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Power System : Analysis Control

Electrical Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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CHAPTER 1

Transmission Line Parameters & Corona

1.1 Transmission Line Parameters

An electric transmission line has four parameters which affect its ability to fulfill its function as part of a power system, resistance, inductance, capacitance, and conductance. The ***"resistance"*** of transmission line conductors is the most important cause of power loss in a transmission line.

"Inductance" is by far the most dominant line parameter from power system engineer's view point which limits the transmission capacity of a line. The inductance of the circuit relates the voltage induced by changing flux to the rate of change of current. The ***"capacitance"*** which exists between the conductors is defined as the charge on the conductors per unit of potential difference between them. The fourth parameter, ***"conductance"***, exists between conductors or between conductors and the ground. Conductance accounts for the leakage current at the insulators of overhead lines and through the insulation of cables which is usually neglected as being very small.

Resistance of a Transmission Line

The term ***"resistance"*** in a transmission line usually means effective resistance.

The effective resistance of a conductor is

$$R = \left(\frac{\text{Power loss in conductor}}{|I|^2}\right)\Omega$$

The effective resistance is equal to the dc resistance of the conductor only if the distribution of current throughout the conductor is uniform.

Direct-current resistance is given by, $R_D = \left(\frac{\rho l}{A}\right)\Omega$

where, ρ = resistivity of conductor
l = length
A = cross-sectional area

The uniform distribution of current throughout the cross-section of a conductor exists only for direct current. As the frequency of alternating current increases, the non-uniformity of distribution becomes more pronounced. An increase in the frequency causes non-uniform current density. This phenomenon is called ***"skin effect"*** which is explained in next articles.

Types of Conductors

In present trend, aluminium conductors have completely replaced copper for overhead lines because of the much lower cost and lighter weight of an aluminium conductor compared with a copper conductor of the same resistance. Symbols identifying different types of aluminium conductors are as follows:

AAC : all-aluminium conductors

AAAC : all-aluminium-alloy conductors

ACSR : aluminium conductor, steel-reinforced

ACAR : aluminium conductor, alloy-reinforced

ACSR

ACSR consists of a central core of steel strands surrounded by layers of aluminium strands. Fig. 1.1 shows the cross-section of a typical ***steel-reinforced aluminium cable*** (ACSR). The conductor shown has 7 steel strands forming a central core, around which there are two layers of aluminium strands. There are 24 aluminium strands in the two outer layers. The conductor stranding is specified as simply 24/7.

$$S = 3y^2 - 3y + 1$$

$S \rightarrow$ Number of strands

$y \rightarrow$ Number of layers ≥ 2 i.e. $y = 1$ for central

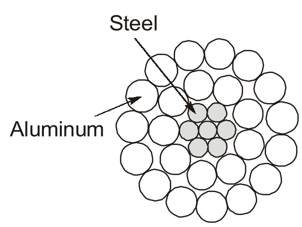


***Fig.1.1 :** Cross-sectional view of a steel-reinforced conductor, 7 steel strands, and 24 aluminium strands*

Overall diameter of stranded conductor,

$$D = (2y - 1)\, d$$

where, $d \rightarrow$ uniform dia of a strand

Remember

- Aluminium-alloy conductors have higher tensile strength than the ordinary conductor grade of aluminium.
- The total number of strands in concentrically stranded cables, where the total annular space is filled with strands of uniform diameter is 7, 19, 37, 61, 91 or more.
- Various tensile strengths, current capacitances, and conductor sizes are obtained by using different combinations of steel and aluminium.
- A type of conductor called ***"expanded ACSR"*** has a filler such as paper separating the inner steel from the outer aluminium strands is being used for EHV lines.

Skin Effect and Proximity Effect

Skin effect: The distribution of current throughout the cross-section of a conductor, is uniform only when DC is passing through it. On the contrary when AC is flowing through a conductor, the current is non-uniformly distributed over the cross-section in a manner that the current density is higher at the surface of the conductor compared to the current density at its center. This effect becomes more pronounced as frequency is increased. This phenomenon is called skin effect. It causes larger power loss for a given rms AC than the loss when the same value of DC is flowing through the conductor. Consequently, the effective conductor resistance is more for AC than for DC.

Imagine a solid round conductor (a round shape is considered for convenience only) to be composed of annular filaments of equal cross-section area as shown in Fig. 1.2. The flux linking the filaments progressively decreases as we move towards the outer filaments for the simple reason that the flux inside a filament does not link it. The inductive reactance of the imaginary filaments therefore decreases outwards with the results that the outer filaments conduct more AC than the inner filaments (filaments being parallel). With the increase of frequency the non-uniformly of inductive reactance of the filaments become more pronounced, so also the non-uniformity of current distribution. For large solid conductors, the skin effect is quite significant even at 50 Hz.

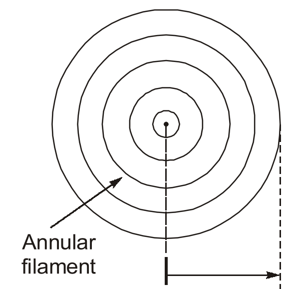


Fig.1.2 : *Skin effect*

Proximity effect: Non-uniform current distribution in a conductor due to presence of other current carrying conductor in the vicinity is called proximity effect.

The fundamental reason for the proximity effect is the change in the net magnetic field around the nearly placed current carrying conductors. Since conductors carrying currents in same direction lead to the reduction in net magnetic field in between them and conductors with opposite currents result in increased field between them. Thus in prior case, current tries to be established densely in adjacent faces of the conductors due to less inductive reactance, while in later condition current density is lesser in adjacent faces w.r.t. remote portion of the conductor. Fig. 1.3 depicts the effect pictorially, for the later condition.

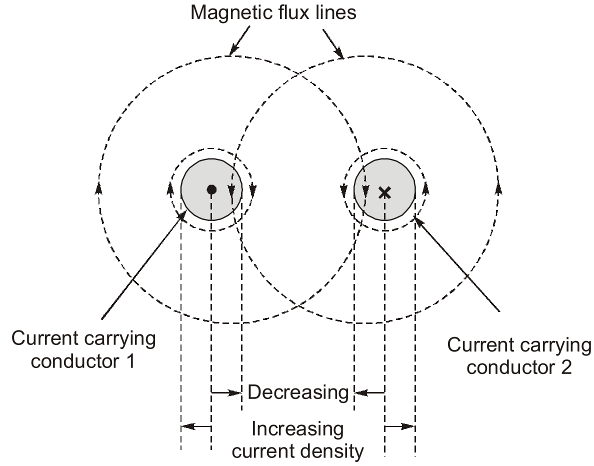


Fig. 1.3 : *Proximity effect*

NOTE

- Skin effect causes larger power loss for a given rms AC than the loss when the same value of DC is flowing through the conductor which is an undesirable effect.
- Due to skin effect, the effective conductor resistance becomes more for AC than for DC.
- *"Proximity effect"* and *"Skin effect"* both results in the increase in the resistance of conductor and decrease of self reactance.
- Both the effects depends on the conductor size, frequency of the supply, resistivity and relative permeability of the material.
- Proximity effect is more pronounced for large conductors, high frequencies and close proximity.
- Proximity effect is more pronounced in case of cables where the spacing between the conductors is small.

Inductance of Transmission Lines

Inductance

- Voltage induced in a circuit is given by $\boxed{e = \frac{d\Psi}{dt} V}$

  where, Ψ represents the flux linkages of the circuit in Weber-turns (*Wb-T*).

  Also, $$\boxed{e = \frac{d\Psi}{dt} \times \frac{di}{di} = L\left(\frac{di}{dt}\right) V}$$

  where, $L = \frac{d\Psi}{dt}$ is defined as the self ***inductance of the circuit*** in henry.
- In a linear magnetic circuit, inductance is constant and is given by

  $$\boxed{L = \frac{\Psi}{i} \text{ Henry}} \quad \text{or} \quad \boxed{\Psi = Li \;\; Wb\text{-}T}$$
- For alternating current, $\boxed{\lambda = LI}$

  where, λ and I are the rms values of flux linkages and current respectively.
- Similarly, ***mutual inductance*** between two circuits is defined as ***"the flux linkages of one circuit due to current in other circuit".***

  i.e., $$\boxed{M_{12} = \frac{\lambda_{12}}{I_2} \text{ H}}$$

  Also, voltage drop in circuit 1 due to current in circuit 2 is $\boxed{V_1 = j\omega M_{12} I_2 = j\omega \lambda_{12} \text{ V}}$.

Inductance of a Conductor Due to Internal Flux

The inductance of a transmission line is calculated as flux per ampere. To obtain an accurate value for the inductance of a transmission line, it is necessary to consider the flux inside each conductor as well as the external flux. Let us consider the long cylindrical conductor whose cross-section is shown in Fig. 1.4. Here, it is assumed that the lines of flux are concentric with the conductor.

By Ampere's law,

$$\text{mmf} = \oint H \cdot dS = I \text{ At} \quad \text{...(1)}$$

where, H = magnetic field intensity, At/m

S = distance along path, m

I = current enclosed, A

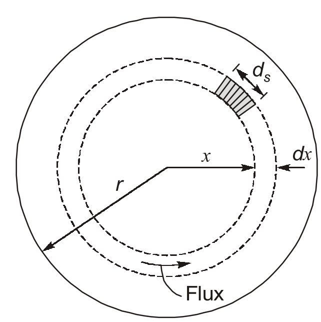


***Fig. 1.4 :** Cross-section of a cylindrical conductor*

Let the field intensity at a distance x meters from the center of the conductor be designated H_x. Since field is symmetrical, H_x is constant at all points equidistance from the center of the conductor.

Using equation (1) for Fig. 1.4, we have

$$\oint H_x \, dS = I_x \quad \text{...(2)}$$

and $$2\pi x H_x = I_x \quad \text{...(3)}$$

where I_x is the current enclosed. Then, assuming uniform current density

$$I_x = \frac{\pi x^2}{\pi r^2} I \quad (I = \text{Total current in the conductor}) \quad \text{...(4)}$$

Substituting I_x from equation (4) into equation (3), we have

$$2\pi x H_x = \frac{\pi x^2}{\pi r^2} I \quad \text{or} \quad H_x = \frac{x}{2\pi r^2} I \text{ At/m}$$

∴ Flux density x meter from the center of the conductor is

$$B_x = \mu H_x = \frac{\mu x I}{2\pi r^2} \text{ Wb/m}^2 \qquad (\mu = \text{permeability of conductor}) \qquad ...(5)$$

The flux per meter of length is, $d\phi = \frac{\mu x I}{2\pi r^2} dx$ Wb/m

∴ Flux linkage, $d\lambda$ per meter of length is

$$d\lambda = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu I x^3}{2\pi r^4} dx \quad \text{Wbt/m} \qquad ...(6)$$

So, $$\lambda_{int} = \int_0^r \frac{\mu I x^3}{2\pi r^4} dx = \frac{\mu I}{8\pi} \quad \text{Wbt/m} \qquad ...(7)$$

Since, $$\mu = 4\pi \times 10^{-7} \text{ H/m}$$

therefore, $$\lambda_{int} = \frac{I}{2} \times 10^{-7} \text{ Wbt/m}$$

So, $$\boxed{L_{int} = \frac{1}{2} \times 10^{-7} \text{ H/m}} \qquad \left(\text{Since, } L = \frac{\lambda_{int}}{I}\right) \qquad ...(8)$$

Flux Linkages Due to Flux between Two Points External to Conductor

For finding the external flux linkages, let us derive an expression for the flux linkages of an isolated conductor due to only that portion of the external flux which lies between two points at D_1 and D_2 meters from the center of the conductor. In Fig. 1.5, P_1 and P_2 are two such points. The conductor carries a current of I A.

At the tabular element, which is x meters from the center of the conductor, the field intensity is H_x.

The mmf around the element is

$$2\pi x H_x = I \quad \text{or} \quad H_x = \frac{I}{2\pi x} \text{ A/m}$$

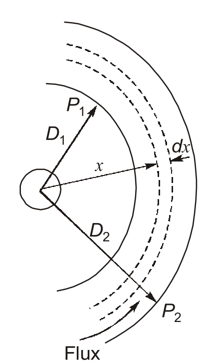


Fig. 1.5 : *A conductor and external points P_1 and P_2*

Now, $$B_x = \mu H_x \quad \text{or} \quad B_x = \mu\left[\frac{I}{2\pi x}\right] \text{Wb/m}^2$$

The flux $d\phi$ in the tabular element of thickness dx is

$$d\phi = \frac{\mu I}{2\pi x} dx \text{ Wb/m}$$

Now, between points P_1 and P_2, the flux linkage are

$$\lambda_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi x} \cdot dx = \frac{\mu I}{2\pi} ln\left(\frac{D_2}{D_1}\right) \quad \text{Wbt/m}$$

or for a relative permeability of 1,

$$\lambda_{12} = 2 \times 10^{-7} I \, ln \frac{D_2}{D_1} \quad \text{Wbt/m}$$

The inductance only due to the flux induced between P_1 and P_2 is

$$\boxed{L_{12} = 2 \times 10^{-7} \, ln\left(\frac{D_2}{D_1}\right) \text{ H/m}} \qquad \text{...(Important result)} \qquad ...(9)$$

Inductance of a Single-phase Two-wire Line

Fig. 1.6 shows a line having two conductors of radii r_1 and r_2. One conductor is the return circuit for the other. Here, D is much greater than r_1 and r_2 to assume that D can be used instead of $(D - r_2)$ or $(D + r_2)$.

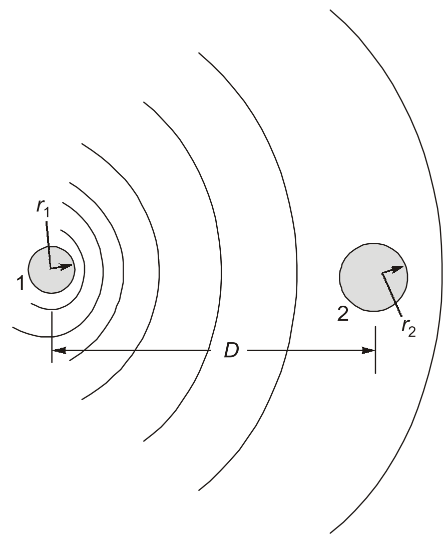


Fig. 1.6 : *Conductors of different radii and the magnetic field due to current in conductor 1 only*

Now, replacing D_1 with r_1 and D_2 with D in equation (9), the inductance due to external flux linkage is

$$L_{ext} = L_{12} = 2 \times 10^{-7} \, ln\left(\frac{D}{r_1}\right) \text{ H/m} \quad \text{and} \quad L_{int} = \frac{1}{2} \times 10^{-7} \text{ H/m}$$

Hence, inductance of the circuit of Fig. 1.6 due to the current in conductor 1 only is,

$$L_1 = \left(\frac{1}{2} + 2 \, ln\frac{D}{r_1}\right) \times 10^{-7} \text{ H/m}$$

or,
$$L_1 = 2 \times 10^{-7}\left(ln\, e^{1/4} + ln\frac{D}{r_1}\right) \text{ H/m} \quad \text{or} \quad L_1 = 2 \times 10^{-7} \, ln\left(\frac{D}{r_1 e^{-1/4}}\right) \text{ H/m}$$

Let, $r_1' = r_1 e^{-1/4}$, then

$$\boxed{L_1 = 2 \times 10^{-7} \, ln\left(\frac{D}{r_1'}\right) \text{H/m}}$$ [Inductance per conductor (if $r_1' = r'$)] ...(10)

NOTE

The radius r_1' is that of a fictitious conductor assumed to have no internal flux but, with the same inductance as the actual conductor of radius r_1.

Here, $e^{-1/4}$ **= 0.7788**.

The multiplying factor of 0.7788 applies only to solid round conductors.

Similarly, the inductance due to current in conductor 2 is

$$L_2 = 2 \times 10^{-7} \, ln\left(\frac{D}{r_2'}\right) \text{ H/m}$$ [Inductance per conductor (if $r_2' = r'$)] ...(11)

For complete circuit,
$$L_1 = L_1 + L_2 = 4 \times 10^{-7} \, ln\left(\frac{D}{\sqrt{r_1' r_2'}}\right) \text{ H/m}$$

If $r_1' = r_2' = r'$, the total inductance reduces to

$$L = 4\times10^{-7}\, ln\left(\frac{D}{r'}\right) \text{ H/m} \qquad \text{...(Inductance per loop meter)}$$

or,

$$L = 0.921 \log\left(\frac{D}{r'}\right) \text{ mH/km} \qquad \text{...(Important result)} \qquad ...(12)$$

Inductance of Composite Conductor Lines

Fig. 1.7 shows a single-phase line comprising composite conductors *A* and *B* with *A* having '*n*' parallel filaments and *B* having *m*′ parallel filaments. It is assumed that the current is equally divided among the filaments of each composite conductor. Hence, each filament of *A* is taken to carry a current I/n, while each filament of conductor '*B*' carries the return current of $-I/m'$.

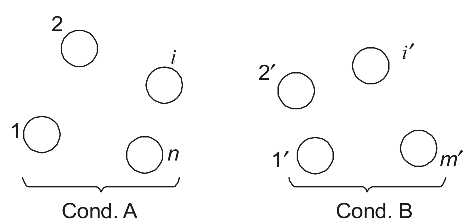


Fig. 1.7 : *Single-phase line consisting of two composite conductors*

Now, the total flux linkages of conductor *i* of composite conductor *A* is given by

$$\lambda_i = 2\times10^{-7}\cdot\frac{I}{n}\left[ln\frac{1}{D_{i_1}} + ln\frac{1}{D_{i_2}} + ... + ln\frac{1}{D_{ii}} + ... + ln\frac{1}{D_{in}}\right]$$

$$-2\times10^{-7}\cdot\frac{I}{m'}\left[ln\frac{1}{D_{i_{1'}}} + ln\frac{1}{D_{i_{2'}}} + ... + ln\frac{1}{D_{ij'}} + ... + ln\frac{1}{D_{im'}}\right]$$

$$= -2\times10^{-7}\, I\, ln\left(\frac{\sqrt[m']{D_{i_{1'}}\, D_{i_{2'}}\, ...D_{im'}}}{\sqrt[n]{D_{i_1}\, D_{i_2}\, ...D_{ii}....D_{in}}}\right) \text{ Wb-T/m}$$

The inductance of filament *i* is then,

$$\frac{\lambda_i}{I/n} = 2n\times10^{-7}\, ln\left(\frac{\sqrt[m']{D_{i1'}....D_{ij}....D_{im'}}}{\sqrt[n]{D_{i1}....D_{ii}....D_{in}}}\right) \text{ H/m} \qquad ...(13)$$

Now, the average inductance of the filament of composite conductor *A* is

$$L_{\text{avg}} = \frac{L_1 + L_2 + L_3 + ... + L_n}{n}$$

Since conductor *A* is composed of *n* filaments electrically in parallel, it's inductance is

$$L_A = \frac{L_{avg}}{n} = \left(\frac{L_1 + L_2 + ... + L_n}{n^2}\right) \qquad ...(14)$$

Substituting the logarithmic expression for filament inductance from equation (13) in equation (14), we have

$$L_A = 2\times10^{-7}\, ln\left(\frac{\sqrt[m'n]{(D_{11'}...D_{1j'}...D_{1m'})\,(D_{i1'}...D_{ij'}...D_{im'})\,(D_{n1'}...D_{nj'}...D_{nm'})}}{\sqrt[n^2]{(D_{11}...D_{1j}...D_{1m})\,(D_{i1}...D_{ii}...D_{in})\,(D_{n1}...D_{nj}...D_{nn})}}\right) \text{ H/m} \qquad ...(15)$$

The numerator of the argument of the logarithm in equation (15) is the $m'n^{th}$ root of the $m'n$ terms i.e. it is the products of all possible mutual distances from the *n* filaments of a conductor *A* to *m*′ filament of conductor *B*. This is called ***"mutual geometric mean distance (mutual GMD)"*** (denoted by D_m).

The denominator is defined as the ***self geometric mean distance (self GMD)*** of conductor. Sometimes, self GMD is also called ***"geometric mean radius (GMR)"*** (denoted by D_s).

In terms of D_m and D_s, equation (15) can be written as

$$L_A = 2\times10^{-7}\, ln\left(\frac{D_m}{D_s}\right) \text{H/m} = 0.461 \log\frac{D_m}{D_s}\ \text{mH/km} \quad \text{...(Important result)} \quad \text{...(16)}$$

The inductance of the composite conductor *B* is determined in a similar manner, and the total inductance of the line is given by

$$L = L_A + L_B$$

NOTE

- The inductance of one conductor of a single-phase line is

$$L(\text{inductance per conductor}) = 2\times10^{-7}\, ln\left(\frac{D}{r'}\right) \text{H/m}$$

$$= 2\times10^{-7}\, ln\left(\frac{\text{Mutual GMD}}{\text{Self GMD}}\right) \text{H/m} \quad \text{...(Important result)}$$

- The inductance of a single-phase line is

$$L(\text{inductance per loop meter}) = 4\times10^{-7}\, ln\left(\frac{\text{GMD}}{\text{GMR}}\right) \text{H/m}$$

or,

$$4\times10^{-7}\, ln\left(\frac{\text{Mutual GMD}}{\text{Self GMD}}\right) \text{H/m} \quad \text{...(Important result)}$$

Inductance of Three-phase Lines

Case-I

Equilateral spacing between conductors:

Fig. 1.8 shows the conductors of a three-phase line spaced at the corners of an equivalent triangle. For balanced three-phase phasor currents, $I_a + I_b + I_c = 0$.

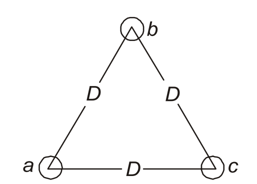


Fig. 1.8 : *Cross-sectional view of the symmetrically/equilaterally spaced conductors of a three-phase line*

Now, flux linkages of conductor *a* is

$$\lambda_a = 2\times10^{-7}\left(I_a\, ln\frac{1}{D_s} + I_b\, ln\frac{1}{D} + I_c\, ln\frac{1}{D}\right) \text{Wbt/m}$$

Since, $I_a = -(I_b + I_c)$, therefore, above equation becomes

$$\lambda_a = 2\times10^{-7}\left(I_a\, ln\frac{1}{D_s} - I_a\, ln\frac{1}{D}\right) = 2\times10^{-7} I_a\, ln\left(\frac{D}{D_s}\right) \text{Wbt/m}$$

So,

$$L_a = 2\times10^{-7}\, ln\left(\frac{D}{D_s}\right) \text{H/m} \quad \text{...(17)}$$

NOTE

- Due to symmetry, the inductance of conductors *b* and *c* will be equal to the inductance of conductor a

So,

$$L_a = L_b = L_c = 2\times10^{-7}\, ln\left(\frac{D}{D_s}\right) \text{H/m}$$

- Since each phase consists of single conductor therefore, the inductance per phase of the three-phase line will be equal to that due to any one phase.

  i.e., $$L_{(3\text{-}\phi\text{ line})} = L_a = L_b = L_c$$
  $$= 2\times10^{-7}\, ln\left(\frac{D}{D_s}\right) \text{ H/m}$$

Case-II

Unsymmetrical spacing between conductors: When the conductors of a three-phase line are unsymmetrically spaced, the flux linkages and inductance of each phase are not the same. The different inductance in each phase give rise to an unbalanced circuit. The three-phase circuit can again be balanced with the help of "***transposition***".

Transposition of Transmission Lines

Transposition of a transmission line (3-phase line) is defined as "exchange of conductor positions at regular intervals along the line so that each conductor occupies the original position of each other conductor over an equal distance."

An unsymmetrically spaced 3-phase conductors with it's transposition cycle is shown in Fig. 1.9.

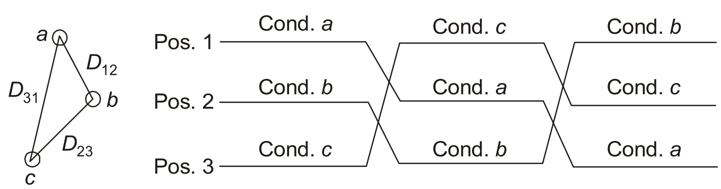


***Fig. 1.9 :** An unsymmetrically spaced 3-phase conductors with it's transposition cycle*

From Fig. 1.9, flux linkage of conductor a in position 1 when *b* is in position 2 and *c* is in position 3 is given by

$$\lambda_{a_1} = 2\times10^{-7}\left(I_a\, ln\frac{1}{D_s} + I_b\, ln\frac{1}{D_{12}} + I_c\, ln\frac{1}{D_{31}}\right) \text{ Wbt/m}$$

when *a* is in position 2, *b* in position 3, and *c* in position 1, then

$$\lambda_{a_2} = 2\times10^{-7}\left(I_a\, ln\frac{1}{D_s} + I_b\, ln\frac{1}{D_{23}} + I_c\, ln\frac{1}{D_{12}}\right) \text{ Wbt/m}$$

when *a* is in position 3, *b* in position 1, and *c* in position 2, then

$$\lambda_{a_3} = 2\times10^{-7}\left(I_a\, ln\frac{1}{D_s} + I_b\, ln\frac{1}{D_{31}} + I_c\, ln\frac{1}{D_{23}}\right) \text{ Wbt/m}$$

Now, the average value of the flux linkage of *a* is, $\lambda_a = \left(\frac{\lambda_{a_1} + \lambda_{a_2} + \lambda_{a_3}}{3}\right)$

or, $$\lambda_a = \frac{2\times10^{-7}}{3}\left(3I_a\, ln\frac{1}{D_s} + I_b\, ln\frac{1}{D_{12}D_{23}D_{31}} + I_C\, ln\frac{1}{D_{12}D_{23}D_{31}}\right)$$

since, $$I_a = -(I_b + I_c)$$

therefore, $$\lambda_a = \frac{2\times10^{-7}}{3}\left[3I_a\, ln\frac{1}{D_s} - I_a\, ln\left(\frac{1}{D_{12}D_{23}D_{31}}\right)\right]$$

$$= 2\times10^{-7} I_a\, ln\left(\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{D_s}\right) \text{ Wbt/s}$$

So, average inductance per phase is

$$L_a = 2\times10^{-7}\, ln\left(\frac{D_{eq}}{D_s}\right) \text{ H/m} \qquad ...(18)$$

where, $D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$

$= D_m =$ Mutual GMD of the three-phase conductors

and $D_s =$ GMR or self GMD of the conductor

Example - 1.1 A three-phase, fully transposed, 50 Hz, 110 kV, transmission line has horizontal spacing of 3.5 m between adjacent conductors of diameter 1.05 cm. Find the capacitance per phase and charging current per km of line.

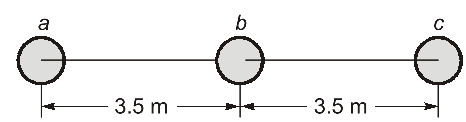
Solution:

The equivalent spacing, $D_{eq} = (3.5 \times 3.5 \times 7)^{1/3} = 4..41$ m

Radius of conductor, $r = \frac{1}{2}\times1.05\times10^{-2} = 5.25\times10^{-3}$ m

Capacitance per phase = Capacitance to neutral (C_n)

$$C_n = \frac{2\pi\varepsilon}{ln\frac{D_m}{r}}$$



Here, D_m = mutual geometric mean distance (= D_{eq})

∴ $D_m = 4.41$ m

$$C_n = \frac{2\pi\varepsilon_0\varepsilon_r}{ln\frac{D_m}{r}} = \frac{2\pi\times8.854\times10^{-12}}{ln\frac{4.41}{5.25\times10^{-3}}} = 8.26\times10^{-12} \text{ F/m}$$

$$= 8.26 \times 10^{-9} \text{ F/km}$$

Capacitive reactance per phase,

$$X_c = \frac{1}{2\pi f C_n} = \frac{1}{2\pi\times50\times8.26\times10^{-9}} = 3.853 \times 10^5 \ \Omega/\text{km}$$

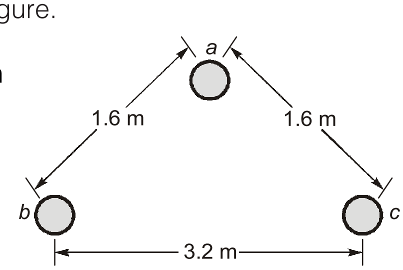
Charging current, $$I_c = \frac{V}{X_c} = \frac{110\times10^3/\sqrt{3}}{3.853\times10^5} = 0.1648 \text{ A/km}$$

Example - 1.2 Determine the inductance of a 3-phase line operating at 50 Hz. Phase conductors of the line having diameter 0.8 cm and arranged at the vertices of an isosceles triangle of sides 1.6 m, 1.6 m and 3.2 m.

Solution:

As given, configuration of three-phase conductors is shown in the figure.

The self GMD of the conductor (D_S) = $\frac{0.7788\times0.8}{2\times100} = 0.003115$ m



The mutual GMD of the conductor (D_m) = $\sqrt[3]{1.6\times3.2\times1.6} = 2.016$ m

∴ Inductance per km = $2\times10^{-7}\, ln\frac{2.016}{0.003115}$

$= 1.294$ mH/km

Important Expressions

1. Inductive per conductor is given by

$$L_1 = 2\times 10^{-7}\, \ln\left(\frac{D}{r_1'}\right) \text{H/m};\quad L_2 = 2\times 10^{-7}\, \ln\left(\frac{D}{r_2'}\right) \text{H/m}$$

Hence, inductance per loop meter is given by

$$L = 4\times 10^{-7}\, \ln\left(\frac{D}{r'}\right) \text{H/m} \qquad (r_1' = r_2' = r)$$

or,
$$L = 0.921 \log\left(\frac{D}{r'}\right) \text{mH/km}$$

2. For a composite conductor,

$$\text{Inductor per loop meter} = 2\times 10^{-7} \ln\left(\frac{GMD}{GMR}\right) \text{H/m} = 2\times 10^{-7} \ln\left(\frac{\text{Mutual } GMD}{\text{Sefl } GMD}\right) \text{H/m}$$

3. Line-to-line capacitance is given by $\left[C_{ab} = \dfrac{\pi\,\varepsilon}{\ln\left(\dfrac{D}{r}\right)} \text{F/m}\right]$

or,
$$\left[C_{ab} = \frac{0.0121}{\log\left(\dfrac{D}{r}\right)} \mu\text{F/km}\right]$$
(Since, $\varepsilon_r = 1$ and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m for air

4. Line-to-line ground capacitance is given by $C_n = C_{an} = C_{bn} = \dfrac{2\pi\varepsilon}{\ln\left(\dfrac{D}{r}\right)}$ F/m $= \dfrac{0.0242}{\log\left(\dfrac{D}{r}\right)}$ μF/km

5. If there is non-uniform distribution of charge on the surface of each conductor then,

$$C_n = \frac{2\pi\,\varepsilon}{\ln\left[\dfrac{D}{2r} + \sqrt{\left(\dfrac{D^2}{4r^2} - 1\right)}\right]} \text{F/m}$$

6. Charging current per phase is given by, $I_{chg} = j\omega C_n V_{an}$ A/m
7. When the effect of earth is taken into account then,

$$C_n = \frac{2\pi\,\varepsilon}{\ln\left[\dfrac{D}{r\left(1 + \dfrac{D^2}{4h^2}\right)}\right]} \text{F/m} \qquad \text{(Line-to-neutral capacitance)}$$

and $$C_{ab} = \frac{\pi\varepsilon}{ln\left[\dfrac{D}{r\left(1+\dfrac{D^2}{4h^2}\right)^{1/2}}\right]} \text{ F/m} \quad \text{(Line-to-line capacitance)}$$

8. For a bundled conductor, $C_n = \dfrac{2\pi\varepsilon}{ln\left(\dfrac{D_{eq}}{D_{SC}^b}\right)}$ F/m

9. Corona:

- Air density factor is, $\delta = \left(\dfrac{3.92h}{t+273}\right)$
- Breakdown strength of air at NTP = 30 kV/cm/peak = 21.1 kV/cm/rms = g_0
- At any temperature and pressure, critical disruptive voltage is given by $\left[V_{d_0} = g_0\delta m_0 r\, ln\left(\dfrac{d}{r}\right)\right]$

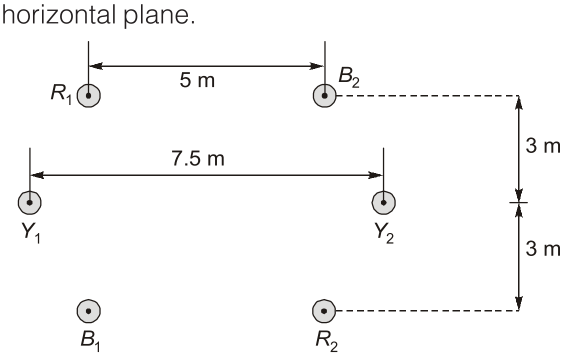
  or, $$\left[V_{d_0} = 21.1\, r\,\delta m_0\, ln\left(\frac{d}{r}\right) \text{ kV/cm/rms per phase}\right]$$

- Visual critical voltage is, $\left[V_V = g_0\,\delta\, r\, M_v\left(1+\dfrac{0.3}{\sqrt{\delta r}}\right) ln\left(\dfrac{d}{r}\right) \text{ kV (rms) to neutral}\right]$
- Corona power loss is given by $\left[P_C = \dfrac{244}{\delta}(f+25)\sqrt{\dfrac{r}{d}}\,(V_{ph}-V_{d0})^2 \times 10^{-5} \text{ kW/km/phase}\right]$

Practice Questions

Q.1 (a) Find the inductance per km of a double circuit 3ϕ line as shown in the figure below. The transmission line is transposed within each circuit and each circuit remains on its ownside. Diameter of each conductor is 15 mm.

(b) Explain why the given arrangment is better as compared to when conductors of the same phase are placed in the same horizontal plane.



Q.2 A 3ϕ 50 Hz, 132 kV transmission line consists of conductors of 1 cm diameter and spaced equilateral at distance of 2 meters. The line conductors have smooth surface with value for m = 0.9. The barometric pressure is 72 cm of Hg and temperature is 25°C. Find:

(a) Critical disruptive voltage

(b) Corona loss per km per phase

Solve this for both conditions (i) fair weather, (ii) bad weather.

Practice Questions Explanations

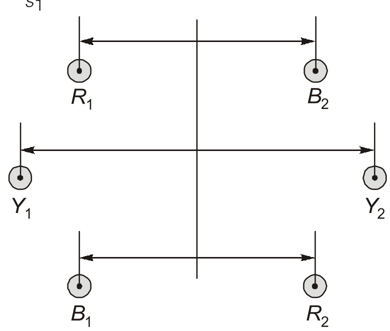
1. Solution :

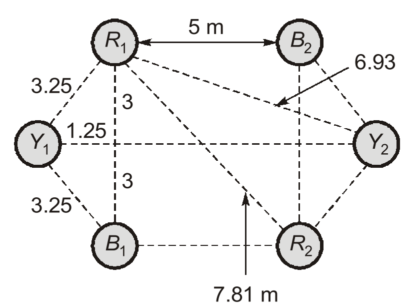
(a) As we know that inductance per phase

$$L_{ph} = 2\times10^{-7}\ln\left(\frac{D_{m_1}D_{m_2}D_{m_3}}{D_{s_1}D_{s_2}D_{s_3}}\right)^{1/3}$$

$$\text{Diameter} = 15\text{ mm},\ r = 7.5\times10^{-3}$$

For D_{m1}, D_{s1}





$$D_{m_1} = \left[\left(D_{R_1Y_1}D_{R_1Y_2}D_{R_1B_1}D_{R_1B_2}\right)\left(D_{R_2Y_1}D_{R_2Y_2}D_{R_2B_1}D_{R_2B_2}\right)\right]^{1/2\times4}$$

$$D_{m_1} = [(3.25\times6.93\times6\times5)\times(6.93\times3.25\times5\times6)]$$

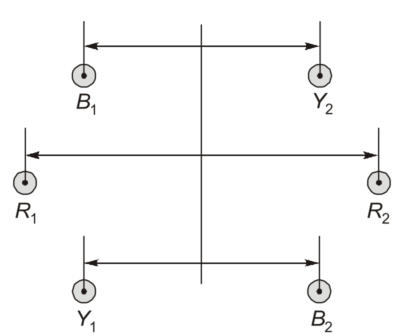
$$= 5.09\text{ m}$$

$$D_{s_1} = \left[\left(D_{R_1R_1}D_{R_1R_2}D_{R_2R_1}D_{R_2R_2}\right)\right]^{1/4}$$

$$= [0.7788\times7.5\times10^{-3}\times7.81\times0.7788\times7.5\times10^{-3}\times7.81]^{1/4}$$

$$D_{s_1} = 0.2135\text{ m}$$

For D_{m2}, D_{s2}



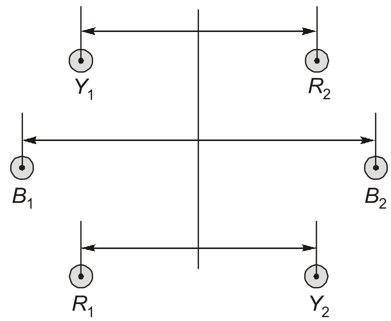
$$D_{m_2} = [(3.25\times6.93\times3.25\times6.93)\times(6.93\times3.25\times6.93\times3.25)]^{1/8}$$

$$= 4.75\text{ m}$$

$$D_{s_2} = [0.7788\times7.5\times10^{-3}\times7.5\times0.7788\times7.5\times10^{-3}\times7.5]^{1/4}$$

$$D_{s_2} = 0.2093\text{ m}$$

For D_{m3}, D_{s3}



$$D_{m3} = [(6 \times 5 \times 3.25 \times 6.93) \times (5 \times 6 \times 6.93 \times 3.25)]^{1/8}$$
$$= 5.09 \text{ m}$$
$$D_{s3} = [0.7788 \times 10^{-3} \times 7.5 \times 7.81 \times 0.7788 \times 7.5 \times 10^{-3} \times 7.81]^{1/4}$$
$$D_{s3} = 0.2135 \text{ m}$$

Now,
$$L_{ph} = 2 \times 10^{-7} \ln\left[\frac{5.09 \times 5.09 \times 4.75}{0.2135 \times 0.2135 \times 0.2093}\right]^{1/3} = 0.63 \text{ mH/km/ph}$$

(b) Conductors are in diagonally placed.

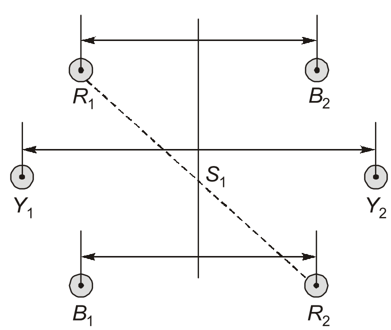


Fig. 1

Conductors in same horizontal line.

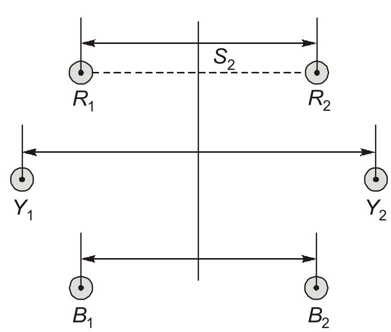


Fig. 2

Comparison:

1. If conductors are diagonally placed as shown in figure, self GMD is increased because spacer length S_1 is more compared to S_2.
2. Inductance per phase is reduced, because of increase in self GMD.
3. Power transfer capability is inversely proportional to inductance so power capability increased.

Because of above advantages instead of horizontal plane, arragement in Fig. 1 is used.