NEXT IAS
BIG LEARNINGS MADE EASY

Strength of Materials

Civil Engineering

Comprehensive Theory

with Solved Examples

Civil Services Examination

MADE EASY Publications Pvt. Ltd.

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station),

New Delhi-110016 | **Ph. :** 9021300500

E-mail: infomep@madeeasy.in | **Web :** www.madeeasypublications.org

Strength of Materials

First Edition: 2019

Reprint : 2020

Reprint : 2021

Second Edition: 2022

Reprint : 2023

Reprint : 2024

Contents

Strength of Materials

Chapter 5

Chapter 6

Chapter 7

Chapter 8

■■■■

Strength of Materials

01

CHAPTER

Properties of Material and Basic Concepts

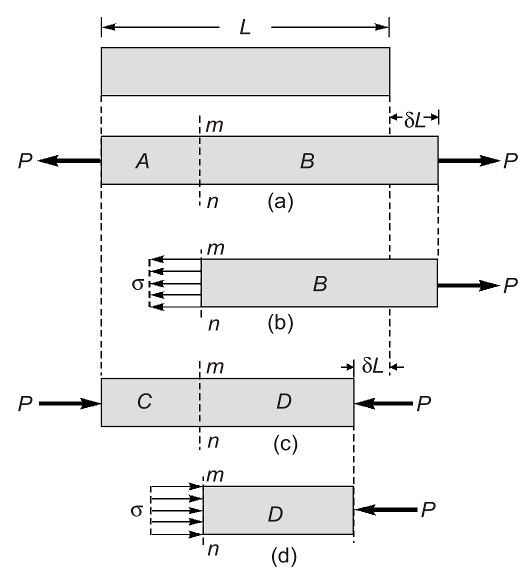
1.1 INTRODUCTION

Strength of materials is a subject that studies the internal effects of stress and strain in a solid body that is subjected to an external loading. Stress is associated with the strength of the material from which the body is made, while strain is a measure of the deformation of the body. A thorough understanding of the fundamentals of this subject is of vital importance because many of the formulas and rules of design cited in engineering codes are based upon the principles of this subject.

1.2 STRESS

- Stress is internal resistance per unit area offered by material against deformation. The unit of stress is N/m^2 or Pa.
- The fundamental concept of stress can be understood by considering a prismatic bar that is loaded by axial forces *P* at the ends as shown in the Fig.

  A prismatic bar is a straight structural member having constant cross-sectional area throughout its length. In the Fig. (a), axial forces produce a uniform stretching of the bar, hence, the bar is said to be in tension and in Fig. (c) forces produce uniform compression of the bar, hence, the bar is said to be in compression.



To investigate the internal stresses produced in the bar by axial forces, we consider a section *mn* as shown in figure. This section is taken perpendicular to the longitudinal axis of bar, hence, it is known as cross-section.

In figure (b), part *B* is subjected to axial force *P* at the right end. In order to keep part *B* in equilibrium, part *A* offers a resistance at section *mn* which is uniformly distributed over cross-sectional area.

Similarly, in fig. (d), part *C* offers resistance at section *mn* to keep part *D* in equilibrium.

This internal resistance offered by the material is called the stress.

Thus, stress can be defined as **"Stress is internal resistance per unit area offered by the material against deformation"**.

- Stress induced in material depends upon the nature of force, point of application and cross-sectional area of material. Stress can be **Tensile** or **Compressive** in nature depending on the nature of load. Generally, stress is represented by the Greek letter σ (sigma). We can calculate stress mathematically as

$$\sigma = \frac{P}{A} \quad \text{...(i)}$$

When a sign convention for normal stresses is required, it is customary to define tensile stresses as positive and compressive stresses or negative.

Units: S.I. units of stress is Newtons per square meter (N/m^2) or pascals (Pa).

$1 \text{ Pa} = 1 \text{ N/m}^2$

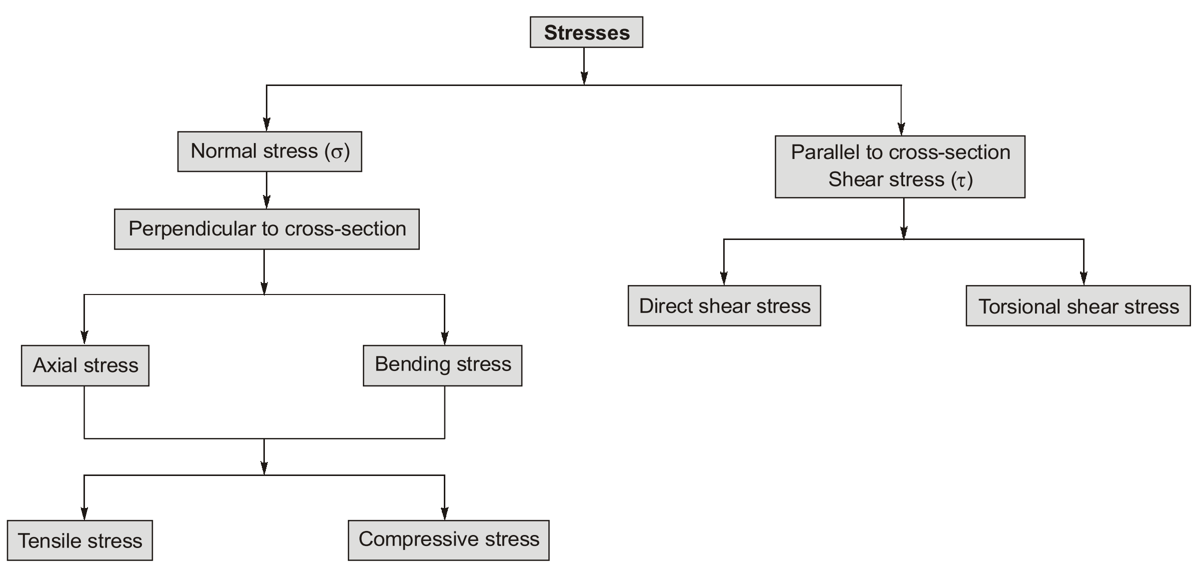
$1 \text{ MPa} = 10^6 \text{ Pa or } 10^3 \text{ kPa}$ also $1 \text{ MPa} = 1 \text{ N/mm}^2$

$1 \text{ GPa} = 10^9 \text{ Pa or } 10^6 \text{ kPa}$ also $1 \text{ GPa} = 1 \text{ kN/mm}^2$

NOTE

- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If body or bar is free to move or free expansion is allowed then no stresses will be induced.
- Pressure has same unit as stress but pressure is magnitude of external forces applied on the cross-section and stress is magnitude of internal resisting force developed on the cross-section.

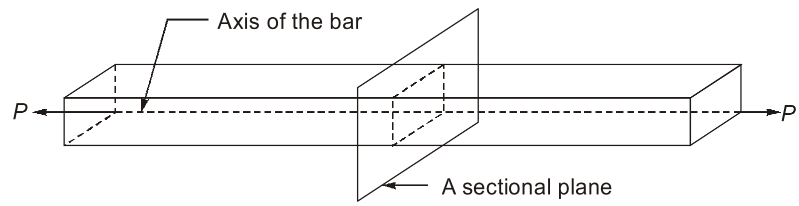
A material is capable of offering the following **types of stresses**:



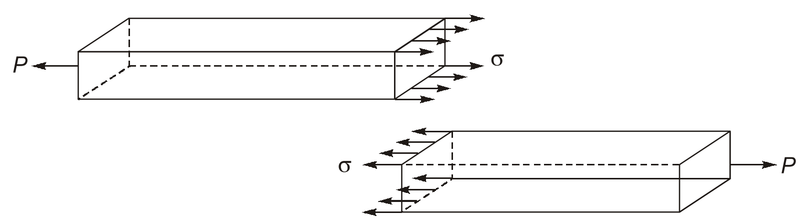
(i) **Normal Stress:** Normal stresses are always normal to the cross-section at any section. Normal stresses are represented by greek letter s. Normal stress may be of two types:

(a) **Direct Stress or Axial Stress:** These stresses are produced when an axial force is acted at *CG* of cross-section. For prismatic body with axial loading, direct stresses are uniform across the cross-section. Generally, tensile stresses are taken positive, and compressive stresses are taken negative.

For example, the stress offered by section *mn* in fig. (b) is tensile stress whereas the stress offered by section *mn* in fig. (d) is compressive stress.

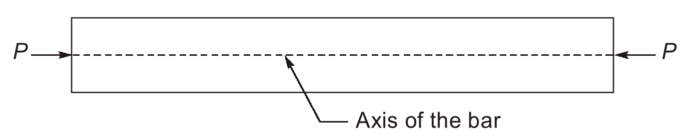


(a) Bar subjected to pulls

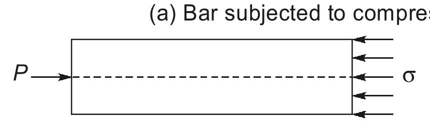


(b) Resisting force developed

Fig. *Tensile stresses*



(a) Bar subjected to compressive force



(b) Resisting force developed

Fig. *Compressive stresses*

On the basis of cross-sectional area considered during calculation of stresses, direct stresses can be of following two types:

(a) Engineering stress or nominal stress

(b) True stress or Actual stress

Engineering stress (Nominal stress)

Mathematically,

$$\sigma = \frac{P}{A_0}$$

where, A_0 = original cross-sectional area of specimen taken

True stress (Actual stress)

Mathematically,

$$\sigma = \frac{P}{A_a}$$

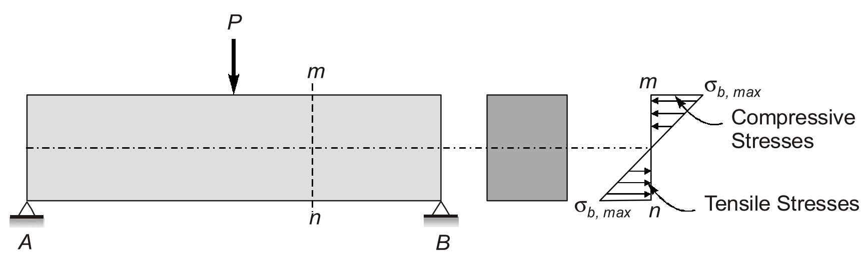
where, A_a = Actual cross-sectional area of specimen at any time of loading i.e. changed area of cross-section due to loading

$$A_a = A_0 \pm \Delta A \quad \text{'+' for Compression, '–' for Tension}$$

REMEMBER
- In tension, true or actual stress is always greater than engineering or nominal stress.
- In compression, true or actual stress is always less than engineering or nominal stress.

(b) Bending Stresses: Bending stresses are produced by bending moment. Bending stresses vary linearly from zero at neutral axis to maximum at farthest fibre from neutral axis.

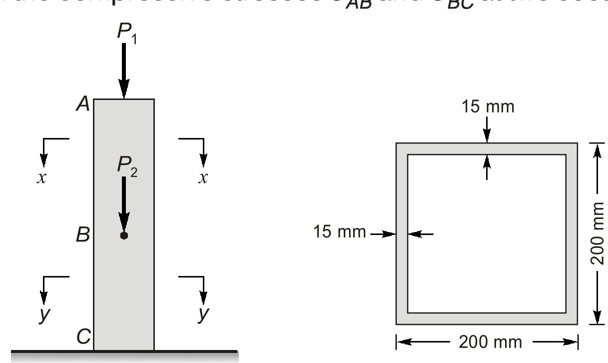
Tensile bending stresses are taken as positive and compressive bending stresses are taken as negative.



This will be discussed in detail in chapter 3.

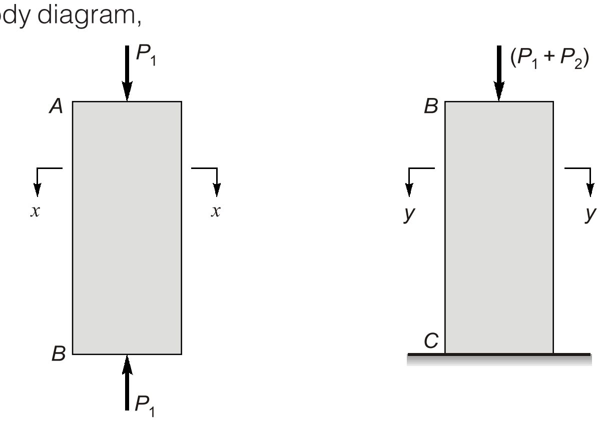
EXAMPLE : 1.1

A two storey column *ABC* in a building is constructed with a hollow square box section as shown below. The roof load at the top of column P_1 = 80 kN, and the floor load at mid height is P_2 = 100 kN. Obtain the compressive stresses σ_{AB} and σ_{BC} at two section *x-x* and *y-y* respectively.



Solution:

Drawing the free body diagram,



$$P_1 = 80 \text{ kN}, \quad P_2 = 100 \text{ kN}$$

Area of cross-section of column,

$$A = (200 \times 200) - [(200 - 30) \times (200 - 30)]$$
$$= 40000 - 28900 = 11100 \text{ mm}^2$$

In portion *AB*, at section *x*-*x*, column is subjected to axial force P_1,

so, stress in portion *AB*, $\sigma_{AB} = \frac{P_1}{A} = \frac{80 \times 10^3}{11100} \text{ N/mm}^2 = 7.21 \text{ N/mm}^2$

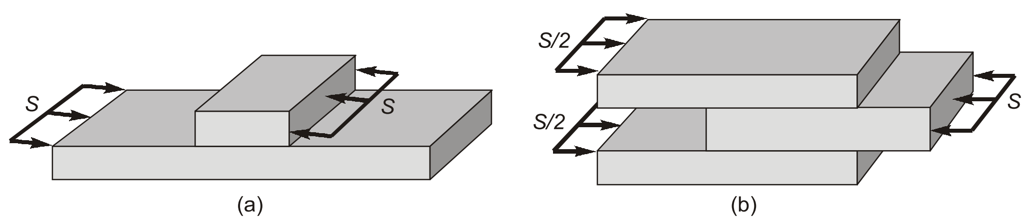
In portion *BC*, at section *y*-*y*, column is subjected to

Total axial force, $P = P_1 + P_2 = 80 + 100 = 180 \text{ kN}$

So, stress in portion *BC*, $\sigma_{BC} = \frac{P}{A} = \frac{180 \times 10^3}{11100} \text{ N/mm}^2 = 16.22 \text{ N/mm}^2$

(ii) **Shear Stress or Tangential Stress:** Shear stress is resistance offered by material against shearing force. Its value is determined by dividing the shear force in the plane of the section by corresponding area. Its unit is N/mm^2.

$$\tau = \frac{S}{A} \text{ N/mm}^2$$



Loading conditions causing shear stresses

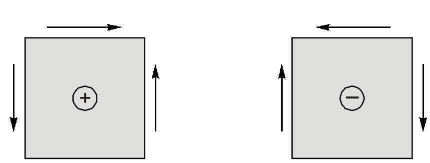
Shear stress may be of following two type:

(a) **Direct shear stress:** Direct shear stress is produced due to direct shear force acting on surface.

(b) **Torsional Stress:** This stress is produced when member is subjected to torsional moment or torque.

REMEMBER A shear stress in a given direction cannot exist without a balancing shear stress of equal intensity in a direction at right angle to it. This stress is called **complementary shear stress**.

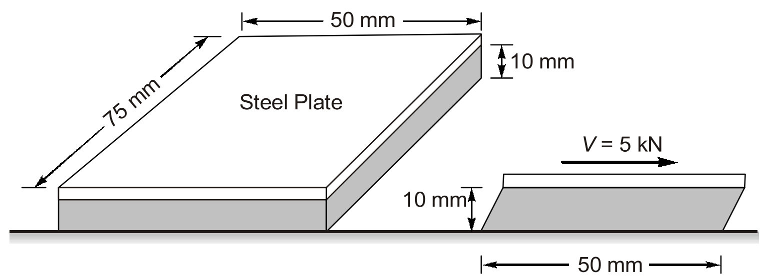
Sign convention:



Positive shear stress   Negative shear stress

EXAMPLE : 1.2

A bearing pad consisting of a flexible material of thickness 10 mm capped by a thin steel plate of dimension 50 × 75 mm is subjected to a horizontal shear force 5 kN. Determine the average shear stress.



(a) (b)

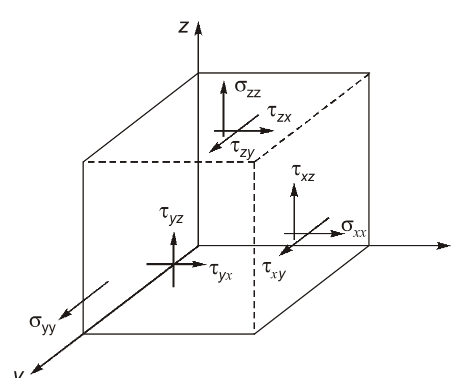
Solution:

The average shear stress will be equal to the shear force divided by the area over which it acts.

$$\tau_{avg} = \frac{V}{A} = \frac{5\times10^3}{50\times75}\ \text{N/mm}^2 = 1.33\ \text{N/mm}^2$$

1.2.1 General State of Stress, Stress Tensor and Matrix Representation of Stress

- If the body is sectioned by planes parallel to the x-z plane, y-z plane and x-y plane, we can cut out a cubic element that represents the state of stress acting on the element.
- Tensor is defined as a mathematical object analogous to but more general than a vector, represented by an array of components that are functions of the coordinates of a space.
- Stress is a second order tensor
- Stress in 3D form can be represented in a matrix form as



Triaxial state of stress at a point

$$[\text{Stress}] = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

- In the above notations σ_{xx} represents normal stress with 1st letter in subscript representing plane of face and the second letter representing the direction of stress $\sigma_{\underbrace{x}_{\text{face}}\ \underbrace{x}_{\text{direction}}}$.
- τ_{xy} represents tangential or shear stress in which x represents plane of face and y represents direction of stress $\tau_{\underbrace{x}_{\text{face}}\ \underbrace{y}_{\text{direction}}}$.

REMEMBER When a point is drawn on any face maximum three stresses can be developed.

1.2.2 Shear Stress Equilibrium

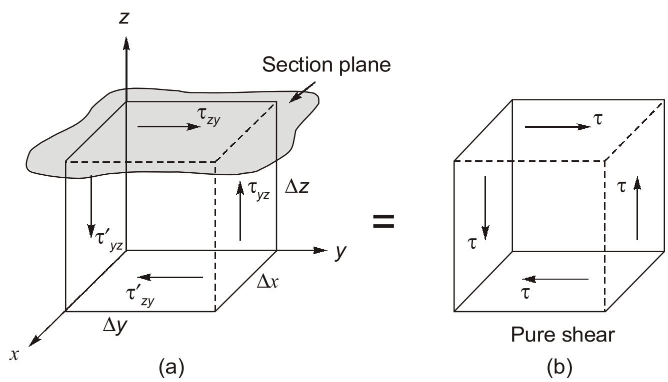


Fig. (a) shows a volume element of material taken at a point located on the surface of a sectioned area which is subjected to a shear stress τ_{zy}. Force and moment equilibrium requires the shear stress acting on this face of the element to be accompanied by shear stress acting on three other faces. To show this we will first consider force equilibrium in the y direction.Then

$\Sigma F_y = 0;$ $\quad \tau_{zy}(\Delta x\,\Delta y) - \tau'_{zy}\,\Delta x\,\Delta y = 0$

$$\tau_{zy} = \tau'_{zy} \qquad \text{...(i)}$$

In a similar manner, force equilibrium in the z-direction yields

$$\tau_{yz} = \tau'_{yz} \qquad \text{...(ii)}$$

Finally, taking moment about the x-axis.

$\Sigma M_x = 0;\ -\tau_{zy}(\Delta x\,\Delta y)\,\Delta z + \tau_{yz}(\Delta x\,\Delta z)\,\Delta y = 0$

$$\tau_{zy} = \tau_{yz} \qquad \text{...(iii)}$$

From eq. (i), (ii) and (iii), $\quad \tau_{zy} = \tau'_{zy} = \tau_{yz} = \tau'_{yz} = \tau$

In other words, all four shear stresses must have equal magnitude and be directed either toward or away from each other at opposite edges of the element as represented in Fig. (b). This is referred to as the complementary property of shear, and under the conditions shown in Fig., the material is subjected to pure shear.

1.2.3 Allowable Stresses

The ability of structure to resist loading is called strength. For safety, material should have much higher strength than the strength required for an external loading. The ratio of the actual strength to the required strength is called the ***factor of safety*** (FOS).

$$\text{FOS}(n) = \frac{\text{Failure stress}}{\text{Working stress}} = \frac{\text{Actual strength}}{\text{Required strength}}$$

So for maintaining required factor of safety, allowable stress can be calculated as

$$\sigma_{\text{allowable}} = \frac{\text{Yield stress}}{\text{FOS}} \quad \text{(Ductile materials)}$$

$$\sigma_{\text{allowable}} = \frac{\text{Ultimate stress}}{\text{FOS}} \quad \text{(Brittle materials)}$$

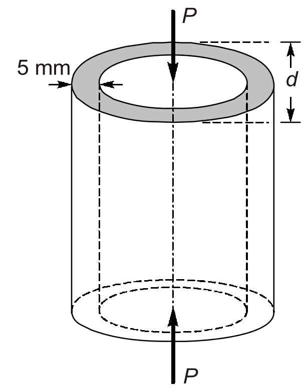
NOTE

- In air craft design generally margin of safety is considered rather than factor of safety.
- The margin of safety is defined as the factor of safety minus one.

$$\boxed{\text{Margin of safety} = n - 1}$$

EXAMPLE : 1.3

A short hollow circular cast iron cylinder support an axial compressive load $P = 50$ kN. The ultimate stress in compression for the material is σ_u = 280 N/mm². It is decided to design the cylinder with a wall thickness 5 mm and factor of safety of 3.0 with respect to the ultimate strength. Compute the minimum required outside diameter of the cylinder.



Solution:

The allowable compressive stress is equal to the ultimate stress divided by the factor of safety.

$$\sigma_{Allowable} = \frac{\sigma_u}{FOS} = \frac{280}{3}\text{ N/mm}^2 = 93.33\text{ N/mm}^2$$

The required cross-sectional area is,

$$A = \frac{P}{\sigma_{allowable}} = \frac{50\times10^3}{93.33} = 535.71\text{ mm}^2$$

The actual cross-sectional area is,

$$A = \frac{\pi d^2}{4} - \frac{\pi}{4}(d-2t)^2 = \frac{\pi}{4}d^2 - \frac{\pi}{4}\left(d^2+4dt+4t^2\right)$$

$$\Rightarrow \quad A = \frac{\pi d^2}{4} - \frac{\pi d^2}{4} + \frac{4\pi dt}{4} - \frac{4\pi t^2}{4} = \pi t\,(d-t)$$

so, $\quad \pi\times5\,(d-5) = 535.71\text{ mm}^2$

$\Rightarrow \quad d = 39.10$ mm

EXAMPLE : 1.4

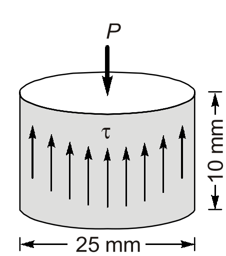
Structural steel has a shearing ultimate strength of approximately 310 MPa. Determine the force P necessary to punch a 25 mm diameter hole through the 10 mm thick plate of steel.

Solution:

Let as assume uniform shearing on a cylindrical surface.

Diameter, D = 25 mm, Thickness, t = 10 mm

For equilibrium of force



P = shear force resisted by material

$P = \tau\times$ surface area

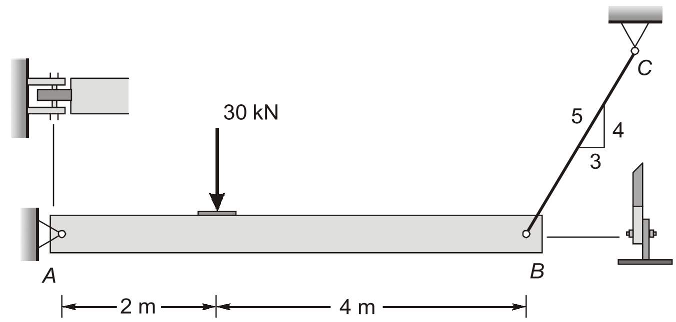
$= 310\times\pi\times D\times t = 310\times\pi\times25\times10$

$= 243.47\times10^3$ N

= 243.47 kN

EXAMPLE : 1.5

Determine the average shear stress in the 20 mm diameter pin at A and the 30 mm diameter pin at B that support the beam in figure.



Solution:

Internal Loadings: The forces on the pins can be obtained by considering the equilibrium of the beam, as shown in Fig. (a).

$\circlearrowleft \Sigma M_A = 0; \quad F_B\left(\frac{4}{5}\right)(6\text{m}) - 30\text{ kN}(2\text{m}) = 0$

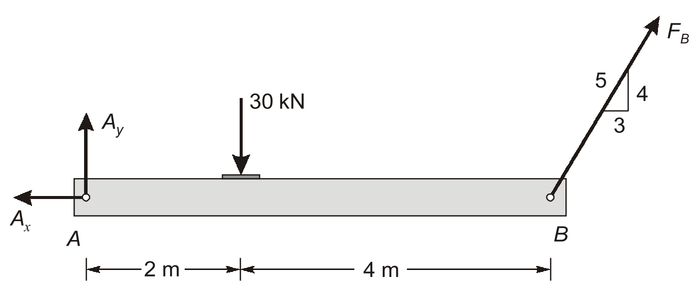
$$F_B = 12.5\text{ kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad (12.5\text{ kN})\left(\frac{3}{5}\right) - A_x = 0$

$$A_x = 7.50\text{ kN}$$

$+\uparrow \Sigma F_y = 0; \quad A_y + (12.5\text{ kN})\left(\frac{4}{5}\right) - 30\text{ kN} = 0$

$$A_y = 20\text{ kN}$$



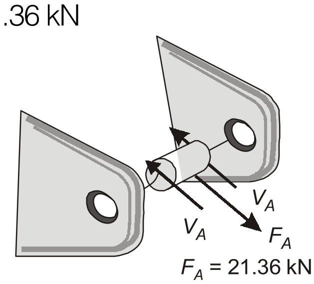
(a)

Thus, the resultant force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(7.50\text{ kN})^2 + (20\text{ kN})^2} = 21.36\text{ kN}$$

The pin at A is supported by two fixed "leaves" and so the free-body diagram of the center segment of the pin shown in Fig. (b) has two shearing surfaces between the beam and each leaf. The force of the beam (21.36 kN) acting on the pin is therefore supported by shear force on each of these surfaces. This case is called double shear. Thus

$$V_A = \frac{F_A}{2} = \frac{21.36\text{ kN}}{2} = 10.68\text{ kN}$$



(b)

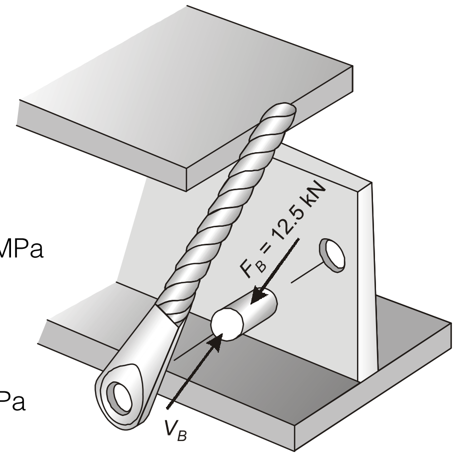
In Fig. (Question), note that pin B is subjected to single shear, which occurs on the section between the cable and beam, Fig. (c). For this pin segment,

$$V_B = F_B = 12.5 \text{ kN}$$

Average shear stress

$$(\tau_A)_{avg} = \frac{V_A}{A_A} = \frac{10.68(10^3)\text{ N}}{\frac{\pi}{4}(0.02\text{ m})^2} = 34.0 \text{ MPa}$$

$$(\tau_B)_{avg} = \frac{V_B}{A_B} = \frac{12.5(10^3)\text{ N}}{\frac{\pi}{4}(0.03\text{ m})^2} = 17.7 \text{ MPa}$$



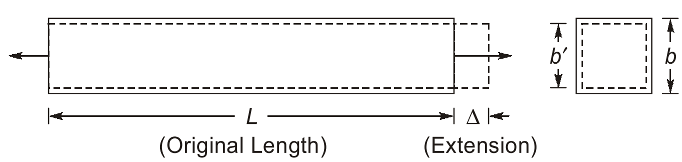
(c)

1.3 STRAIN

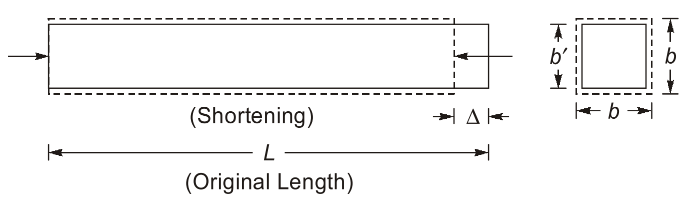
Strain is a measure of deformation representing the displacement between particles in the body relative to a reference length.

Mathematically strain can be calculated as

$$\in = \frac{\Delta}{L}$$



(a)



(b)

Unit: Strain is dimensionless quantity. It is always expressed in the form of number. If the member is in tension (a), the strain is called a tensile strain. If the member is in compression (b), the strain is called a compressive strain.

On the basis of length of member used in calculation of strain, strain can be of following two types:

(a) Engineering or nominal strain

(b) True or Actual strain

Engineering or Nominal Strain

Engineering or nominal strain is strain calculated, when length of member is taken as original length

$$\in_0 = \frac{\Delta l}{l_0} \qquad l_0 = \text{original length of member}$$

True or Actual Strain

True or actual strain is strain calculated, when length of member is taken as actual length of member at loading

$$\in_a = \frac{\Delta l}{l_a} \qquad l_a = \text{Actual length of member}$$

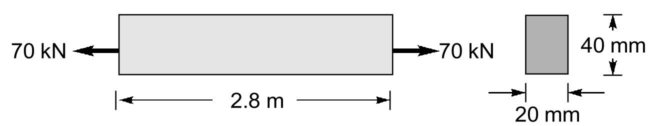
$$l_a = l_0 \pm \Delta l$$

Sign convention:

- Tensile strains are positive where as compressive strains are negative.

EXAMPLE : 1.6

A prismatic bar with rectangular cross-section (20 mm × 40 mm), length L = 2.8 m is subjected to an axial tensile force of 70 kN. The measured elongation of the bar is 1.2 mm. Calculate the tensile stress and strain in the bar.



Solution:

Assuming that force acts at CG of section. We know that,

$$\text{Stress, } \sigma = \frac{P}{A} = \frac{70 \times 10^3 \text{N}}{20 \times 40 \text{ mm}^2} = 87.5 \text{ MPa}$$

and

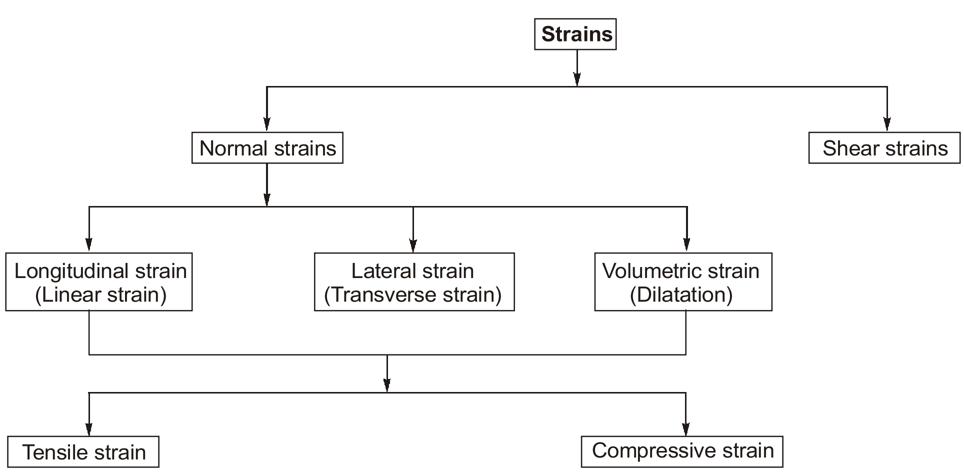
$$\text{Strain, } \in = \frac{\Delta L}{L_0} = \frac{1.2 \text{ mm}}{2.8 \times 1000 \text{ mm}} = 4.285 \times 10^{-4}$$

1.3.1 Deformation

Whenever a force is applied to a body, it will tend to change the body's shape and size. These changes are referred to as deformations.

These changes may be due to change in axial length or distortion of body. Based on different types of deformation, strains can be of the following types.

Types of Strains



Strains: Strains are of the following types.

(*i*) Axial strain ($\in$) or longitudinal strain

(*ii*) Lateral strain ($\in_L$) or transverse strain
(*iii*) Volumetric strain ($\in_v$)
(*iv*) Shear strain (ϕ)

(*i*) Axial Strain ($\in$): Strain in the direction of applied force is known as axial strain. It is the ratio of change in the linear dimension to original linear dimension of the member.

$$\text{Axial strain, } \in = \frac{\text{Change in linear dimension}}{\text{Original linear dimension}} = \frac{\Delta L}{L}$$

(*ii*) Lateral Strain ($\in_L$): Strain in the perpendicular direction to the direction of applied force is known as lateral strain. It is the ratio of change in lateral dimension to the original lateral dimension of the member,

$$\text{Lateral strain, } \in_L = \frac{\text{Change in lateral dimension}}{\text{Original lateral dimension}}$$

Consider, a copper rod as shown in Fig. of length 'L' and cross-section ($B \times B$) is pulled with an axial force 'P'. The length of rod increases due to applied force but width and depth decrease.

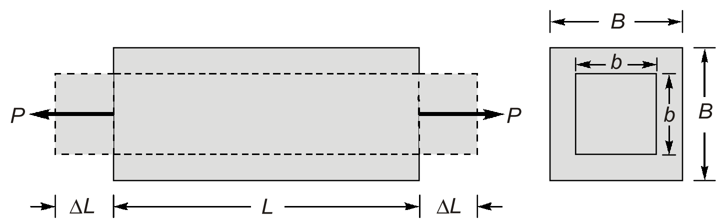


Fig.

In this case,

$$\text{Lateral strain, } \in_L = \frac{\text{Change in lateral dimension}}{\text{Original lateral dimension}} = \frac{\Delta B}{B} = \frac{B - b}{B}$$

(*iii*) Volumetric Strain ($\in_v$) : Volumetric strain is defined as the ratio of change in volume to the original volume

$$\text{Volumetric strain, } \in_v = \frac{\Delta V}{V}$$

where, ΔV = Change in volume, V = Original volume

(*iv*) Shear Strain (ϕ): Shear strains are angular deformations caused by shearing force. These are represented by ϕ.

$$\text{Shear strain, } \phi = \frac{\Delta}{h}$$

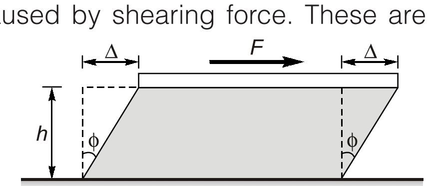
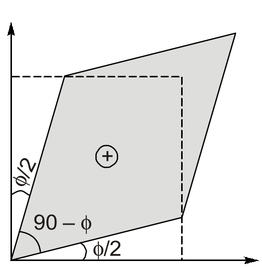


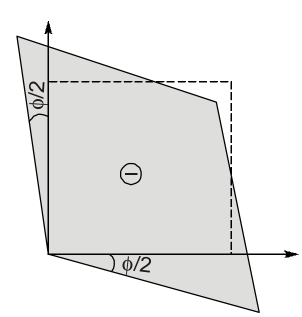
Fig.

Shear strain can be positive or negative.



Termed as positive shear strain
(Positive shear strains are produced by positive shear components).

Fig.



Termed as negative shear strain
(Negative shear strains are produced by negative shear components).

Fig.

1.3.2 Matrix Representation of Strains

$$[\text{Strain}] = \begin{bmatrix} \in_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{xz}}{2} \\ \frac{\phi_{yx}}{2} & \in_{yy} & \frac{\phi_{yz}}{2} \\ \frac{\phi_{zx}}{2} & \frac{\phi_{zy}}{2} & \in_{zz} \end{bmatrix}$$

where, $\in_{xx}$, $\in_{yy}$ and $\in_{zz}$ are linear strains in x, y and z directions respectively and $\frac{\phi_{xy}}{2}, \frac{\phi_{xz}}{2}$ and $\frac{\phi_{yz}}{2}$ are shear strain components in x-y, x-z and y-z planes respectively.

$$\text{Total shear strain in } x\text{-}y \text{ plane} = \frac{\phi_{xy}}{2} + \frac{\phi_{yx}}{2} = \phi_{xy}$$

$$\text{Total shear strain in } x\text{-}z \text{ plane} = \frac{\phi_{xz}}{2} + \frac{\phi_{zx}}{2} = \phi_{xz}$$

$$\text{Total shear strain in } y\text{-}z \text{ plane} = \frac{\phi_{zy}}{2} + \frac{\phi_{yz}}{2} = \phi_{yz}$$

NOTE

Under pure normal stress, the volume change may occur but shape change will not occur. Similarly under pure shear stress, shape change will occur but no volume change will occur.

1.3.3 Differential Form of Strains

Consider a point *P*, whose position is defined by coordinate x, y and z. If some force F is applied at *P*, then point *P* moves to position u, v and w in coordinate system.

Then linear strains in x, y and z directions are given by,

in x-direction, $\in_{xx} = \frac{\partial u}{\partial x}$; in y-direction, $\in_{yy} = \frac{\partial v}{\partial y}$; in z-direction, $\in_{zz} = \frac{\partial w}{\partial z}$

The shear strains at point *P* are given as

(*i*) $(\phi_{xy} = \phi_{yx}) = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ (*ii*) $(\phi_{yz} = \phi_{zy}) = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$ (*iii*) $(\phi_{zx} = \phi_{xz}) = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$

1.3.4 Elastic Constants

1. **Young's Modulus (*E*):** Young's modulus is slope of stress-strain curve under direct loading.

$$E = \frac{\text{Direct stress}}{\text{Linear strain}} = \frac{\sigma}{\in}$$

2. **Shear Modulus (*G*):** When a body is subjected to shearing stresses, the shape of the body gets distorted. The measurement of this distortion is done by angle of distortion.

If under the shear, the shear strain is ϕ then the linear strain in the diagonal of the specimen is given by $\in = \frac{\phi}{2}$ i.e. **linear strain of diagonal is half of the shear strain in the body.**

Proof: Consider a cube of side '*a*' subjected to shear and complimentary shear as shown in Fig.

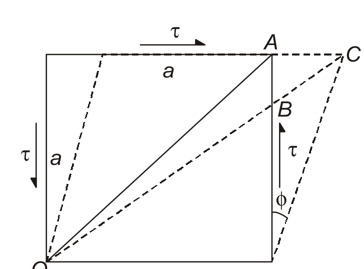


Fig.

Assuming that the strain are small and the angle ACB is 45°,

$$\text{Strain in diagonal } OA = \frac{BC}{OA} = \frac{AC\cos 45°}{OA}$$

$$= \frac{AC}{a\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{AC}{2a}$$

But, $\quad AC = a\phi$

$$\therefore \quad \text{Strain in diagonal} = \frac{AC}{2a} = \frac{a\phi}{2a} = \frac{\phi}{2}$$

Shear modulus is defined as the ratio of shear stress to shear strain

$$G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi} \qquad \text{...(ii)}$$

It is also called ***Modulus of Rigidity.***

3. **Bulk Modulus (*K*):** Bulk modulus is defined as the ratio of direct stress to the volumetric strain. It is represented by K.

$$\text{Bulk modulus, } K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\epsilon_V}$$

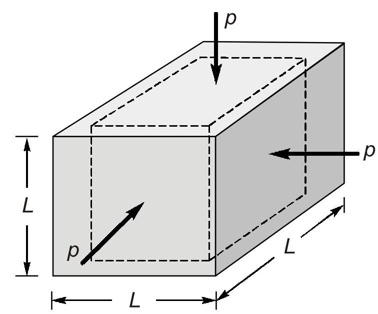


Fig.

- Bulk modulus is valid under hydrostatic stress condition or spherical state of stress condition.
- A point is said to be under hydrostatic condition, when it is subjected to equal and like normal stress in 3-mutual perpendicular direction.

  i.e., ($\sigma_x = \sigma_x = \sigma_y = \sigma$) and ($\tau_{xy} = \tau_{yz} = \tau_z = 0$, no shear stress)
- Under hydrostatic condition, only volume is changes i.e., No distortion.

Significance of Bulk modulus is with respect to compressibility.
In 3D hydrostatic loading,

$$\sigma_x = \sigma_y = \sigma_z = p$$

and Volumetric strain, $\quad \epsilon_V = \dfrac{\Delta V}{V}$

then Bulk modulus, $\quad K = \dfrac{p}{\left(\dfrac{\Delta V}{V}\right)} \qquad \text{...(iii)}$

REMEMBER Bulk modulus is inversely proportional to compressibility.

4. **Poisson's Ratio:** Poisson's ratio is defined as ratio of lateral strain to longitudinal strain within elastic zone under direct loading. It is represented by ν.

$$\text{Poission's ratio, } \nu = \frac{-(\text{Lateral strain})}{(\text{Longitudinal strain})} \text{ or } -\frac{\epsilon_L}{\epsilon} \qquad \text{...(iv)}$$

$$= \frac{10}{4} = 2.5t = 2500 \text{ kg}$$

$\therefore$ Stress intensity in the lower washer

$$= \frac{2500}{12.76} = 195.92 \text{ kg/cm}^2$$

(ii) When the nuts are tightened the compressive load in the upper washer = Tension in the bolt

$$= 0.4t = 400 \text{ kg}$$

$$\text{Area of upper washer} = \frac{\pi}{4}\left(4^2 - 2^2\right) \text{ cm}^2 = 9.4248 \text{ cm}^2$$

$\therefore$ Stress intensity in the upper washer $= \dfrac{400}{9.4248} = 42.44 \text{ kg/cm}^2$

Now, compressive load on the lower washer = 2500 + 400 = 2900 kg

$\therefore$ Stress intensity in the lower washer $= \dfrac{2900}{12.76} = 227.27 \text{ kg/cm}^2$

Question : 2

A steel bar of square cross-section 40 mm × 40 mm and 12 cm long is subjected to an axial compressive load of 30 tonnes. The later at strain is prevented by the application of uniform external pressure.

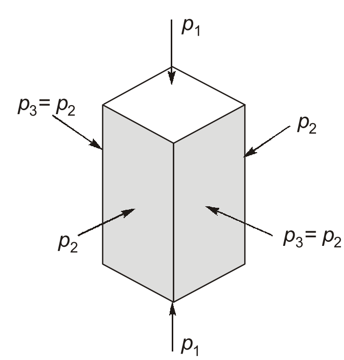
(i) If $\mu = 0.3$ and $E = 2 \times 10^6$ kg/cm^2, find the change in the length of the bar.

(ii) If only one-half the lateral strain is prevented, what would be the change in the length of the bar?

[12 Marks]

Solution:

The adjoining figure shows the bar with its axis placed vertical.



(i) Axial stress on the normal

$$\text{Section,} \quad P_1 = \frac{30}{4 \times 4} = 1.875 \text{ t/cm}^2$$

Let the compressive stress applied on the side face be P_2 and P_3.

Due to symmetry, $P_2 = P_3$

$$\text{Strain on either side} = \frac{P_2}{E} - \frac{\mu P_2}{E} - \frac{\mu P_1}{E} = 0$$

$$\Rightarrow \quad \frac{1}{E}\left[P_2 - 0.3P_2 - 0.3 \times 1.875\right] = 0$$

$$\Rightarrow \quad 0.7P_2 = 0.5625$$

$$\Rightarrow \quad P_2 = 0.8036 \text{ t/cm}^2 \quad \text{(Compressive)}$$

$$\text{Strain in the length} = \frac{P_1}{E} - \frac{2\mu P_2}{E} = \frac{1}{E}\left[1.875 - 2 \times 0.3 \times 0.8036\right]$$

$$= \frac{1}{2000}\left[1.875 - 0.482\right] = 6.9643 \times 10^{-4} \quad \text{(Compressive strain)}$$

$$\therefore \quad \text{Decrease in length} = (6.9643 \times 10^{-4}) \times l$$

$$= 6.9643 \times 10^{-4} \times 12$$

$$= 0.008357 \text{ cm}$$

(ii) When only one-half the lateral strain is prevented

If lateral stresses had not been applied, then the lateral strain would be $\frac{\mu P_1}{E}$. Since half of this strain is prevented by the application of lateral stress,

We have,

$$\text{Lateral strain} = \frac{P_2}{E} - \frac{\mu P_2}{E} - \frac{\mu P_1}{E} = -\frac{1}{2}\frac{\mu P_1}{E}$$

$$\therefore \quad P_2 - \mu P_2 = \frac{\mu P_1}{2}$$

$$\Rightarrow \quad P_2(1-0.3) = \frac{0.3}{2} \times 1.875$$

$$\Rightarrow \quad P_2 = 0.4018 \text{ t/cm}^2 \quad \text{(Compressive)}$$

$$\text{Now, strain in the length} = \frac{P_1}{E} - \frac{2\mu P_2}{E} = \frac{1}{E}[1.875 - 2 \times 0.3 \times 0.4018]$$

$$= \frac{1}{2000}[1.875 - 0.241] = 8.1696 \times 10^{-4} \quad \text{(Compressive strain)}$$

$$\therefore \quad \text{Decrease in length} = (8.1696 \times 10^{-9}) \times 12 = 0.0098 \text{ cm}$$

Question : 3

A steel rod 25 mm in diameter passes centrally through a steel tube of 30 mm internal diameter and 35 mm external diameter. The tube is 900 mm long and is closed by rigid washers of negligible thickness which are fastened by nuts threaded on the rod. The nuts are tightened until the compressive load on the tube is 25 kN. Calculate the stresses in the tube and the rod.

Find the increase in these stresses when one nut is tightened by one-quarter of a turn relative to the other. There are 4 threads per 10 mm. Take E = 2 × 10^5 N/mm^2.

[10 Marks]

Solution:

Given data : Diameter of rod = 25 mm,

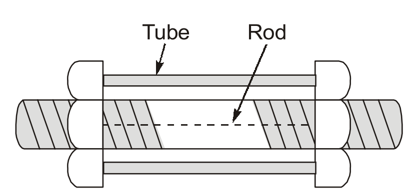
$$\therefore \text{ Area of rod,} \quad A_r = \frac{\pi}{4}(25)^2 = 156.25\pi \text{ mm}^2$$

$$\text{Area of tube,} \quad A_t = \frac{\pi}{4}(35^2 - 30^2) = 81.25\pi \text{ mm}^2$$

Length, l = 900 mm

Compressive load on tube, P_t = 25 kN = 25 × 10^3 N

Value of E = 2 × 10^5 N/mm^2



When the nuts are tightened, the tube is compressed and the rod is elongated. This means that the tube will be under compression and rod will be under tension. Since no external forces have been applied, the compressive load on the tube must be equal to the tensile load on the rod.

Let σ_t = Stress in the tube and σ_r = Stress in the rod

Now, tensile load on the rod = Compressive load on the tube

$$\therefore \quad \sigma_r \times A_r = \sigma_t \times A_t$$

$$\Rightarrow \quad \sigma_r = \frac{A_t}{A_r} \times \sigma_t = \frac{81.25\pi}{156.25\pi} \times \sigma_t = 0.52\sigma_t \quad \text{...(i)}$$