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Structural Analysis

Civil Engineering

Comprehensive Theory

with Solved Examples

Civil Services Examination

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Structural Analysis

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Structural Analysis

01
CHAPTER

Stability and Indeterminacy

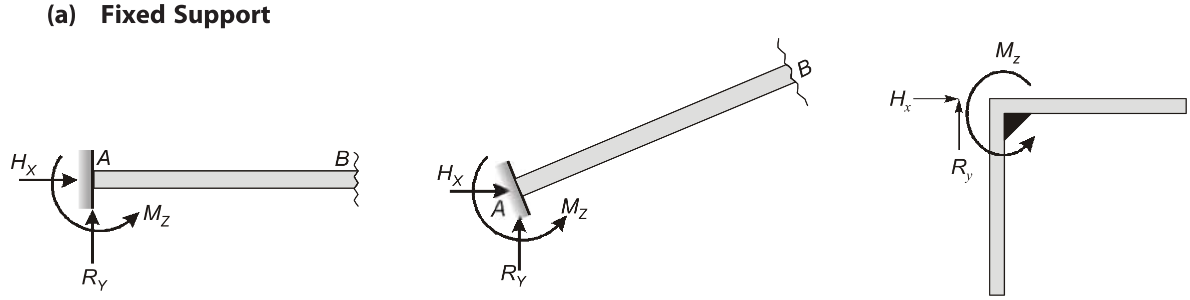
1.1 INTRODUCTION

Before starting the analysis of any structure, it is important to know what kind of structure it is. This is because the structure is analysed based on its type. For example, only static equilibrium is used to analyse determinate structures, whereas both static equilibrium and compatibility relationships together are needed to find the internal forces in an indeterminate structure. Also, the structure need to be stable as an unstable structure cannot recover static equilibrium after a disturbance. Hence, the concepts of stability, determinacy and indeterminacy will be discussed in this chapter.

1.2 SUPPORT SYSTEM

1.2.1 2-D Supports

(a) Fixed Support



(i) *Number of reaction = 3* ***(ii)*** *Number of reactions = 3* ***(iii)*** *Number of reactions = 3*

Fig. *(i)*

At 2-D fixed support, there can be three reactions:

(*i*) one vertical reaction (R_y) (*ii*) one horizontal reaction (H_x) (*iii*) one moment reaction (M_z)

(b) Hinge Support

Hinge support is represented by the symbol △.

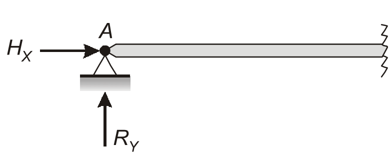


Fig. *(i)* *Number of reactions = 2*

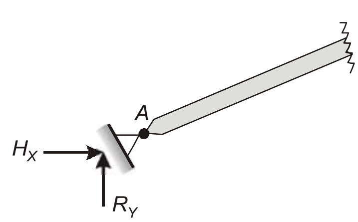


Fig. *(ii)* *Number of reactions = 2*

At hinged support, there can be two reactions:

(*i*) one horizontal reaction (H_x)

(*ii*) one vertical reaction (R_y)

(c) Roller Support

Roller support is represented by the symbol  or .

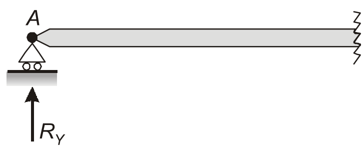


Fig. (*i*) *Number of reactions = 1*

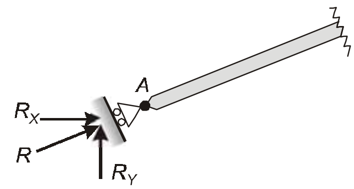


Fig. (*ii*) *Number of reactions = 1*

At roller support there can be only one externally independent reaction which is normal to the contact surface.

(d) Guided Roller Support

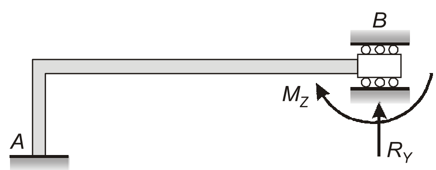


Fig. *Number of reactions = 2*

At guided roller supports there can be two reactions:

(*i*) one vertical reaction (R_y) (*ii*) one moment reaction (M_z)

1.2.2 2-D Internal Joints

(a) Internal Hinge

At internal hinge bending moment will be zero.

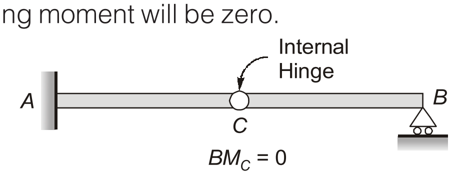
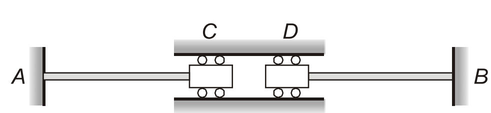


Fig.

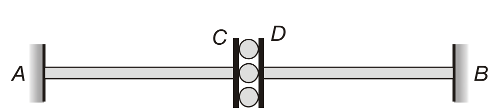
REMEMBER An internal hinge provides one additional equilibrium equation for structures.

(b) Internal Roller

At internal roller either axial force or shear force will be zero.



In above Fig., axial force at *C* and *D* is zero.



In above Fig., shear force at *C* and *D* will be zero i.e., $S_C = S_D = 0$

(c) Internal Link

If any member is connected by hinges at its end and subjected to no external loading in between then it can be termed as internal link and carry axial force only.

Here BC is a link, link BC carry only axial force

Also $BM_B = 0$ and $BM_C = 0$

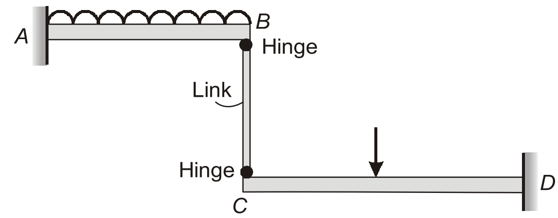


Fig.

REMEMBER Internal link also provides additional equation for analysis of structure.

1.2.3 3-D Supports

(a) Fixed Support

At 3-D fixed support there can be six reactions:

(*i*) three reactions R_x, R_y and R_z

(*ii*) three moment reactions M_x, M_y and M_z

The fixed support is also called ***Built-in support***

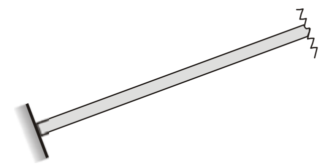


Fig. *Number of reactions = 6*

(b) 3-D Hinged Support

3-D hinged support there can be three reactions:

(*i*) R_x

(*ii*) R_y

(*ii*) R_z

The 3-D hinged support is also called **'ball and socket joint'**.

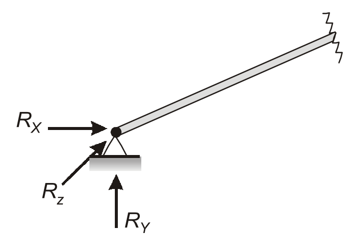


Fig. *Number of reactions = 3*

(c) Roller Support

At 3-D roller support there can be only one externally independent reaction which is perpendicular to the contact surface.

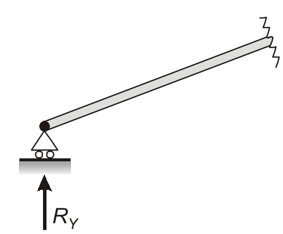


Fig. *Number of reactions = 1*

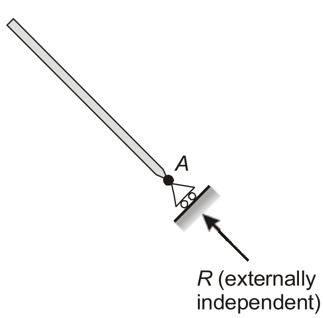


Fig. (i)

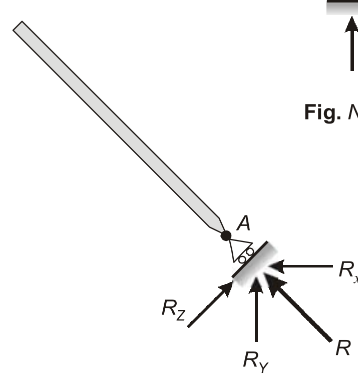


Fig. (ii)

in Fig. (ii), reactions at roller support A, R_x, R_y and R_z are externally dependent reactions which depends on reaction R.

1.3 STRUCTURE

Structure is defined as a system of interconnected members that are assembled in a stable configuration and used to support loads under the equilibrium of external forces and internal reactions.

1.3.1 Elements of Structure

Some of the major elements of structure by which structures are fabricated are as follows:

(a) Beams: Beams are structural members which are predominantly subjected to bending. On the basis of support system, beams can be classified as:

(*i*) Simply supported beam

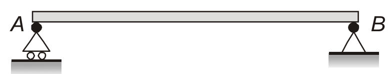


Fig.

(*ii*) Cantilever beam

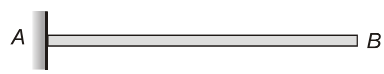


Fig.

(*iii*) Propped cantilever

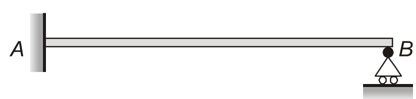


Fig.

(*iv*) Fixed beam

Fig.

(*v*) Continuous beam

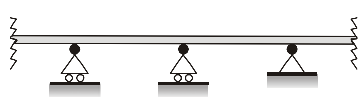


Fig.

(b) Columns: A column is a vertical compression member which is slender and straight. Generally, columns are subjected to axial compression and bending moment as shown in figure.

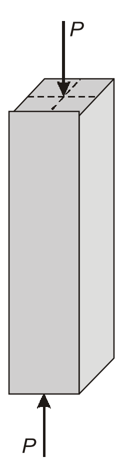


Fig. (i)

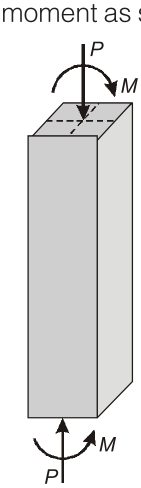


Fig. (ii)

(c) Tie Members: Tie members are tension members of trusses and frame, which are subjected to axial tensile force. (above Fig.)

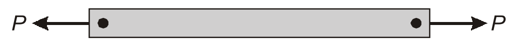


Fig. *Tie Rod*

1.3.2 Types of Structures

(a) Trusses: A truss is constructed from pin jointed slender members, usually arranged in triangular manner. In trusses, loads are applied on joints due to which each member of truss is subjected to only axial forces i.e., either axial compression or axial tension. Generally, trusses are used when span of structure is large.

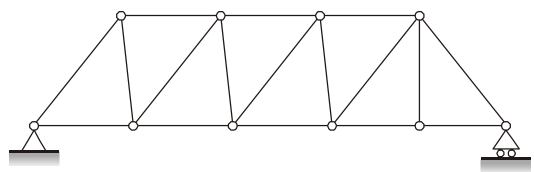


Fig. *Truss*

(b) Frames: A frame is constructed from either pin jointed or fixed jointed beam and columns. Generally, loads are applied on beams and this loading causes axial force, shear force and bending to the members of frame.

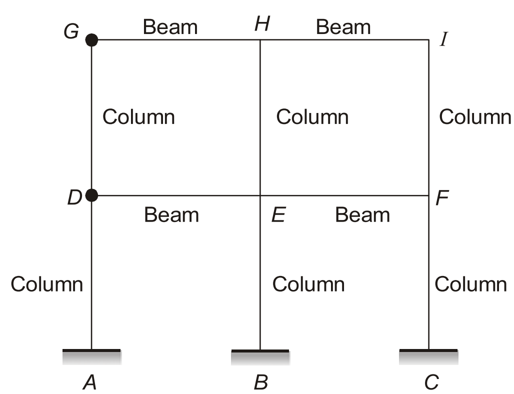


Fig. *Frames*

(c) Arches: Arches are used in bridges, dome roof, auditorium, where span of structures are relatively more due to external loading, Arch can be subjected to axial compression, shear force or bending moment.

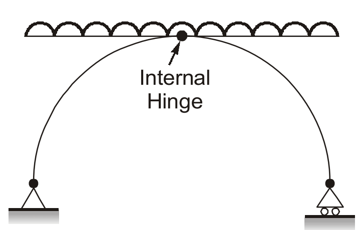


Fig. ***(i)*** *Three Hinge Arch*

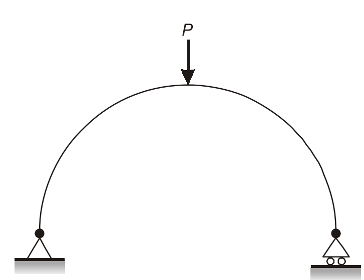


Fig. ***(ii)*** *Two Hinge Arch*

(d) Cables: Cables are used to support long span bridges. Cables are flexible members and due to external loading it is subjected to axial tension only.

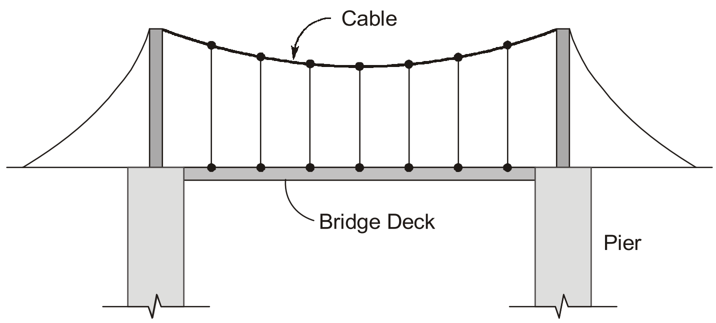


Fig. *Cable and Bridge*

1.4 TYPES OF LOADING

(a) Point load: A point load is considered to be acting at a point. It is also called concentrated load. In actual practice point loads are distributed load which are distributed over very small area.

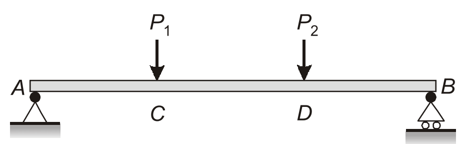


Fig. *Point Load*

(b) Distributed loads: Distributed loads are those loads, which acts over some measurable length. Distributed loads are measured by the intensity of loading per unit length along the beam.

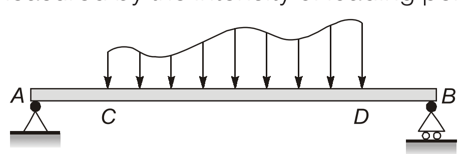


Fig. *Distributed Loads*

(c) Uniformly distributed loads: Uniformly distributed loads are those distributed loads which have uniform intensity of loading over the length.

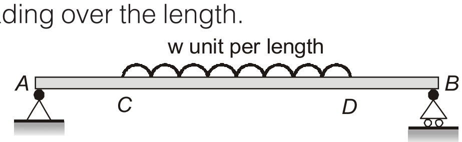


Fig. *Uniformly Distributed Loads*

(d) Uniformly varying loads : A uniformly varying load, commonly abbreviated as UVL, is the one in which the intensity of loading varies linearly from one end to other. For example, in Fig., intensity is zero at *C* and *W* at *D*.

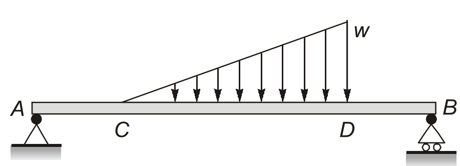


Fig. *Uniformly Varying Loads*

(e) Couple : A system of forces with resultant moment, but no resultant force is called couple. It is statically equivalent to force times the offset distance.

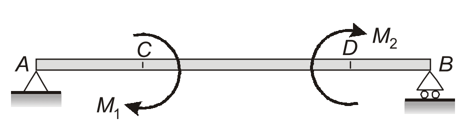


Fig. *Couple*

1.5 STABILITY OF STRUCTURES

Structural stability is the major concern of the structural designer. To ensure the stability, a structure must have enough support reaction along with proper arrangement of members. The overall stability of structures can be divided into

(*i*) External stability (*ii*) Internal stability

1.5.1 External Stability

(a) 2-D Structures: For stability of 2-D structures there should be no rigid body movement of structure due to loading, so it should have support in x-direction, y-direction and no rotation in x-y plane. So there should be enough reactions to restrain the rigid body motion.

For stability of 2-D structures, following three conditions of static equilibrium should be satisfied.

(*i*) $\Sigma F_x = 0$ (To prevent Δ_x)

(*ii*) $\Sigma F_y = 0$ (To prevent Δ_y)

(*iii*) $\Sigma M_z = 0$ (To prevent θ_z)

For stability in 2-D structures following conditions should also be satisfied:

(*i*) There should be minimum three number of externally independent support reactions.

(*ii*) All reactions should not be parallel, otherwise linear unstability will set up.

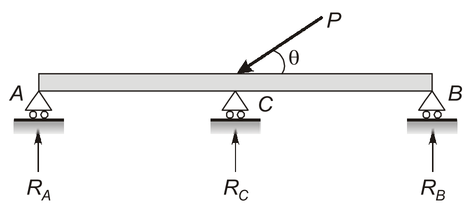


Fig. *Unstable*

(*iii*) All reactions should not be linearly concurrent otherwise rotational unstability will set up.

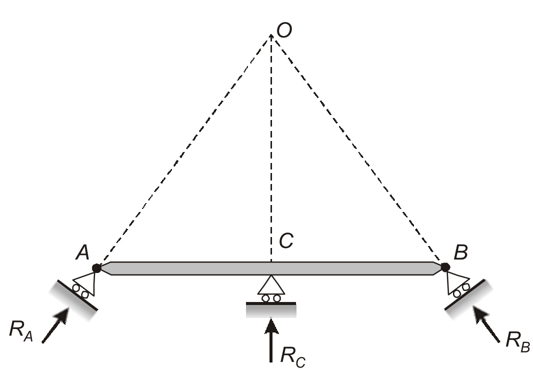


Fig. ***(i)*** *Unstable*

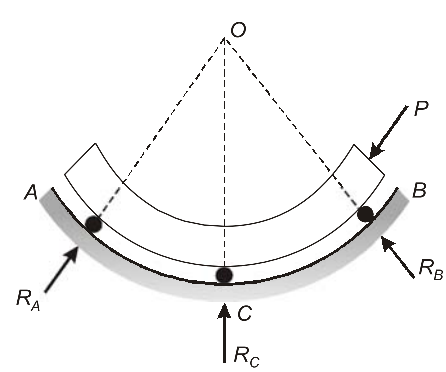


Fig. ***(ii)*** *Unstable*

(*iv*) Reactions should be non-trival i.e. there should be enough magnitude and enough difference between them.

(b) 3-D Structures: In case of 3-D structures, there should be a minimum of six independent external reactions to prevent rigid body displacement of structure. The displacement to be prevented are: Δ_x, Δ_y, Δ_z, θ_x, θ_y and θ_z. Therefore, there will be six equations of static equilibrium.

(*i*) $\Sigma F_x = 0$ (*ii*) $\Sigma F_y = 0$ (*iii*) $\Sigma F_z = 0$

(*iv*) $\Sigma M_x = 0$ (*v*) $\Sigma M_y = 0$ (*vi*) $\Sigma M_z = 0$

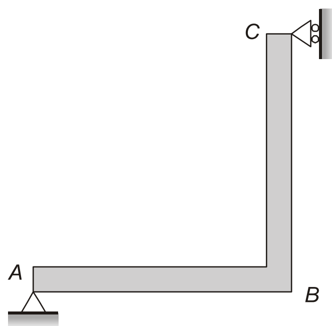
For stability in 3-D structures, all the reactions should be non-coplanar, non-concurrent and non-parallel.

REMEMBER If a structure is constructed from elastic members then small elastic displacement may be permitted but small rigid body displacement will not be permitted.

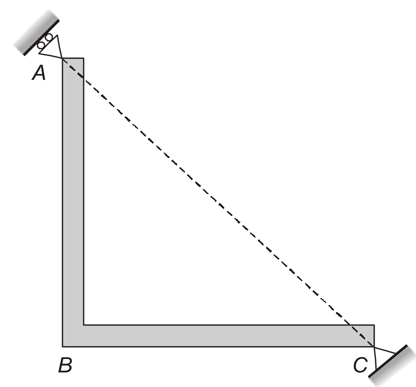
EXAMPLE - 1.1

Which one of the following structures is stable?

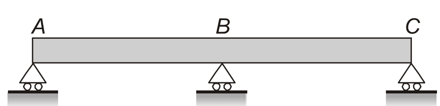
(a)



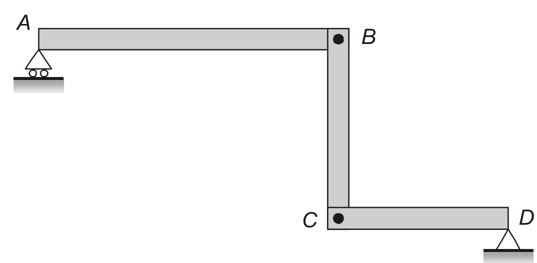
(b)



(c)

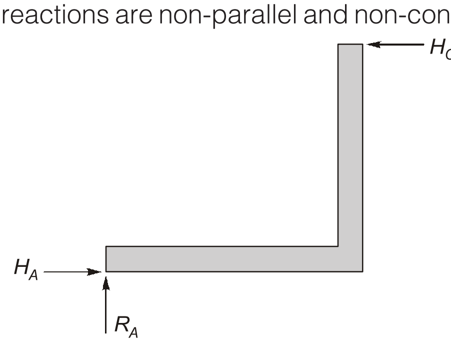


(d)

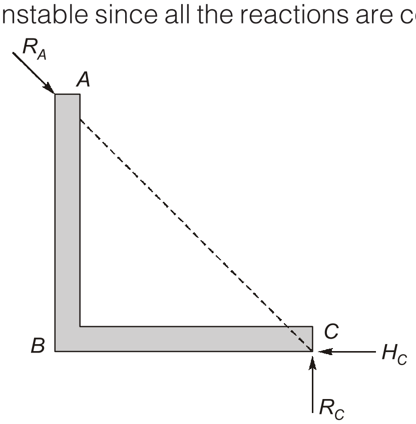


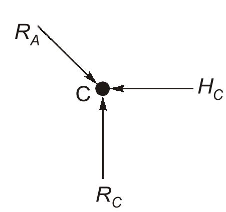
Ans. (a)

Member (a) is stable, since reactions are non-parallel and non-concurrent.

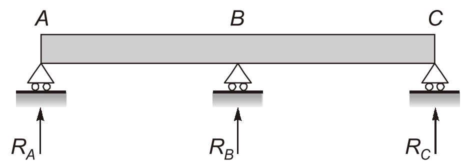


Member (b) is unstable since all the reactions are concurrent at *C*.

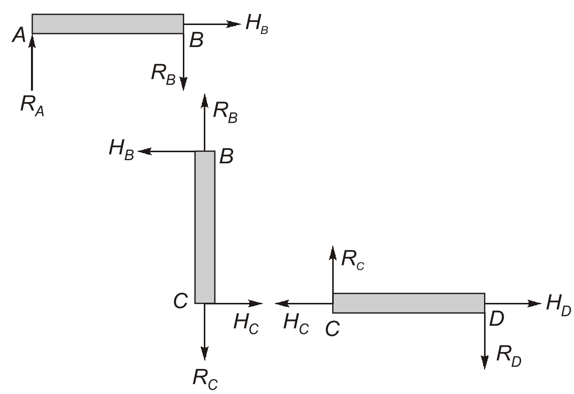




Beam (c) is unstable, since all three reactions are parallel.



Structure (d) is unstable, since the member AB can move horizontally without any restrain. i.e. $\Sigma F_x \neq 0$



1.5.2 Internal Stability

For the internal stability, no part of the structure can move rigidly relative to the other part so that geometry of the structure is preserved, however small elastic deformations are permitted. To preserve geometry, enough number of members and their adequate arrangement is required. For the geometric stability, there should not be any condition of mechanism. Mechanism is formed when there are three collinear hinges, hence to preserve geometric stability there should not be three collinear hinges.

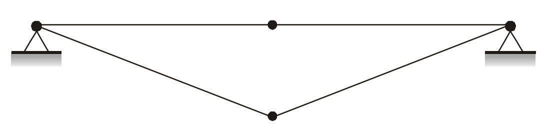


Fig.

For 2-D truss the minimum number of members needed for geometric stability are:

$$m = 2j - 3$$

and for 3-D truss,

$$m = 3j - 6$$

where, j = Number of joints in truss

m = Number of members required for geometrical stability.

All the members should be arranged in such a way that truss can be divided into triangular blocks. i.e. no rectangular or polygonal blocks.

Hence, for overall geometrical stability of truss:

(i) Minimum number of member should be present

$$m = 2j - 3 \quad \text{(2-D truss)}$$

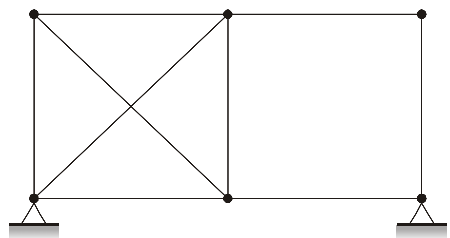
and

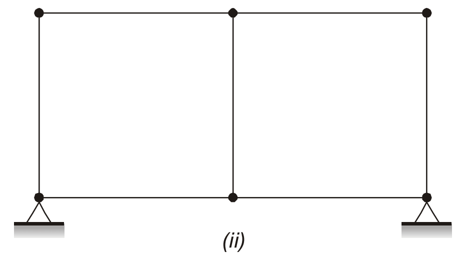
$$m = 3j - 6 \quad \text{(3-D truss)}$$

(ii) There should be no condition of mechanism i.e. no three collinear hinges.

EXAMPLE - 1.2

Check geometrical stability for given trusses.


(i)


(ii)

Solution:

(*i*) In case (*i*), arrangement of members is not adequate, hence right panel is unstable and left panel is over stiff. For geometric stability, all panels of truss should be stable so given truss is geometrical unstable.

For right panel: $j = 4$

Number of member present, $m = 4$

But minimum number of member needed $= 2j - 3 = 2 \times 4 - 3 = 5$

Hence Right panel is deficient.

For left panel: $j = 4$

Number of member present, $m = 6$

But minimum number of member needed $= 2j - 3 = 2 \times 4 - 3 = 5$

Hence left panel is over stiff.

(*ii*) $j = 6$

Number of members present, $m = 7$

But minimum number of member needed $= 2j - 3 = 2 \times 6 - 3 = 9$

Hence, above truss is geometrically unstable and it can be called 'deficient structure'.

$\therefore$ Number of deficiency $= 2$

1.5.3 Overall Stability

For overall stability, external stability is compulsory. In some cases structure is overall stable but it may be over stiff externally or deficient internally. It mean support reactions are more than three and number of member are less then $2j - 3$.

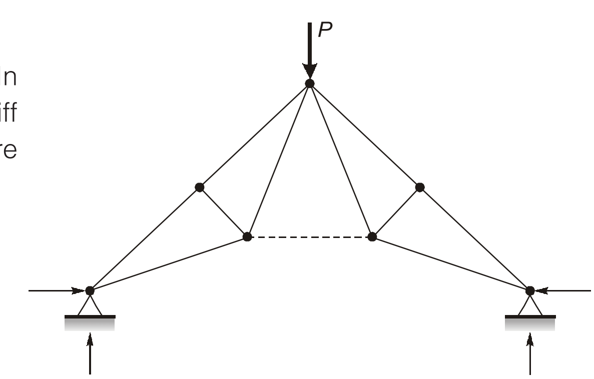


Fig.

Consider a truss shown in above Fig.,

Here,

External reaction, $r_e = 4$

Number of member present, $m = 10$

But,min. number of members needed

$$= 2j - 3 = 2 \times 7 - 3 = 11$$

It means truss is deficient to 1 degree.

But above truss is overall stable because there is one extra redundant reaction which prevent geometric deficiency.

In adjoining Fig., external reactions,

$r_e = 8$

Number of member present, $m = 8$

But min. no. of member needed $= 2j - 3 = 2 \times 8 - 3 = 13$

it means truss is deficient to 5 degree

But above structure is overall stable because there are five extra redundant reaction which prevent geometric deficiency.

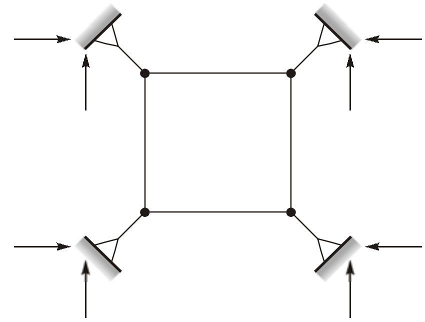
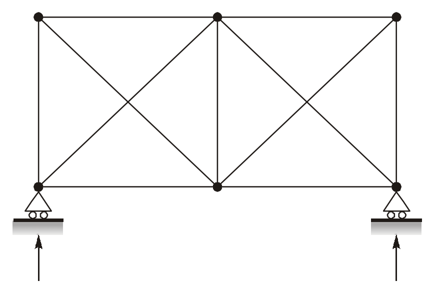


Fig.

EXAMPLE - 1.3

Comment on the stability of pin-jointed frame shown in figure.



Solution:

External reaction, $r_e = 2$

No. of member present = 11

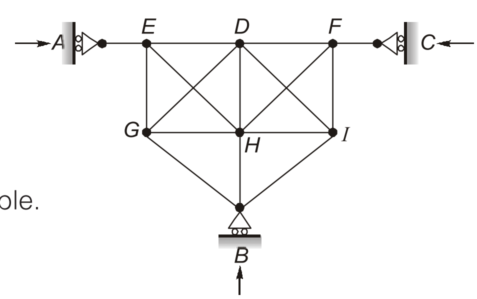
No. of joints = 6

Min. no. of members needed $= 2j - 3 = 2 \times 6 - 3 = 9$

Above truss is internally stable (over stiff) but externally unstable. Hence this truss is overall unstable.

EXAMPLE - 1.4

Comment on the stability of pin-jointed frame shown in figure.



Solution:

External stability:

Number of external reaction = 3

All three reactions are non-parallel but all three reactions are concurrent at point *D*, hence given frame is externally unstable.

Internal stability:

Number of joint, $j = 9$

Number of member present, $m = 16$

Number of member needed $= 2j - 3$

$= 2 \times 9 - 3 = 15$

Above frame is internally stable (over stiff).

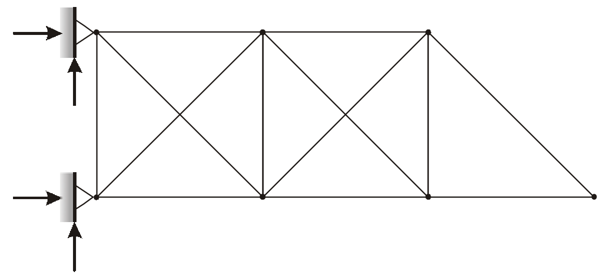
Since frame is externally unstable. Hence given frame is overall unstable.

REMEMBER It is desirable for overall stability, structure should be stable externally and internally both.

PRACTICE QUESTIONS

Question : 1

What is the total degree of static indeterminacy (both internal and external) of the cantilever plane truss shown in the figure below?



Solution:

$$m = 13$$
$$j = 7$$
$$r_e = 4$$

First approach:

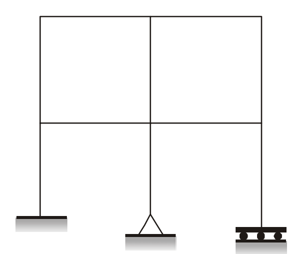
$$D_{Se} = r_e - 3 = 4 - 3 = 1$$
$$D_{Si} = m - (2j - 3)$$
$$= 13 - (2 \times 7 - 3) = 2$$
$$\therefore \quad D_S = D_{Se} + D_{Si}$$
$$D_S = 1 + 2 = 3$$

Second approach:

$$D_S = m + r_e - 2j$$
$$= 13 + 4 - 2 \times 7 = 3$$

Question : 2

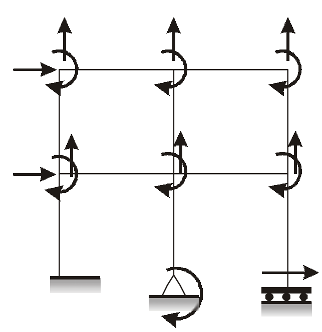
Rigid jointed frame shown in figure. Find degree of kinematic indeterminacy. Assuming beams are axially inextensible.



Solution:

Here, $j = 9$

$$r_e = 3 + 2 + 2 = 7$$
$$m'' = \text{Axially rigid members} = 4$$
$$D_K = 3j - r_e - m'' + r_r$$
$$\therefore \quad D_K = 3 \times 9 - 7 - 4$$
$$= 27 - 7 - 4$$
$$= 16$$



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