $$\in\ = \frac{[I^2T^2]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}I^2T^4]$$

and $$\text{Voltage} = [V]$$
$$= [I^{-1}ML^2T^{-3}];\ d = [L]$$

we get, $$[ML^{-1}T^{-2}] = [I^2M^{-1}L^{-3}T^4]^a[I^{-1}ML^2T^{-3}]^b[L]^c$$
$$= [I]^{2a-b}[M]^{-a+b}[L]^{-3a+2b+c}[T]^{4a-3b}$$

Balancing the dimensions on both sides, we have,

$$0 = 2a - b$$
$$\Rightarrow \quad b = 2a \quad \ldots(i)$$
$$1 = -a + b \quad \ldots(ii)$$
$$-1 = -3a + 2b + c \quad \ldots(iii)$$
$$-2 = 4a - 3b \quad \ldots(iv)$$

From equations (i) and (ii),

$$1 = -a + 2a$$
$$\Rightarrow \quad a = 1\ ;\ b = 2$$
and $$-1 = -3 + 4 + c$$
$$\Rightarrow \quad c = -2$$

From above; $a = 1$; $b = 2$ and $c = -2$

Hence, energy stored per unit volume is

$$\boxed{\omega = K \in V^2 d^{-2} = \frac{K \in V^2}{d^2}}$$

3. *Solution*:

Thousand is set at 4000 Ω and error

$$= \pm 4000 \times \frac{0.02}{100} = \pm 0.8\ \Omega$$

$$\text{For hundred error} = \pm 300 \times \frac{0.05}{100} = \pm 0.15\ \Omega$$

Similarly, $$\text{For ten error} = \pm 20 \times \frac{0.1}{100} = \pm 0.02\ \Omega$$