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Signals and Systems

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Contents
Signals and Systems

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CHAPTER 1

Introduction to Signals

Introduction

A signal is any quantity having information associated with it. It may also be defined as a function of one or more independent variables which contain some information. A function defines a relationship between two sets i.e. one is domain and another is range.

It means function defines mapping from one set to another and similarly a signal may also be defined as mapping from one set (domain) to another (range). e.g.

- A speech signal would be represented by acoustic pressure as a function of time.
- A monochromatic picture would be represented by brightness as a function of two spatial variables.
- A voltage signal is defined by a voltage across two points varying as function of time.

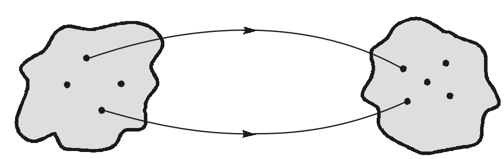


Fig. 1.1

- A video signal, in which color and intensity as a function of 2-dimensional space (2D) and 1-dimensional time (i.e. hybrid variables).

Note: In this course of "signals and systems", we shall focus on signals having only one variable and will consider 'time' as independent variable.

1.1 Elementary Signals

These signals serve as basic building blocks for construction of somewhat more complex signals. The list of elementary signals mainly contains singularity functions and exponential functions. These elementary signals are also known as basic signals/standard signals. Let us discuss these basic signals one-by-one.

1.1.1 Unit Impulse Function

A continuous-time unit impulse function $\delta(t)$, also called as dirac delta function is defined as

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

The unit-impulse function is represented by an arrow with strength of '1' which represents its 'area' or 'weight'.

The above definition of an impulse function is more generalised and can be represented as limiting process without any regard to shape of a pulse. For example, one may define impulse function as a limiting case of rectangular pulse, triangular pulse, Gaussian pulse, exponential pulse and sampling pulse as shown below:

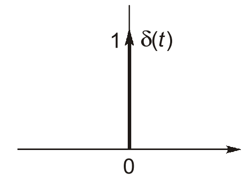


Fig. 1.2

1. Rectangular Pulse

$$\delta(t) = \lim_{\varepsilon \to 0} p(t)$$

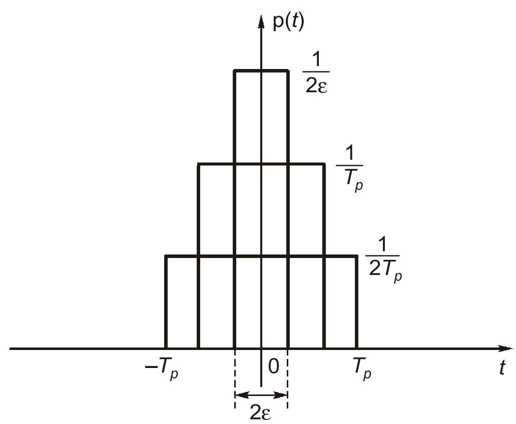


Fig. 1.3

2. Triangular Pulse

$$\delta(t) = \begin{cases} \lim_{\tau \to 0} \frac{1}{\tau}\left[1 - \frac{|t|}{\tau}\right] & ; \quad |t| < \tau \\ 0 & ; \quad |t| > \tau \end{cases}$$

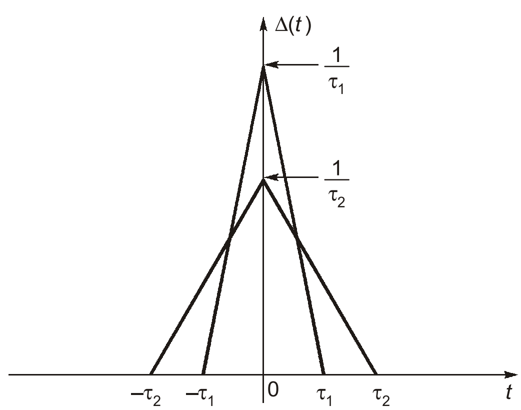


Fig. 1.4

3. Gaussian Pulse

$$\delta(t) = \lim_{\tau \to 0} \frac{1}{\tau}\left[e^{-\pi t^2/\tau^2}\right]$$

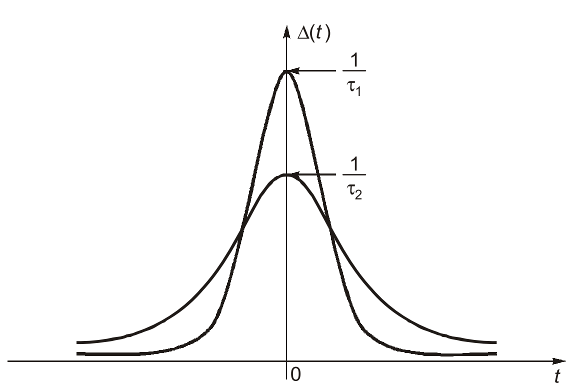


Fig. 1.5

4. Exponential Pulse

$$\delta(t) = \lim_{\tau \to 0} \frac{1}{2\tau}\left[e^{-|t|/\tau}\right]$$

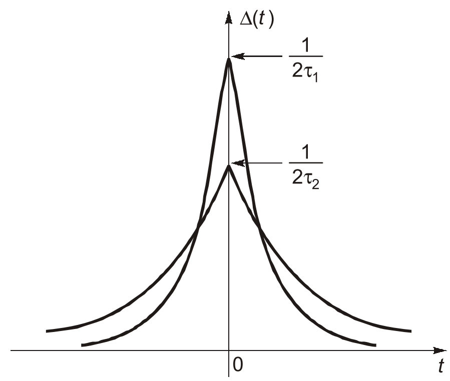


Fig. 1.6

5. Sampling Function

$$\int_{-\infty}^{\infty} \frac{k}{\pi} Sa(kt)dt = 1$$

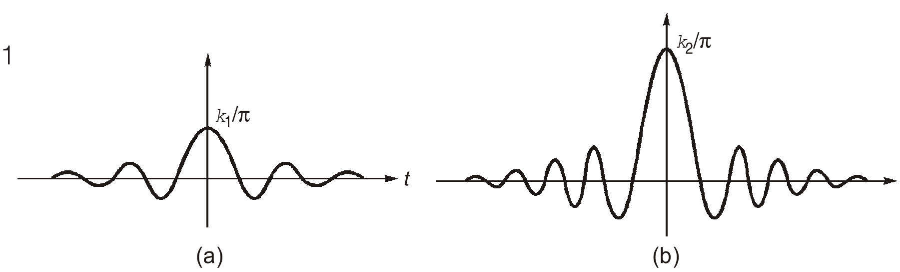


Fig. 1.7

Properties of Continuous Time Unit Impulse Function

1. Scaling property:

$$\delta(at) = \frac{1}{|a|}\delta(t) \quad ; \quad \text{'}a\text{' is a constant, positive or negative}$$

Proof:

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Integrating above equation on both the sides with respect to 't'.

$$\int_{-\infty}^{+\infty} \delta(at)\,dt = \int_{-\infty}^{+\infty} \frac{1}{|a|}\delta(t)\,dt$$

Let, $at = \tau$

$a \cdot dt = d\tau$; 'a' is a constant, positive or negative

or $|a| \cdot dt = d\tau$

Now, $$\int_{-\infty}^{+\infty} \delta(at)\,dt = \int_{-\infty}^{+\infty} \delta(\tau)\cdot\frac{d\tau}{|a|} = \int_{-\infty}^{+\infty} \frac{1}{|a|}\delta(t)\cdot dt$$

By definition, $$\int_{-\infty}^{+\infty} \delta(t)dt = \int_{-\infty}^{+\infty} \delta(\tau)d\tau = 1$$

- $\delta(at \pm b) = \frac{1}{|a|}\delta\left(t \pm \frac{b}{a}\right)$
- $\delta(-t) = \delta(t)$ $\cdots$ $\delta(t)$ is an even function of time.

2. Product property/multiplication property:

$$x(t)\delta(t-t_o) = x(t_o)\,\delta(t-t_o)$$

Proof :

The function $\delta(t-t_o)$ exists only at $t = t_o$. Let the signal $x(t)$ be continuous at $t = t_o$.

Therefore, $x(t)\,\delta(t-t_o) = x(t)\big|_{t=t_o}\cdot\delta(t-t_o) = x(t_o)\,\delta(t-t_o)$

- $x(t)\,\delta(t) = x(0)\,\delta(t)$

3. Sampling property:

$$\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_o)\,dt = x(t_o)$$

Proof :

Using product property of impulse function

$$x(t)\,\delta(t-t_o) = x(t_o)\,\delta(t-t_o)$$

Integrating above equation on both the sides with respect to 't'.

$$\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_o)dt = \int_{-\infty}^{+\infty} x(t_0)\,\delta(t-t_o)dt$$

$$= x(t_o)\int_{-\infty}^{+\infty} \delta(t-t_o)dt = x(t_o)$$

- $\displaystyle\int_{-\infty}^{+\infty} x(t)\delta(t)\,dt = x(0)$

4. The first derivative of unit step function results in unit impulse function.

$$\delta(t) = \frac{d}{dt}u(t)$$

Proof :

Let the signal $x(t)$ be continuous at t = 0.

Consider the integral, $\displaystyle\int_{-\infty}^{+\infty}\frac{d}{dt}[u(t)]\,x(t)dt = [u(t)\,x(t)]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} x'(t)\,u(t)\,dt$

$$= x(\infty) - \int_{0}^{\infty} x'(t)\,d(t)$$

$$= x(\infty) - [x(t)]_0^{\infty}$$

$$= x(0) \quad \text{...(i)}$$

We know from sampling property, $\displaystyle x(0) = \int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt$ ...(ii)

From equations (i) and (ii), we get

$$\int_{-\infty}^{+\infty}\frac{d}{dt}[u(t)]\,x(t)dt = \int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt$$

On comparing, we get $\displaystyle\delta(t) = \frac{d}{dt}u(t)$

5. Derivative property:

$$\int_{t_1}^{t_2} x(t)\delta^n(t-t_o)dt = (-1)^n x^n(t)\Big|_{t=t_0} \;;\quad t_1 < t_0 < t_2 \text{ and suffix } n \text{ means } n^{th} \text{ derivative}$$

Proof:

Let the signal $x(t)$ be continuous at $t = t_0$ where $t_1 < t_0 < t_2$.

Consider the derivative $\frac{d}{dt}[x(t)\,\delta(t-t_0)] = x(t)\,\delta'(t-t_0) + x'(t)\,\delta(t-t_0)$

Integrating above equation on both the sides with respect to 't'.

$$\int_{t_1}^{t_2} \frac{d}{dt}[x(t)\,\delta(t-t_0)]dt = \int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt + \int_{t_1}^{t_2} x'(t)\,\delta(t-t_0)dt$$

$$[x(t)\,\delta(t-t_0)]_{t_1}^{t_2} = \int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt + \int_{t_1}^{t_2} x'(t)\,\delta(t-t_0)dt$$

$$[x(t_2)\,\delta(t_2-t_0) - x(t_1)\,\delta(t_1-t_0)] = \int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt + \int_{t_1}^{t_2} x'(t)\,\delta(t-t_0)dt$$

Here, $\delta(t_1 - t_0) = 0$ and $\delta(t_2 - t_0) = 0$ because $t_0 \neq t_1$ or $t_0 \neq t_2$

So,
$$0 = \int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt + \int_{t_1}^{t_2} x'(t)\delta(t-t_0)dt$$

$$\int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt = (-1)\int_{t_1}^{t_2} x'(t)\delta(t-t_0)dt$$

($\because$ using sampling property)

$\Rightarrow$
$$= (-1)\,x'(t_0)$$

Hence,
$$\int_{t_1}^{t_2} x(t)\,\delta'(t-t_0)dt = (-1)^1\,x'(t_0)$$

If same procedure is repeated for second derivative, we get

$$\int_{t_1}^{t_2} x(t)\,\delta''(t-t_0)dt = (-1)^2\,x''(t_0)$$

On generalising aforementioned results, we get

$$\int_{t_1}^{t_2} x(t)\,\delta^n(t-t_0)dt = (-1)^n\,x^n(t_0)$$

6. Shifting Property:

According to shifting property, any signal can be produced as combination of weighted and shifted impulses.

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

Proof:

Using product property, $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

Replacing t_0 by τ, $x(t)\,\delta(t-\tau) = x(\tau)\,\delta(t-\tau)$

Integrating above equation on both the sides with respect to 'τ'.

$$\int_{-\infty}^{+\infty} x(t)\,\delta(t-\tau)d\tau = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)d\tau$$

$$x(t)\int_{-\infty}^{+\infty} \delta(t-\tau)d\tau = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)d\tau$$

$$x(t)\cdot 1 = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)d\tau$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)d\tau$$

7. The derivative of impulse function is known as *doublet* function:

$$\delta'(t) = \frac{d}{dt}\delta(t)$$

Graphically,

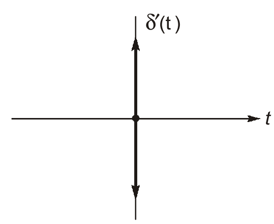


Fig. 1.8

Area under the ***doublet*** function is always zero.

Discrete-Time Case

The discrete time unit impulse function δ[n], also called unit sample sequence or delta sequence is defined as,

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

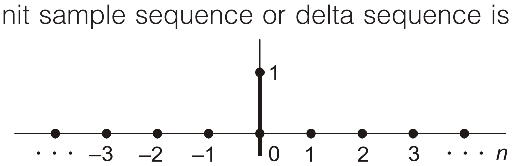


Fig. 1.9

It is also known as *Kronecker delta*.

Properties of Discrete Time Unit Impulse Sequence

1. Scaling property:

$$\delta[kn] = \delta[n]\,;\ k \text{ is an integer}$$

Proof:

By definition of unit impulse sequence,

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Similarly,

$$\delta[kn] = \begin{cases} 1, & kn = 0 \\ 0, & kn \neq 0 \end{cases}$$

$$= \begin{cases} 1, & n = \dfrac{0}{k} = 0 \\ 0, & n \neq \dfrac{0}{k} \neq 0 \end{cases} = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} = \delta[n]$$

2. Product property:

$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$$

From definition,

$$\delta[n-n_0] = \begin{cases} 1, & n = n_0 \\ 0, & n \neq n_0 \end{cases}$$

We see that impulse has a non zero value only at $n = n_0$

Therefore, $$x[n]\,\delta[n-n_0] = x[n]\big|_{n=n_0}\,\delta[n-n_0]$$

$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$$

3. Shifting property:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

Proof:

From product property, $x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$

Replacing n_0 by 'k', $x[n]\,\delta[n-k] = x[k]\,\delta[n-k]$

$$\Rightarrow \quad \sum_{k=-\infty}^{+\infty} x[n]\,\delta[n-k] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

$$\Rightarrow \quad x[n]\sum_{k=-\infty}^{+\infty} \delta[n-k] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

$$\Rightarrow \quad x[n]\cdot 1 = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

4. The first difference of unit step sequence results in unit impulse sequence.

$$\delta[n] = u[n] - u[n-1]$$

Proof:

By definition of unit step sequence,

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] \qquad \text{...(i)}$$

$$= \delta[n] + \sum_{k=1}^{\infty} \delta[n-k]$$

But, $$u[n-1] = \sum_{k=1}^{\infty} \delta[n-k]$$

$$u[n] = \delta[n] + u[n-1]$$

We get, $$\delta[n] = u[n] - u[n-1]$$

Graphically we can see,

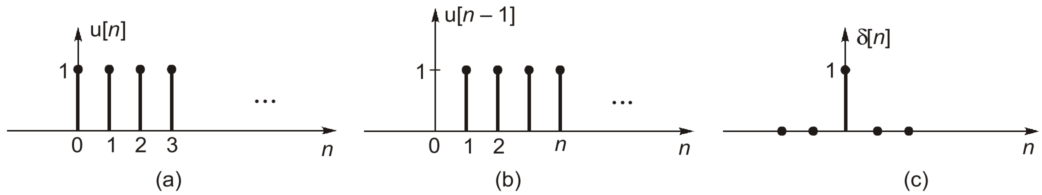


Fig. 1.10

Summary Table:

S.No.	Properties of CT unit Impulse Function	Properties of DT unit impulse sequence
1.	$\delta(t)=\begin{cases}\infty, & t=0\\ 0, & \text{otherwise}\end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)dt=1$	$\delta[n]=\begin{cases}1, & \text{if } n=0;\\ 0, & \text{otherwise}\end{cases}$
2.	$x(t)\,\delta(t-t_0)=x(t_0)\delta(t-t_0)$	$x[n]\delta[n-k]=x[k]\delta[n-k]$
3.	$\delta(t)=\dfrac{d}{dt}u(t)$	$\delta[n]=u[n]-u[n-1]$
4.	$\int_0^{\infty}\delta(t-\tau)d\tau=u(t)$	$\sum_{k=0}^{\infty}\delta[n-k]=u[n]$
5.	$x(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau$	$x[n]=\sum_{k=-\infty}^{\infty}x[k]\,\delta[n-k]$
6.	$\int_{-\infty}^{\infty}x(t)\delta(t-t_0)dt=x(t_0)$	$\sum_{n=-\infty}^{\infty}x[n]\delta[n-n_0]=x[n_0]$
7.	$\delta(at)=\dfrac{1}{\lvert a\rvert}\delta(t)$ $\delta(at\pm b)=\dfrac{1}{\lvert a\rvert}\delta\left(t\pm\dfrac{b}{a}\right)$ $\delta(-t)=\delta(t)$	$\delta[kn]=\delta[n]$ $\delta[-n]=\delta[n]$
8.	$\int_{t_1}^{t_2}x(t)\delta(t)dt=\begin{cases}x(0), & t_1<0<t_2\\ 0, & \text{otherwise}\end{cases}$	
9.	$\int_{t_1}^{t_2}x(t)\delta^n(t-t_0)dt=(-1)^n\ x^n(t_0),\ t_1<t_0<t_2$ where suffix n mean n^{th} derivative	
10.	$\delta'(t)=\dfrac{d}{dt}\delta(t)$	

1.1.2 Unit Step Function

The continuous-time unit step function, also called **"Heaviside"** unit function, is defined as,

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

Graphically,

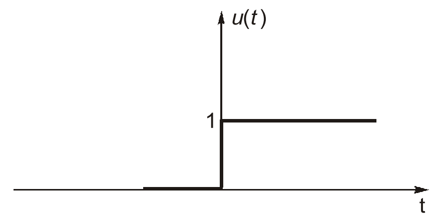


Fig. 1.11

The function value at $t = 0$ is indeterminate (discontinous)

Properties of unit step function:

(i) The unit step function can be represented as integral of weighted, shifted impulses.

$$u(t) = \int_0^{\infty} \delta(t - \tau) d\tau$$

Proof:

According to the shifting property, $x(t) = \int_{-\infty}^{+\infty} x(\tau)\, \delta(t - \tau) d\tau$

Let, $x(t) = u(t)$

$$u(t) = \int_{-\infty}^{+\infty} u(\tau)\, \delta(t - \tau) d\tau = \int_0^{+\infty} \delta(t - \tau)\, d\tau$$

Since, $u(\tau) = 0 \;;\; -\infty < \tau < 0$

$u(\tau) = 1 \;;\; \tau \geq 0$

(ii) Scaling property:

$$u(at) = u(t)$$

NOTE

- The unit step function is continuous for all t, except for $t = 0$ where sudden change take place (i.e. discontinuity).
- $u(0) = \frac{1}{2}$ (The average value)

Discrete-Time Case

The discrete time unit-step sequence $u[n]$ is defined as,

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Graphically,

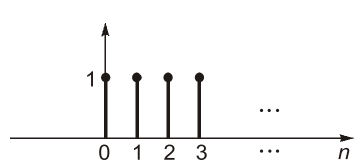


Fig. 1.12

1.1.3 Unit Ramp Function:

A continuous time unit ramp function is defined as

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Also, $$r(t) = tu(t)$$

Graphically,

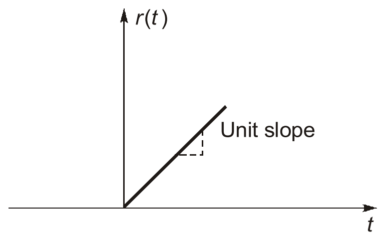


Fig. 1.13

Discrete-Time Case

A discrete-time unit ramp sequence is defined as

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Also, $$r[n] = nu[n]$$

Graphically,

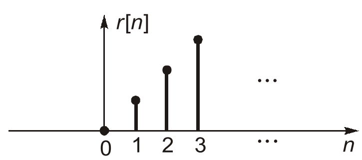


Fig. 1.14

- $r(t) = \int_{-\infty}^{t} u(t)\, dt$
- $r(t) = \int_{-\infty}^{t} \int_{-\infty}^{\alpha} \delta(\tau)\, d\tau\, d\alpha$
- $r[n] = nu[n]$

1.1.4 Unit Parabolic Function:

A continuous-time unit parabolic function $p(t)$ (unit acceleration function) is defined as

$$p(t) = \begin{cases} \dfrac{t^2}{2}, & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Also, $$p(t) = \frac{tr(t)}{2} = \frac{t^2}{2}u(t)$$

Graphically,

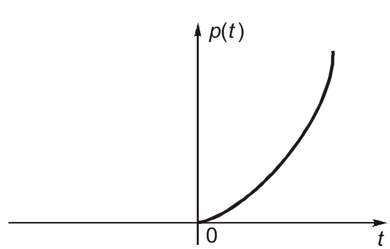


Fig. 1.15

Discrete-Time Case

The discrete time unit parabolic sequence $p[n]$ is defined as,

$$p[n] = \begin{cases} \dfrac{n^2}{2}, & n \ge 0 \\ 0, & n < 0 \end{cases}$$

Also, $$p[n] = \frac{nr[n]}{2} = \frac{n^2u[n]}{2}$$

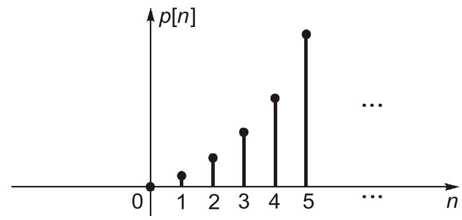


Fig. 1.16

1.1.5 Signum Function

The continuous time signum function, sgn(t) is defined as,

$$\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$$

We see, $$u(t) - u(-t) = \text{sgn}(t)$$

Also $$u(t) + u(-t) = 1$$

We get, $$\text{sgn}(t) = 2u(t) - 1$$

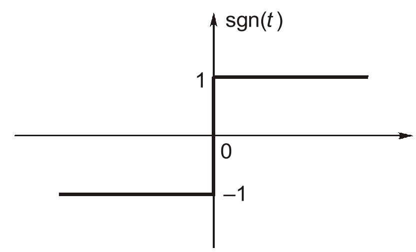


Fig. 1.17

Keeping following facts in the mind that is

(i) $\lim\limits_{\alpha \to 0} e^{-\alpha t} = 1, \; t > 0$

(ii) $\lim\limits_{\alpha \to 0} e^{\alpha t} = 1, \; t < 0$

The positive half of signum function can be represented as $\lim\limits_{\alpha \to 0} e^{-\alpha t}u(t)$ and the negative half, as $\lim\limits_{\alpha \to 0} e^{\alpha t}u(-t)$.

Mathematically, sgn(t) can be represented as limiting case of exponential as

$$\text{sgn}(t) = \lim_{\alpha \to 0} \left[e^{-\alpha t}u(t) - e^{\alpha t}u(-t)\right]$$

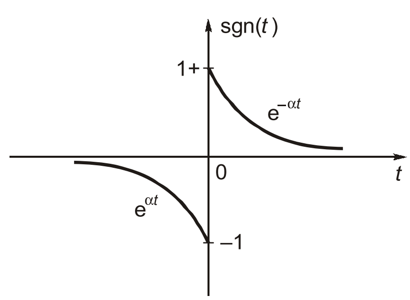


Fig. 1.18 : sgn(t) as limiting curve of exponential function

The discrete-time signum sequence is defined

$$\text{sgn}[n] = \begin{cases} -1, & n < 0 \\ 0, & n = 0 \\ 1, & n > 0 \end{cases}$$

Also, $$\text{sgn}[n] = u[n-1] - u[-n-1]$$

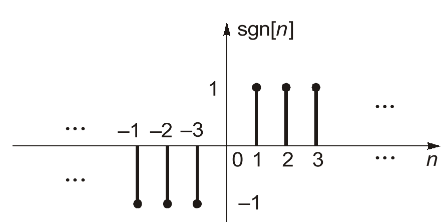


Fig. 1.19

1.1.6 Exponentials and Sinusoidal Signal

A general form of complex exponential signal is

$$x(t) = Ce^{\alpha t}$$

Depending upon values of C and α we further classify complex exponential as

1.1.6.1 Real exponential : Both C and α are real.

1.1.6.2 Periodic complex exponential : C is real α is purely imaginary.

1.1.6.3 Sinusoidal

1.1.6.4 Complex exponential : Both C and α are complex.

(i) Real Exponential Signal

Continuous-time case

A continuous-time real exponential signal, in general form can be defined as

$$x(t) = Ce^{\alpha t} \ ; \text{ both } C \text{ and } \alpha \text{ are real}$$

- If α is positive, the signal is growing exponential signal.
- If α is negative, the signal is decaying exponential signal.
- For $\alpha = 0$, $x(t)$ is constant signal.

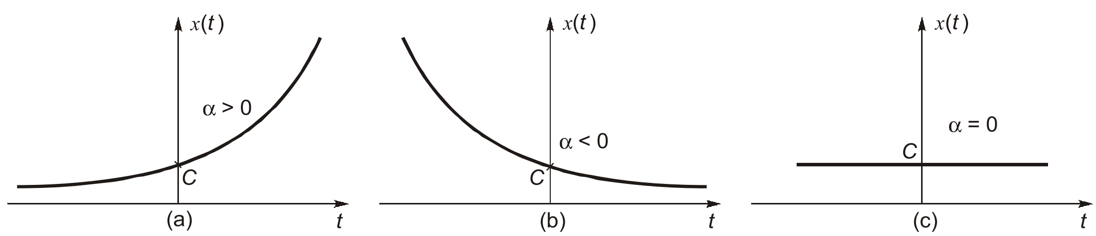


Fig. 1.20

Discrete-time case

The discrete-time real exponential sequence, in general form can be defined as

$$x[n] = Ca^n, \quad \text{for all } n$$

Where $a = e^{\beta}$ and β is real.

Discrete-time real exponential are often used to describe population growth as a function of generation and investments as a function of month, or year.

Let us draw discrete-time real exponential for different values of 'a' for $C = 1$.

(a) $x[n] = a^n \ ; \ 0 < a < 1$

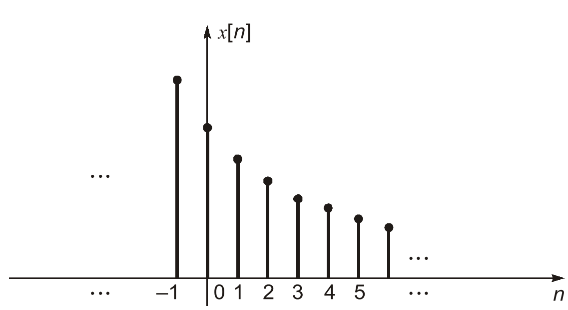


Fig. 1.21 *Decaying exponential for $0 < a < 1$*

(b) $x[n] = a^n$; $a > 1$

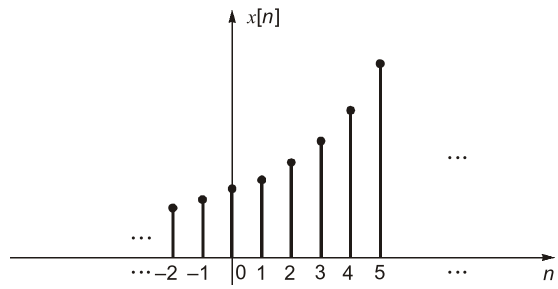


Fig. 1.22 *Growing exponential for a > 1*

(c) $x[n] = a^n$; $-1 < a < 0$

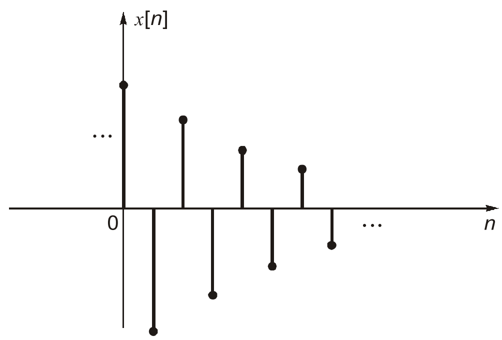


Fig. 1.23 *Decaying alternating exponential for –1 < a < 0*

(d) $x[n] = a^n$; $a < -1$

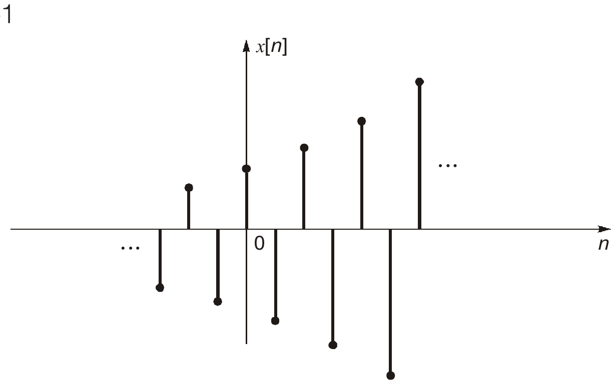


Fig. 1.24 *Growing alternating exponential for a < –1*

(ii) Periodic Complex Exponential

The signal $x(t) = Ce^{\alpha t}$ with α be purely imaginary results in periodic complex exponential.

i.e. $$x(t) = e^{j\omega_0 t} \text{ with period } T$$

where fundamental period of $x(t)$ is T_0 and is given as

$$T_0 = \frac{2\pi}{\omega_0}$$

- The signal $e^{j\omega_0 t}$ is always periodic with period $T = 2\pi/\omega_0$ for any value of ω_0.
- The signal $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ have same fundamental period.
- For $\omega_0 = 0$, $x(t)$ is constant and therefore is periodic with period T for any value of T.
- Fundamental frequency of constant signal is zero i.e. a constant signal has a zero rate of oscillation.

By using Euler's relation, $$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$$

Discrete-Time Case

The signal $x[n] = Ca^n$ where $a = e^{\beta}$ and β be purely imaginary results in discrete-time periodic signal

$$x[n] = e^{j\Omega_0 n}$$

The discrete-time periodic complex exponential is periodic with period $N = 2\pi m/\Omega_0$.

Comparison table of CT exponential and DT exponential

$e^{j\omega_0 t}$	$e^{j\Omega_0 n}$
Distinct signals for any value of ω_0	Identical signals for values of Ω_0 separated by multiples of 2π
Periodic for any value of ω_0	Periodic only if $\Omega_0 = 2\pi m/N$ for some pair of integers $N > 0$ and m.
Fundamental frequency ω_0	Fundamental frequency Ω_0/m (N and m do not have any common factor)
Fundamental period $= \begin{cases} \text{Undefined}, & \omega_0 = 0 \\ \dfrac{2\pi}{\omega_0}, & \omega_0 \neq 0 \end{cases}$	Fundamental period $= \begin{cases} \text{Undefined}, & \Omega_0 = 0 \\ m\left(\dfrac{2\pi}{\Omega_0}\right) & \Omega_0 \neq 0 \end{cases}$

*CT = continuous-time and DT = discrete time

(iii) Sinusoidal signal

A closely related to continuous-time periodic complex exponential is sinusoidal signal.

$$x(t) = A\sin(\omega_0 t + \phi)$$

Like periodic complex exponential signal, the sinusoidal signal is periodic with fundamental period T_0 and is given as,

$$T_0 = \frac{2\pi}{\omega_0}$$

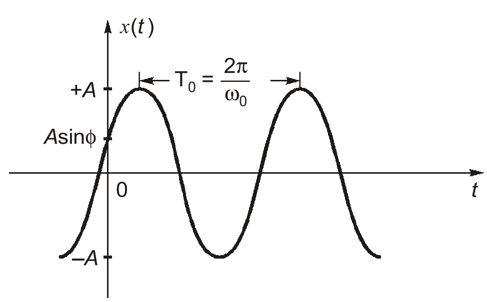


Fig. 1.25

Discrete-Time Case

A closely related to discrete-time periodic complex exponential is sinusoidal signal

$$x[n] = A\sin(\Omega_0 n + \phi)$$

Example-1.1 Sketch the following signals:

(a) $x_1(t) = \delta(\cos t)$

(b) $x_2(t) = \text{sgn}\left(\sin \frac{\pi}{T} t\right)$

(c) $x_3(t) = t\,\text{sgn}\,(\cos t) \quad 0 \le t \le 2\pi$

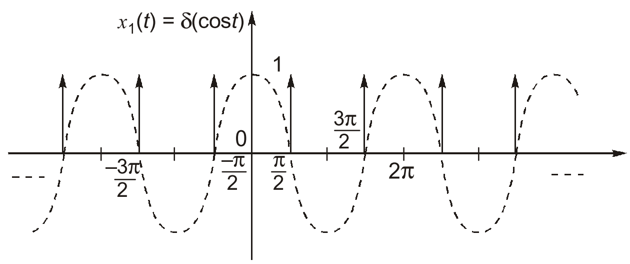
(d) $x_4(t) = u\left(\sin \frac{\pi}{T} t\right) - u\left(-\sin \frac{\pi}{T} t\right)$

Solution:

(a) Since, $\delta(t) = 0, \quad t \neq 0$

we get, $x_1(t) = \delta(\cos t) = 0, \ \cos t \neq 0$

$x_1(t)$ is shown in figure (a).

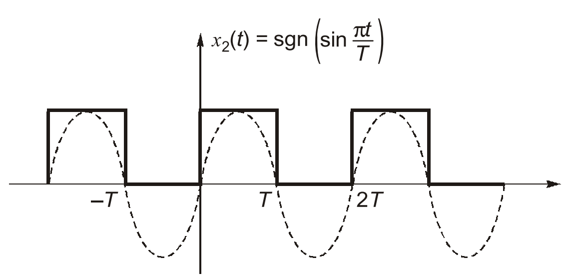


(a)

(b) Since, $\text{sgn} = \begin{cases} 1; & t > 0 \\ -1; & t < 0 \end{cases}$

we get, $x_2(t) = \text{sgn}\left(\sin \frac{\pi}{T} t\right) = \begin{cases} 1; & \sin \frac{\pi}{T} t > 0 \\ -1; & \sin \frac{\pi}{T} t < 0 \end{cases}$

$x_2(t)$ is shown in figure (b).

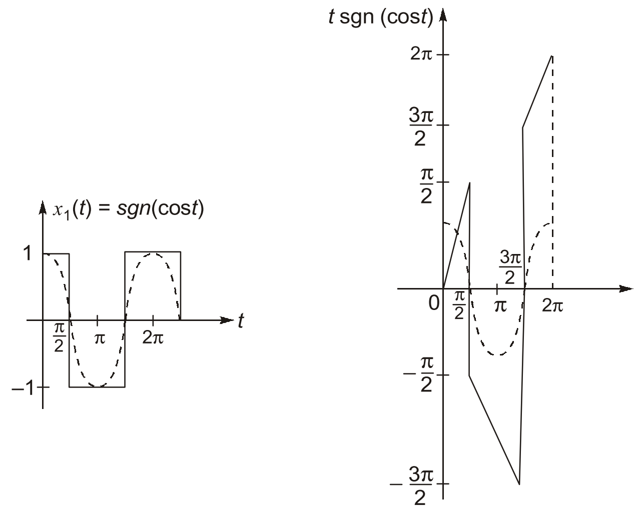


(b)

(c) Since, $\text{sgn}(t) = \begin{cases} 1; & t > 0 \\ -1; & t < 0 \end{cases}$

we get, $t\,\text{sgn}\,(\cos t) = \begin{cases} t; & \cos t > 0 \\ -t; & \cos t < 0 \end{cases}$

The signal $t\,\text{sgn}\,(\cos t)$ is shown in figure (c).



(c)

(d) Since,

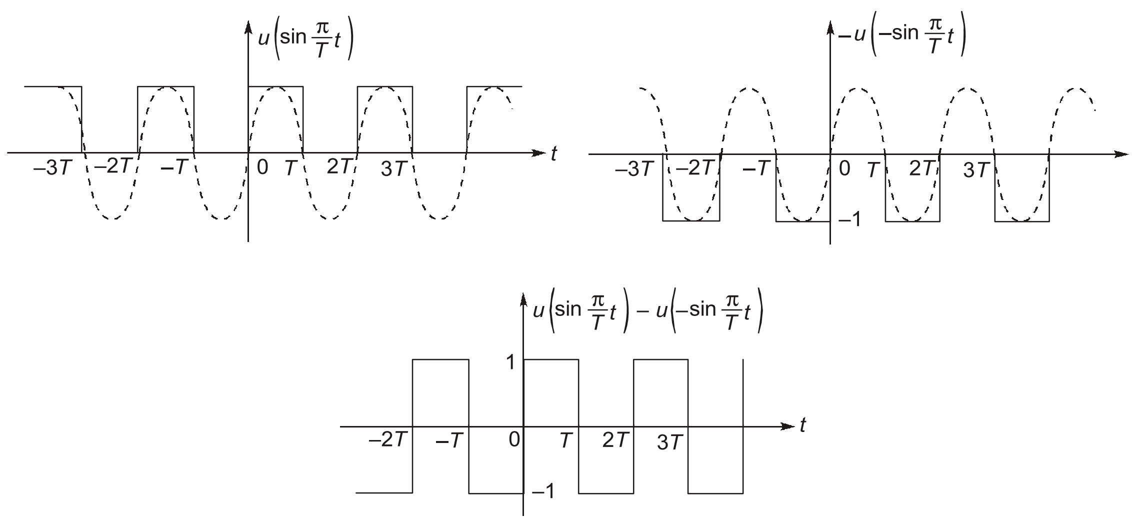
$$u(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0 \end{cases}$$

we get,

$$u\left(\sin\frac{\pi}{T}t\right) = \begin{cases} 1; & \sin\frac{\pi}{T}t > 0 \\ 0; & \sin\frac{\pi}{T}t < 0 \end{cases}$$

$$u\left(\sin\frac{\pi}{T}t\right) - u\left(-\sin\frac{\pi}{T}t\right) = \begin{cases} 1; & \sin\frac{\pi}{T}t > 0 \\ -1; & \sin\frac{\pi}{T}t < 0 \end{cases}$$

The signal $u\left(\sin\frac{\pi}{T}t\right) - u\left(-\sin\frac{\pi}{T}t\right)$ is shown in figure (d).



(d)