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BIG LEARNINGS MADE EASY

Strength of Materials

Mechanical Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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Strength of Materials

First Edition : 2018
Second Edition : 2019
Reprint : 2020
Reprint : 2021
Reprint : 2022
Reprint : 2023

Reprint : 2024

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CHAPTER 01

Properties of Material and Basic Concepts

1.1 Introduction

Strength of materials is a subject that studies the internal effects of stress and strain in a solid body that is subjected to an external loading. Stress is associated with the strength of the material from which the body is made, while strain is a measure of the deformation of the body. A thorough understanding of the fundamentals of this subject is of vital importance because many of the formulas and rules of design cited in engineering codes are based upon the principles of this subject.

1.1.1 Equations of Equilibrium

Equilibrium of a body requires both a balance of forces, to prevent the body from translating or having accelerated motion along a straight or curved path, and a balance of moments, to prevent the body from rotating. These conditions can be expressed mathematically by two vector equations.

$$\Sigma F = 0$$

$$\Sigma M_O = 0 \quad \text{...(i)}$$

Here, ΣF represents the sum of all the forces acting on the body and ΣM_O is the sum of the moments of all the forces about any point O either on or off the body. If an x, y, z co-ordinate system is established with the origin at point O, the force and moment vectors can be resolved into components along each coordinate axis and the above two equations can be written in scalar form as six equations, namely

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \quad \text{...(ii)}$$

Often in engineering practice, the loading on a body can be represented as a system of coplanar forces. If this is the case, and the forces lie in the x-y plane, then the conditions for equilibrium of the body can be specified with only three scaler equilibrium equations, that is,

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M_O = 0 \quad \text{...(iii)}$$

Here all the moments are summed about point O, and so they will be directed along the z-axis.

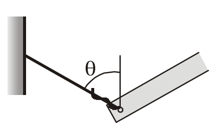
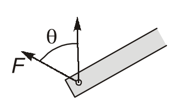
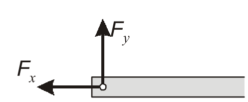
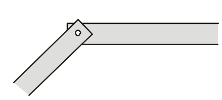
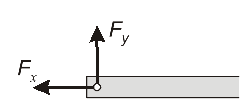
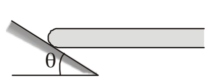
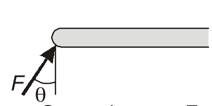
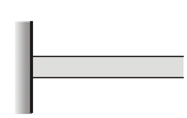
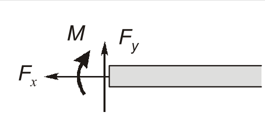
Successful application of the equations of equilibrium requires complete specification of all the known and unknown forces that act on the body and so the best way to account for all these forces is to draw the body's free body diagram.

1.1.2 Body Forces and Support Reactions

Body forces: A body force is developed when one body exerts a force on another body without direct physical contact between the bodies. Examples include the effects caused by the earth's gravitation or its electromagnetic field. Although body forces affect each of the particles composing the body, these forces are normally represented by a single concentrated force acting on the body. In the case of gravitation, this force is called the **weight** of the body and acts through the body's center of gravity.

Support Reactions: The surface forces that develop at the supports or points of contact between bodies are called **reactions**. For two dimensional problems, i.e., bodies subjected to coplanar force systems, the supports most commonly encountered are shown in Table. Note carefully the symbol used to represent each support and the type of reactions it exerts on its contacting member. As a general rule, if the support prevents translation in a given direction, then a force must be developed on the member in that direction. Likewise, if rotation is prevented, a couple moment must be exerted on the member. For example, the roller support only prevents translation perpendicular or normal to the surface. Hence, the roller exerts a normal force *F* on the member at its point of contact. Since the member can freely rotate about the roller, a couple moment cannot be developed on the member.

TABLE

Type of connection	**Reaction**	**Type of connection**	**Reaction**
 Cable	 One unknownn: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: *F*	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: *F*	 Fixed support	 Three unknowns: F_x, F_y, M

1.1.3 Types of Forces

1. Normal force (N): This force acts perpendicular to the area. It is developed whenever the external loads tend to push or pull on the two segments of the body.
2. Shear force (V): The shear force lies in the plane of the area and it is developed when the external loads tend to cause the two segments of the body to slide over one another.

3. **Torsional moment or Torque (T):** This effect is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

4. **Bending moment (M):** The bending moment is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.

1.1.4 Coplanar Loadings

If the body is subjected to a coplanar system of forces as shown in Fig., then only normal force, shear force and bending moment components will exist at the section. If we use the x, y and z coordinate axes, as shown on the left segment, then N can be obtained by applying $\Sigma F_x = 0$, and V can be obtained by $\Sigma F_y = 0$. Finally, the bending moment M_O can be determined by summing moments about point O (the z-axis), $\Sigma M_O = 0$, in order to eliminate the moments caused by the unknowns N and V.

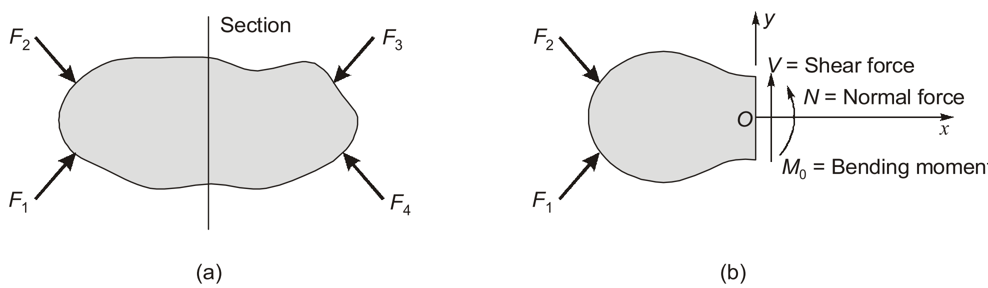
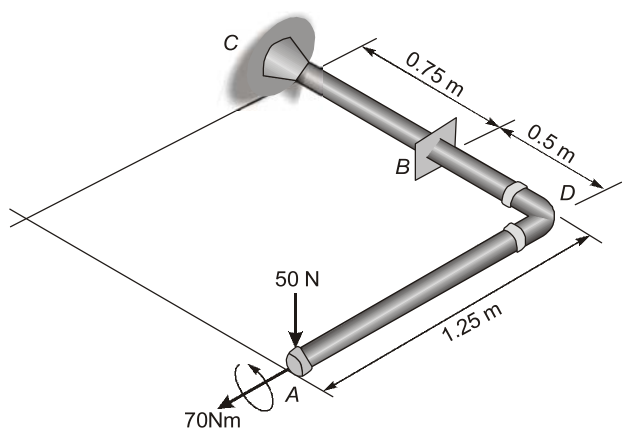


Fig.

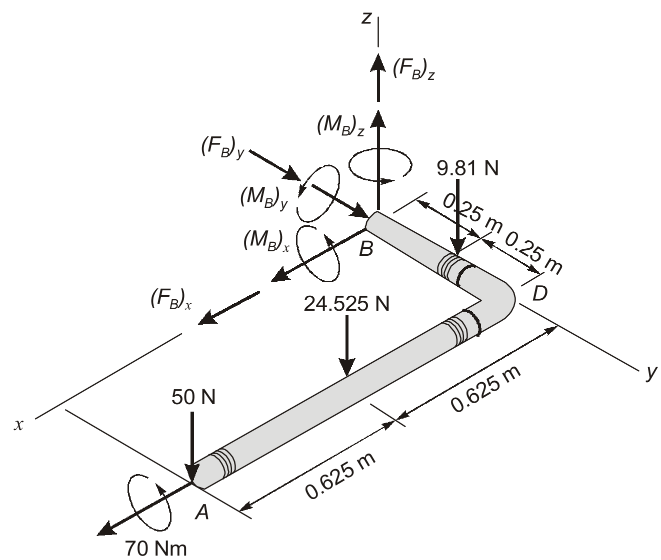
Example 1.1 Determine the resultant internal loadings acting on the cross section at B of the pipe shown in figure.The pipe has a mass of 2 kg/m and is subjected to both a vertical force of 50 N and a couple moment of 70 Nm at its end A. It is fixed to the wall at C.



Solution:

The problem can be solved by considering segment AB, so we do not need to calculate the support reactions at C.

Free-Body Diagram: The x, y, z axes are established at B and the free-body diagram of segment AB is shown in figure. The resultant force and moment components at the section are assumed to act in the positive coordinate directions and to pass through the *centroid* of the cross-sectional area at B. The weight of each segment of pipe is calculated as follows:



$$W_{BD} = (2\text{ kg/m})\,(0.5\text{ m})\,(9.81\text{ N/kg})$$
$$= 9.81\text{ N}$$
$$W_{AD} = (2\text{ kg/m})\,(1.25\text{ m})\,(9.81\text{ N/kg})$$
$$= 24.525\text{ N}$$

These forces act through the centre of gravity of each segment.

Equations of Equilibirum: Applying the six scalar equations of equilibrium, we have

$\Sigma F_x = 0;$ $\quad (F_B)_x = 0$

$\Sigma F_y = 0;$ $\quad (F_B)_y = 0$

$\Sigma F_z = 0;$ $\quad (F_B)_z - 9.81\text{ N} - 24.525\text{ N} - 50\text{ N} = 0$

$$(F_B)_z = 84.3\text{ N}$$

$\Sigma (M_B)_x = 0;$ $\quad (M_B)_x + 70\text{ Nm} - 50\text{ N}\,(0.5\text{ m})$

$$-24.525\text{ N}\,(0.5\text{ m}) - 9.81\text{ N}\,(0.25\text{ m}) = 0$$

$$(M_B)_x = -30.3\text{ Nm}$$

$\Sigma (M_B)_y = 0;$ $\quad (M_B)_y + 24.525\text{ N}\,(0.625\text{ m}) + 50\text{ N}\,(1.25\text{ m}) = 0$

$$(M_B)_y = -77.8\text{ Nm}$$

$\Sigma (M_B)_z = 0;$ $\quad (M_B)_z = 0$

NOTE

What do the negative signs for $(M_B)_x$ and $(M_B)_y$ indicate?

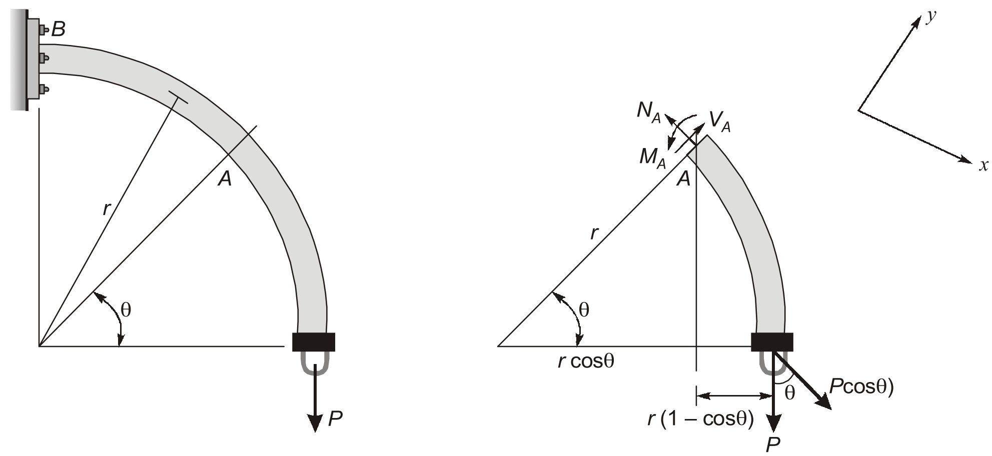
Note that the normal force $N_B = (F_B)_y = 0$,

whereas the shear force is $V_B = \sqrt{(0)^2 + (84.3)^2} = 84.3$ N.

Also, the torsional moment is $T_B = (M_B)_y = 77.8$ Nm

and the bending moment is $M_B = \sqrt{(30.3)^2 + (0)^2} = 30.3$ Nm.

Example 1.2 The curved rod has a radius r and is fixed to the wall at B. Determine the resultant internal loadings acting on the cross-section through A which is located at an angle θ from the horizontal.



Solution:

Equations of Equilibrium: For point *A*

$\searrow + \Sigma F_x = 0;$ $\quad P\cos\theta - N_A = 0$

$$N_A = P\cos\theta$$

$\nearrow + \Sigma F_y = 0;$ $\quad V_A - P\sin\theta = 0$

$$V_A = P\sin\theta$$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A - P[r(1 - \cos\theta)] = 0$

$$M_A = Pr(1 - \cos\theta)$$

1.2 Stress

- Stress is internal resistance per unit area offered by material against deformation. The unit of stress unit is N/m^2 or Pa.

  A material is capable of offering the following types of stresses:

  1. Normal stress
  2. Shear stress

- The fundamental concept of stress can be understood by considering a prismatic bar that is loaded by axial forces *P* at the ends as shown in the Fig.

  A prismatic bar is a straight structural member having constant cross-sectional area throughout its length. In the Fig. (a), axial forces produce a uniform stretching of the bar, hence, the bar is said to be in tension and in Fig. (c) forces produce uniform compression of the bar, hence, the bar is said to be in compression.

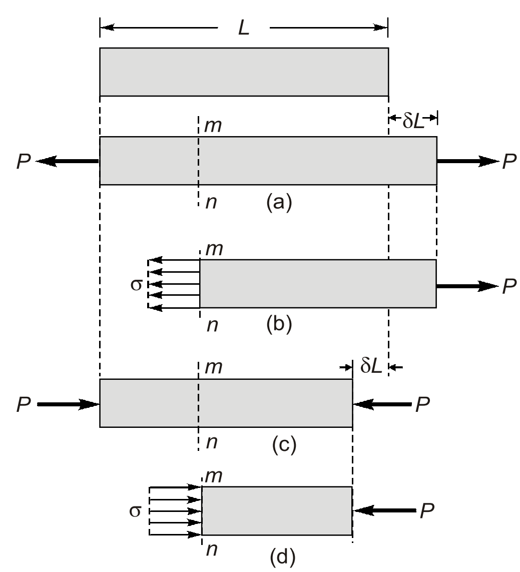


Fig.

To investigate the internal stresses produced in the bar by axial forces, we make an imaginary cut at section *mn* FIg. (b & d). This section is taken perpendicular to the longitudinal axis of bar, hence, it is known as cross-section.

Now isolate the part of the bar to the right of the cut and consider the right of the cut as a free body. The force *P* has a tendency to move free body in the direction of load, so to restrict the motion of bar an internal force is induced which is uniformly distributed over cross-sectional area. The intensity of force that is force per unit area is called the **stress**.

Thus, stress can be defined as **"Stress is internal resistance of material offered against deformation which is force per unit area"**.

- Stress induced in material depends upon the nature of force, point of application and cross-sectional area of material. Stress can be **Tensile** or **Compressive** in nature depending on the nature of load. Generally, stress is represented by the Greek letter σ (sigma). We can calculate stress mathematically as

$$\sigma = \frac{P}{A} \qquad \text{...(i)}$$

When a sign convention for normal stresses is required, it is customary to define tensile stresses as positive and compressive stresses or negative.

Units: S.I. units of stress is Newtons per square meter (N/m^2) or pascals (Pa).

1 Pa = 1 N/m^2

1 MPa = 10^6 Pa or 10^3 kPa   also   1 MPa = 1 N/mm^2

1 GPa = 10^9 Pa or 10^6 kPa   also   1 GPa = 1 kN/mm^2

NOTE

- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If body or bar is free to move or free expansion is allowed then no stresses will be induced.
- Pressure has same unit as stress but pressure is magnitude of external forces applied on the cross-section and stress is magnitude of internal resiting force developed on the cross-section.

On the basis of cross-sectional area considered during calculation of stresses, direct stresses can be of following two types:

1. Engineering stress or nominal stress
2. True stress or Actual stress

- **Engineering stress (Nominal stress)**

Mathematically,

$$\sigma = \frac{P}{A_0}$$ where, A_0 = original cross-sectional area of specimen taken

- **True stress (Actual stress)**

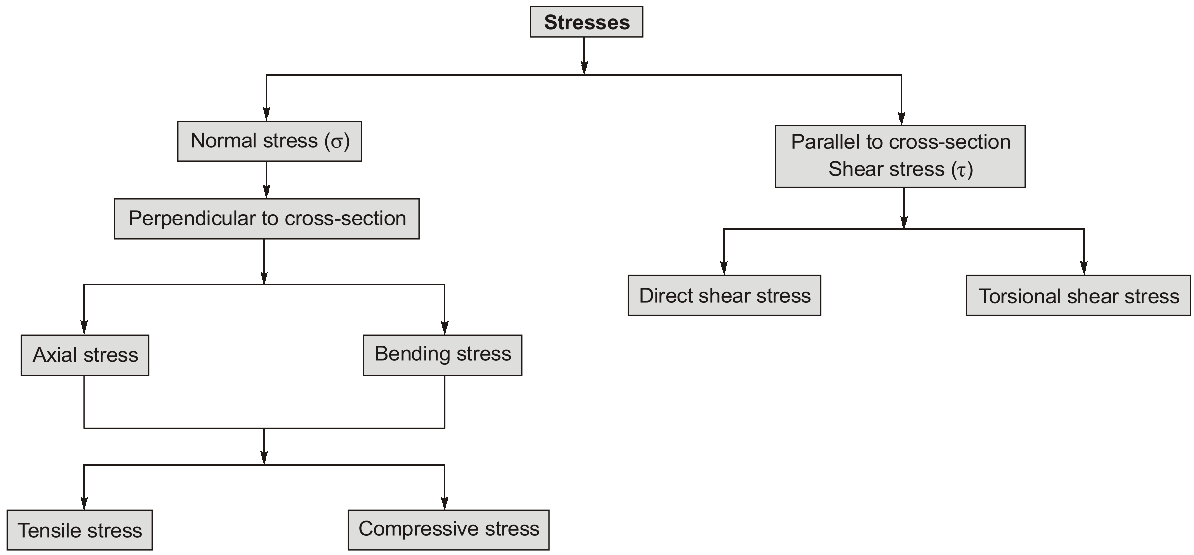
Mathematically,

$$\sigma = \frac{P}{A_a}$$ where, A_a = Actual cross-sectional area of specimen at any time of loading i.e. changed area of cross-section due to loading

$A_a = A_0 \pm \Delta A$ '+' for Compression, '–' for Tension

Remember:
- In tension, true or actual stress is always greater than engineering or nominal stress.
- In compression, true or actual stress is always less than engineering or nominal stress.

Types of Stresses



1. Normal Stress

Normal stresses are always normal to the cross-section at any section. Normal stresses are represented by greek letter σ. Normal stress may be of two type:

(a) **Direct Stress or Axial Stress:** These stresses are produced when an axial force is acted at *CG* of cross-section. For prismatic body with axial loading, direct stresses are uniform across the cross-section. Generally, tensile stresses are taken positive, and compressive stresses are taken negative.

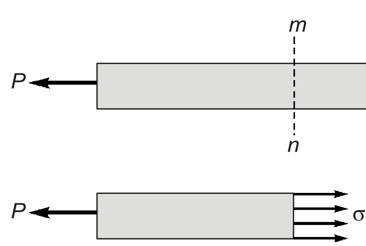


Fig.

(b) **Bending Stresses:** Bending stresses are produced by bending moment. Bending stresses vary linearly from zero at neutral axis to maximum at farthest fibre from neutral axis.

Tensile bending stresses are taken as positive and compressive bending stresses are taken as negative.

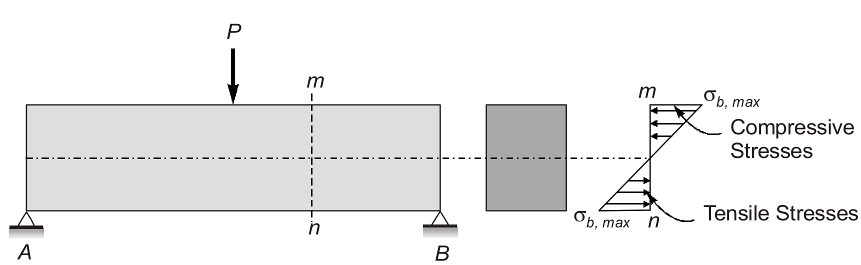
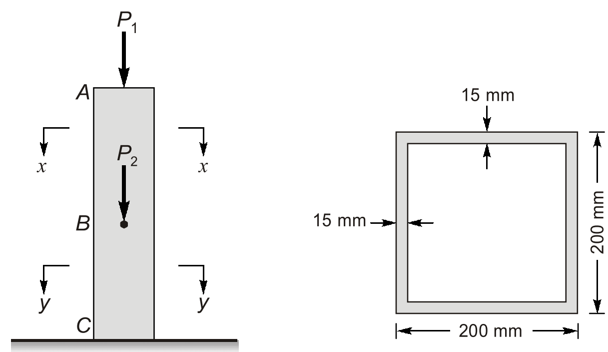


Fig.

Example 1.3 A two storey column *ABC* in a building is constructed with a hollow square box section as shown below. The roof load at the top of column P_1 = 80 kN, and the floor load at mid height is P_2 = 100 kN. Obtain the compressive stresses σ_{AB} and σ_{BC} at two section *x-x* and *y-y* respectively.



Solution:

P_1 = 80 kN, P_2 = 100 kN

Area of cross-section of column,

$$A = (200 \times 200) - [(200 - 30) \times (200 - 30)]$$

$$= 40000 - 28900 = 11100 \text{ mm}^2$$

In portion *AB*, at section *x-x*, column is subjected to axial force P_1,

so, stress in portion *AB*, $$\sigma_{AB} = \frac{P_1}{A} = \frac{80 \times 10^3}{11100} \text{ N/mm}^2 = 7.21 \text{ N/mm}^2$$

In portion *BC*, at section *y-y*, column is subjected to

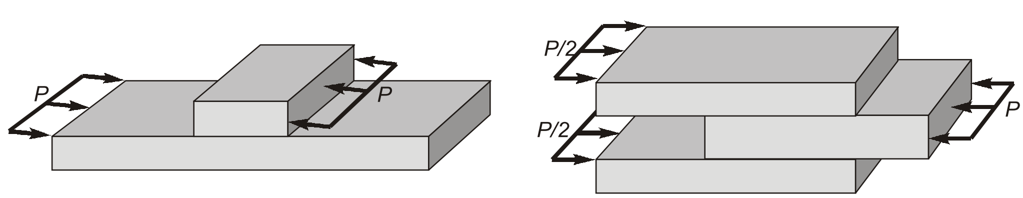
Total axial force, $P = P_1 + P_2 = 80 + 100 = 180$ kN

So, stress in portion *BC*, $$\sigma_{BC} = \frac{P}{A} = \frac{180 \times 10^3}{11100} \text{ N/mm}^2 = 16.22 \text{ N/mm}^2$$

2. Shear Stress or Tangential Stress

Shear stress is resistance offered by material against shearing force. Its value is determined by dividing the shear force in the plane of the section by corresponding area. Its unit is N/mm^2.

$$\tau = \frac{S}{A} \text{ N/mm}^2$$



(a) (b)

Fig. *Loading conditions causing shear stresses*

Shear stress may be of following two type:

(a) Direct shear stress: Direct shear stress is produced due to direct shear force acting on surface.

(b) Torsional Stress: This stress is produced when member is subjected to torsional moment or torque.

Do you know?: A shear stress in a given direction cannot exist without a balancing shear stress of equal intensity in a direction at right angle to it. This stress is called **complementary shear stress**.

Sign convention:

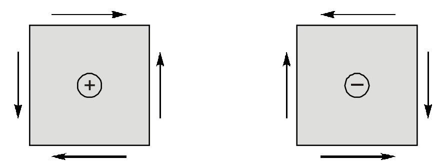
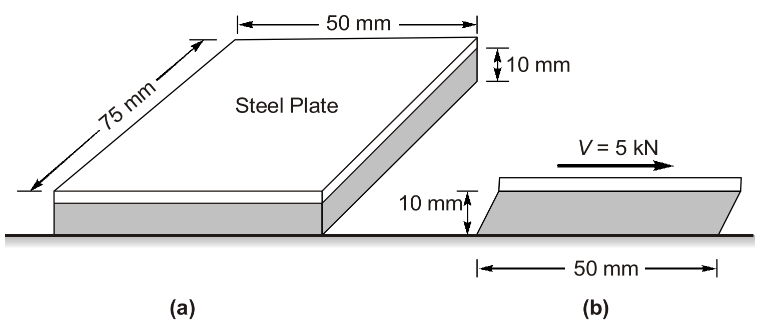


Fig.

Example 1.4 A bearing pad consisting of a flexible material of thickness 10 mm capped by a thin steel plate of dimension 50 × 75 mm is subjected to a horizontal shear force 5 kN. Determine the average shear stress.



Solution:

The average shear stress will be equal to the shear force divided by the area over which it acts.

$$\tau_{avg} = \frac{V}{A} = \frac{5\times 10^3}{50\times 75}\text{ N/mm}^2 = 1.33 \text{ N/mm}^2$$

1.2.1 General state of stress, stress tensor and matrix representation of stress

- If the body is sectioned by planes parallel to the x-z plane, y-z plane and x-y plane, we can cut out a cubic element that represents the state of stress acting on the element.
- Tensor is defined as a mathematical object analogous to but more general than a vector, represented by an array of components that are functions of the coordinates of a space.
- Stress is a second order tensor
- Stress in 3D form can be represented in a matrix form as

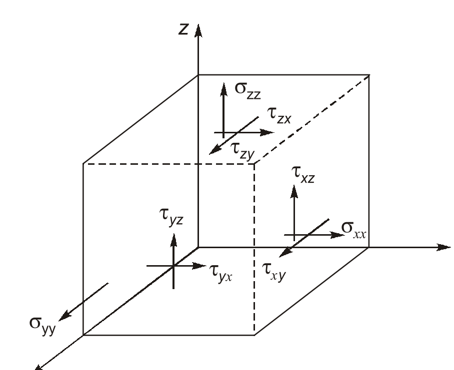


Fig. *Triaxial state of stress at a point*

$$[\text{Stress}] = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

- In the above notations σ_{xx} represents normal stress with 1st letter in subscript representing plane of face and the second letter representing the direction of stress $\sigma \underbrace{x}_{\text{face}} \underbrace{x}_{\text{direction}}$.
- τ_{xy} represents tangential or shear stress in which x represents plane or face and y represents direction of stress $\tau \underbrace{x}_{\text{face}} \underbrace{y}_{\text{direction}}$.

NOTE: When a point drawn on any face maximum three stresses can be developed.

1.2.2 Shear stress equilibrium

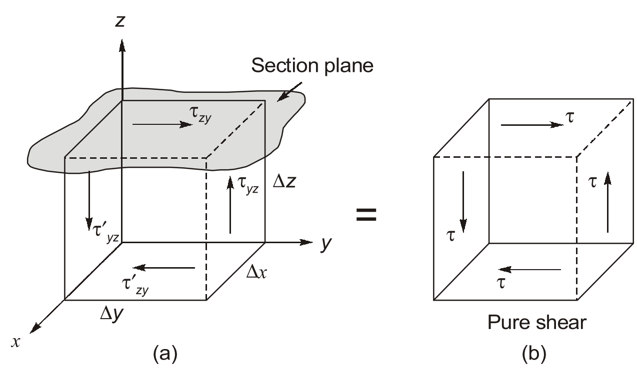


Fig.

Fig. (a) shows a volume element of material taken at a point located on the surface of a sectioned area which is subjected to a shear stress τ_{zy}. Force and moment equilibrium requires the shear stress acting on this face of the element to be accompanied by shear stress acting on three other faces. To show this we will first consider force equilibrium in the y direction.Then

$$\Sigma F_y = 0; \qquad \tau_{zy}(\Delta x\ \Delta y) - \tau'_{zy}\ \Delta x\ \Delta y = 0$$

$$\tau_{zy} = \tau'_{zy}$$

In a similar manner, force equilibrium in the z-direction yields $\tau_{zy} = \tau'_{zy}$. Finally, taking moment about the x-axis.

$$\Sigma M_x = 0; -\tau_{zy}(\Delta x\ \Delta y)\ \Delta z + \tau_{yz}(\Delta x\ \Delta z)\ \Delta y = 0$$

$$\tau_{zy} = \tau_{yz}$$

So that $\qquad \tau_{zy} = \tau'_{zy} = \tau_{yz} = \tau'_{yz} = \tau$

In other words, all four shear stresses must have equal magnitude and be directed either toward or away from each other at opposite edges of the element as represented in Fig. (b). This is referred to as the complementary property of shear, and under the conditions shown in Fig., the material is subjected to pure shear.

1.2.3 Allowable Stresses

The ability of structure to resist loading is called strength. For safety, material should have much higher strength than the strength required for a external loading. The ratio of the actual strength to the required strength is called the ***factor of safety*** (FOS).

$$\text{FOS}(n) = \frac{\text{Failure stress}}{\text{Working stress}} = \frac{\text{Actual strength}}{\text{Required strength}}$$

So for maintaining required factor of safety, allowable stress can be calculated as

$$\sigma_{\text{allowable}} = \frac{\text{Yield stress}}{\text{FOS}} \quad \text{(Ductile materials)}$$

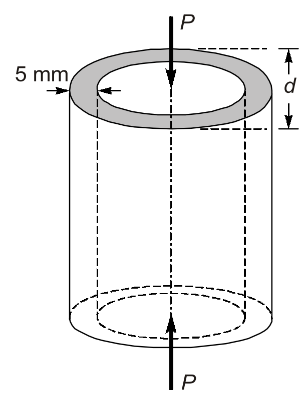
$$\sigma_{\text{allowable}} = \frac{\text{Ultimate stress}}{\text{FOS}} \quad \text{(Brittle materials)}$$

Do You Know?

- In air craft design generally margin of safety is considered rather than factor of safety.
- The margin of safety is defined as the factor of safety minus one.

$$\boxed{\text{Margin of safety} = n - 1}$$

Example 1.5 A short hollow circular cast iron cylinder support an axial compressive load $P = 50$ kN. The ultimate stress in compression for the material is $\sigma_u = 280$ N/mm². It is decided to design the cylinder with a wall thickness 5 mm and factor of safety of 3.0 with respect to the ultimate strength. Compute the minimum required outside diameter of the cylinder.



Solution:

The allowable compressive stress is equal to the ultimate stress divided by the factor of safety.

$$\sigma_{\text{Allowable}} = \frac{\sigma_u}{\text{FOS}} = \frac{280}{3}\text{ N/mm}^2 = 93.33\text{ N/mm}^2$$

The required cross-sectional area is,

$$A = \frac{P}{\sigma_{\text{allowable}}} = \frac{50 \times 10^3}{93.33} = 535.71\text{ mm}^2$$

The actual cross-sectional area is

$$A = \frac{\pi d^2}{4} - \frac{\pi d^2}{4} + \frac{4\pi dt}{4} - \frac{4\pi t^2}{4} = \pi t(d - t)$$

so, $\pi \times 5(d - 5) = 535.71\text{ mm}^2$

$\Rightarrow$ $d = 39.10$ mm

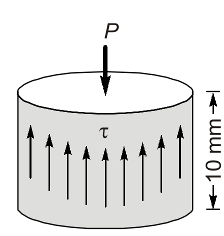
Example 1.6 Structural steel has a shearing ultimate strength of approximately 310 MPa. Determine the force P necessary to punch a 25 mm diameter hole through the 10 mm thick plate of steel.

Solution:

Let as assume uniform shearing on a cylindrical surface.

Diameter, $D = 25$ mm, Thickness, $t = 10$ mm

For equilibrium of force



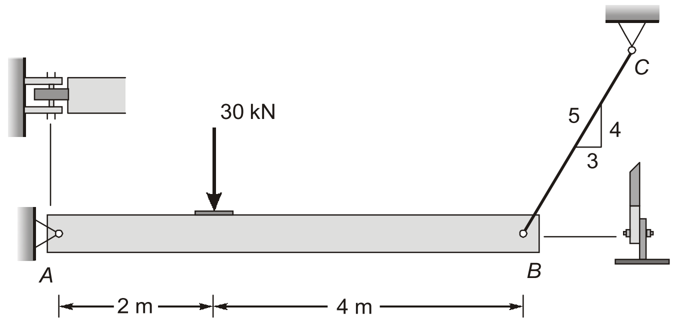
P = shear force resisted by material

$P = \tau \times$ surface area

$= 310 \times \pi \times D \times t = 310 \times \pi \times 25 \times 10$

$= 243.47$ kN

Example 1.7 Determine the average shear stress in the 20 mm diameter pin at *A* and the 30 mm diameter pin at *B* that support the beam in figure.



Solution:

Internal Loadings: The forces on the pins can be obtained by considering the equilibrium of the beam, as shown in Fig. (a).

$\circlearrowleft \Sigma M_A = 0;$ $$F_B\left(\frac{4}{5}\right)(6\text{m}) - 30\text{ kN}(2\text{m}) = 0$$

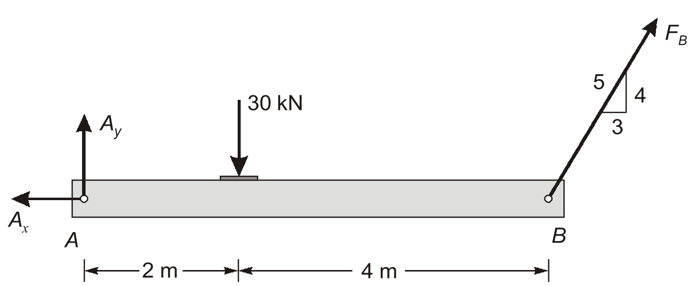
$$F_B = 12.5\text{ kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $$(12.5\text{ kN})\left(\frac{3}{5}\right) - A_x = 0$$

$$A_x = 7.50\text{ kN}$$

$+\uparrow \Sigma F_y = 0;$ $$A_y + (12.5\text{ kN})\left(\frac{4}{5}\right) - 30\text{ kN} = 0$$

$$A_y = 20\text{ kN}$$

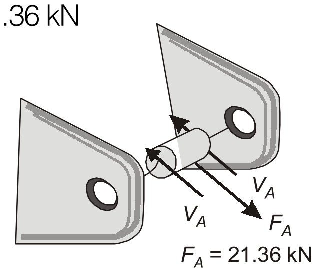


(a)

Thus, the resultant force acting on pin *A* is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(7.50\text{ kN})^2 + (20\text{ kN})^2} = 21.36\text{ kN}$$

The pin at A is supported by two fixed "leaves" and so the free-body diagram of the center segment of the pin shown in Fig. (b) has two shearing surfaces between the beam and each leaf. The force of the beam (21.36 kN) acting on the pin is therefore supported by shear force on each of these surfaces. This case is called double shear. Thus



(b)

$$V_A = \frac{F_A}{2} = \frac{21.36\text{ kN}}{2} = 10.68\text{ kN}$$

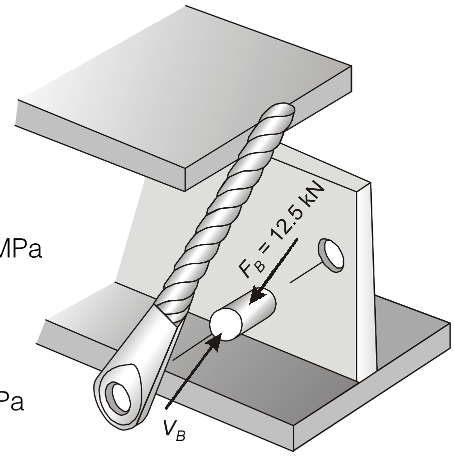
In Fig. (Question), note that pin B is subjected to single shear, which occurs on the section between the cable and beam, Fig. (c). For this pin segment,

$$V_B = F_B = 12.5 \text{ kN}$$

Average shear stress

$$(\tau_A)_{avg} = \frac{V_A}{A_A} = \frac{10.68(10^3) \text{ N}}{\frac{\pi}{4}(0.02 \text{ m})^2} = 34.0 \text{ MPa}$$

$$(\tau_B)_{avg} = \frac{V_B}{A_B} = \frac{12.5(10^3) \text{ N}}{\frac{\pi}{4}(0.03 \text{ m})^2} = 17.7 \text{ MPa}$$



(c)

1.3 Strain

Strain is a measure of deformation representing the displacement between particles in the body relative to a reference length.

Mathematically strain can be calculated as

$$\in = \frac{\Delta L}{L}$$

Unit: Strain is dimensionless quantity. It is always expressed in the form of number. If the member is in tension, the strain is called a tensile strain. If the member is in compression, the strain is called a compressive strain.

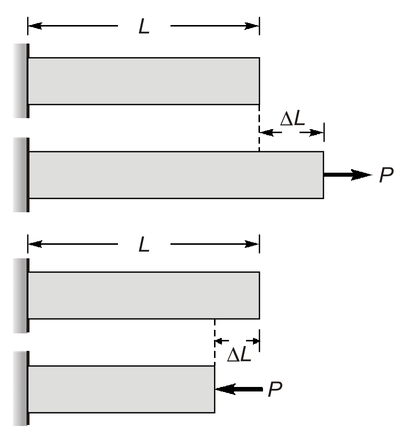


Fig.

On the basis of length of member used in calculation of strain, strain can be of following two types:

1. Engineering or nominal strain
2. True or Actual strain

Engineering or Nominal Strain

Engineering or nominal strain is strain calculated, when length of member is taken as original length

$$\in_0 = \frac{\Delta l}{l_0} \qquad l_0 = \text{original length of member}$$

True or Actual Strain

True or actual strain is strain calculated, when length of member is taken as actual length of member at loading

$$\in_a = \frac{\Delta l}{l_a} \qquad l_a = \text{Actual length of member}$$

$$l_a = l_0 \pm \Delta l$$

Sign convention:

- Tensile strains are positive where as compressive strains are negative.