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Theory of Machines

Mechanical Engineering

Comprehensive Theory *with* Solved Examples

Civil Services Examination

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Theory of Machines

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Contents

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CHAPTER 1

Simple Mechanism

1.1 Introduction

The theory of machines is an applied science which is used to understand the relationships between the geometry and relative motions of the parts of a machine or mechanism and the forces which produce these motions. It comprises the study of relative motion between the various parts of a machine and the study of the forces that act on those parts.

The major objectives of the subject Theory of Mechanisms and Machines are to provide the engineers the necessary tools to systematically synthesize a system which means scientifically arriving at the critical shapes and dimensions of the bodies constituting the system.

1.2 Elements or Links

A link or element is a rigid body which possesses at least two nodes which are points for attachments to other links.

OR

A link (or element or kinematic link) is a resistant body (or assembly of resistant bodies) constituting a part (or parts) of the machine, connecting other parts, which have motion, relative to it.

1.2.1 Types of Links

Links can be classified as binary, ternary, or quaternary depending upon the ends on which revolute or turning pairs can be placed as shown in figure.

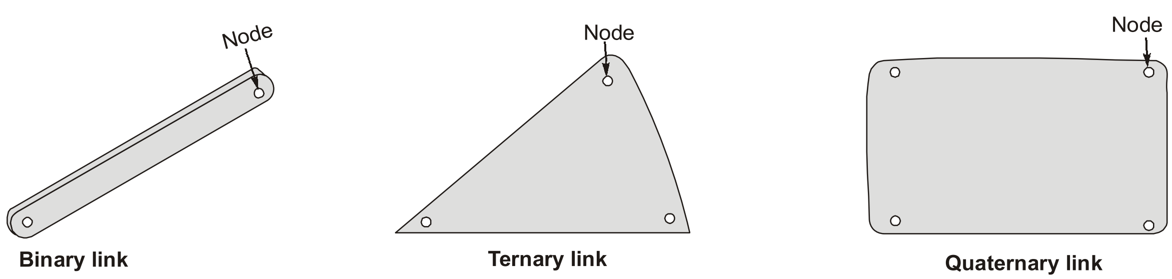


Fig. *Types of Links*

Binary Link : Rigid body with two nodes
Ternary Link : Rigid body with three nodes
Quaternary Link : Rigid body with four nodes
Kinematic links can be divided into three types :

- **Rigid Link :** It does not undergo any deformation while transmitting motion. Links in general are elastic in nature. Links are considered rigid if they do not undergo appreciable deformation while transmitting motion, e.g. connecting rod, crank, valve stem of camshaft etc.
- **Flexible Link :** It is one which is partly deformed in a manner not to affect the transmission of motion, e.g. belts, ropes, springs etc.
- **Fluid Link :** It is deformed by having fluid in a closed vessel and the motion is transmitted through the fluid by pressure, e.g. hydraulic jack, hydraulic brake in automobiles etc.

1.3 Kinematic Joint

A kinematic joint is the connection between two links by a pin. There is ample clearance between the pin and the hole in the ends of the links being connected to provide free motion of the links.

The usual types of joints in a chain are as shown in figure.

- **Binary joint** : Two links are connected at the same joint by a pin.
- **Ternary joint** : Three links are connected at the same joint by a pin.
- **Quaternary joint** : Four links are connected at the same joint by a pin.

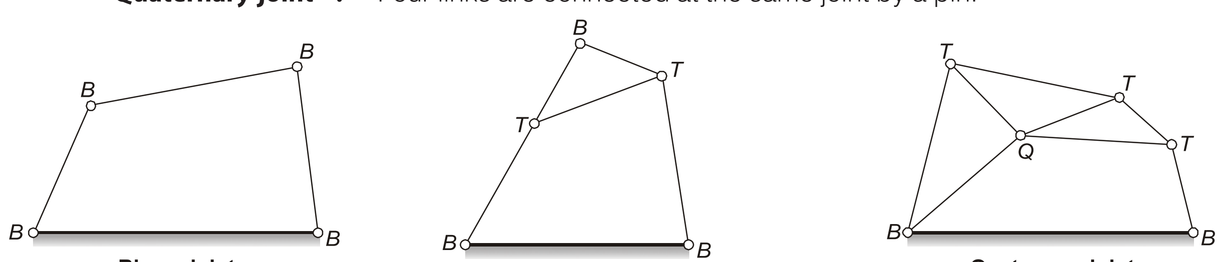


Fig. *Type of Kinematic joints*

NOTE : If n number of links are connected at a joint, it is equivalent to $(n - 1)$ binary joint.

1.4 Kinematic Pair

The two links of a machine, when in contact with each other, are said to form a pair. A kinematic pair consists of two links that have relative motion between them. The links of a mechanism must be connected together in such a manner that these transmit motion from the driver or input link to the follower or output link. Two elements or links which are connected or joined together in such a way that their relative motion is completely constrained forms a kinematic pair.

Kinematic pairs can be classified according to :

- Nature of contact
- Nature of mechanical constraint
- Nature of relative motion

1.4.1 Kinematic Pairs according to Nature of Contact

Lower Pair : When the two elements have surface (or area) contact while in motion and the relative motion is purely turning or sliding, they are called as lower pair. All sliding pairs, turning pairs and screw pairs are lower pairs (given in table (a)).

Examples :

- Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint etc.

Table (a)

Name	Relative Motion	Degree of Freedom
Rigid joint	0 rotation 0 translation	0
Revolute	1 rotation 0 translation	1
Prismatic	0 rotation 1 translation	1
Helical	1 rotation 1 translation	2
Cylindrical	1 rotation 1 translation	2
Spherical	3 rotation 0 translation	3
Planar	1 rotation 2 translation	3

Higher Pair : A pair of links having a point or line contact between the members is called a higher pair shown in table (b). The contact surfaces of the two links are dissimilar.

Examples:

- Contact between cam and follower, contact between two mating gears, a wheel rolling on a rail, ball rolling on a flat surface, ball and roller bearings.

Table (b)

Description	Degree of Freedom
Cylindrical surface on a plane without slipping	1
Cylindrical surface on a plane with slipping	2
Ball on a plane without slipping	3
Point on a plane with slipping	4

1.4.2 Kinematic Pair according to the Relative Motion

- **Sliding pair :** If two links have a sliding motion relative to each other, they form a sliding pair.
  Examples : Rectangular rod in a rectangular hole in a prism, piston and cylinder of an engine, cross-head and guides of a steam engine, ram and its guide in shaper etc.
- **Turning (revolute pair) :** When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair.
  Examples : Four bar chain, crankshaft turning in a bearing etc.
- **Rolling pair :** When the links of a pair have a rolling motion relative to each other, they form a rolling pair.
  Examples : Ball and roller bearings, wheel rolling on flat surface etc.

- **Screw pair (Helical pair) :** If two mating links have a turning as well as sliding motion between them, they form a screw pair.
  Examples : Bolt with a nut, lead screw and nut of a lathe etc.
- **Spherical pair :** When one element in the form of a sphere turns about the other fixed element, it forms a spherical pair. **Examples :** Ball and socket joint.

1.4.3 Kinematic pairs according to Nature of Mechanical Constraint

- **Closed pair :** When two elements of a pair are held together mechanically, it forms a closed pair. All the lower pairs and some of the higher pairs are closed pairs.
  Examples : Sliding pairs, turning pairs, spherical pairs, screw pairs.
- **Open pair (Unclosed pair) :** When two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be forced-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

NOTE

(i) Screw pair (helical pair) has only one degree of freedom because the sliding and rotational motions are related by the helix angle of the thread. If the helix angle is made zero, the screw pair becomes a turning pair and if it is made 90°, the screw pair becomes a sliding pair.
(ii) Rotating pin joint and translating slider joint are also referred as full joints (i.e. full = 1 DOF) and they are lower pairs.
(iii) Two-freedom joint (2 DOF) is sometimes referred to as a "half joint". The half joint is also called a roll-slide joint because it allows both rolling and sliding.

1.5 Degrees of Freedom (DOF)

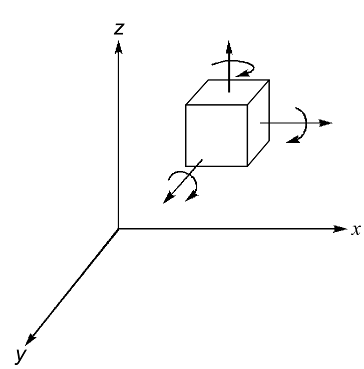


Fig. *Degrees of Freedom*

An unconstrained rigid body moving in space possesses six degrees of freedom, as shown in figure.

Definition of DOF :

The number of inputs which need to be provided in order to create a predictable output.

OR

The number of independent coordinates required to define the position of mechanism.

1.6 Machine, Mechanism and Kinematic Chain

Machine : A machine is defined as a combination of resistant bodies arranged to compel the mechanical forces of nature to do work accompanied by determinate motions.

Mechanism : A mechanism is defined as a kinematic chain in which at least one link has been "grounded" or attached, to the frame of reference (which itself may be in motion.

Kinematic Chain : A kinematic chain is defined as an assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied input motion.

1.6.1 Machines: The layout of machines classification is shown in figure

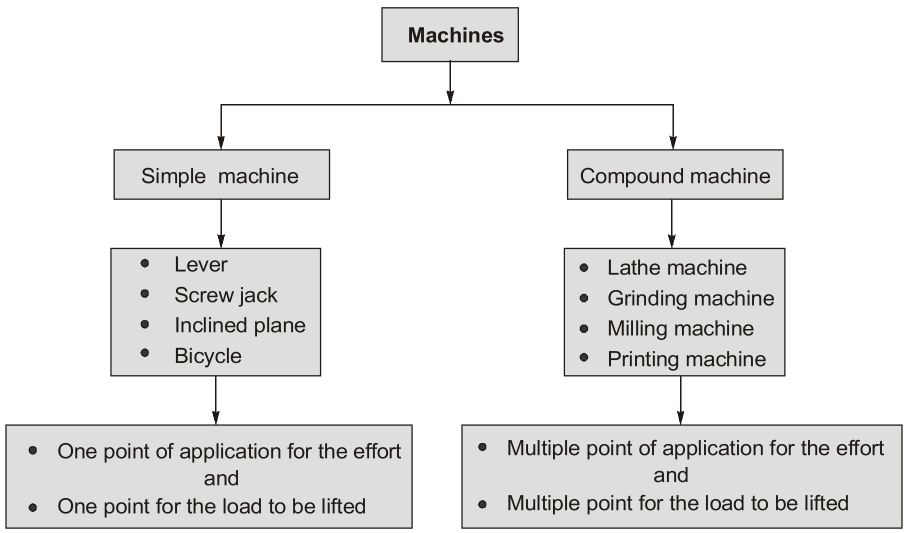


Fig.

1.6.2 Kinematic Chain

A kinematic chain may be defined as an assembly of links in which the relative motion of the links is possible and the motion of each link relative to the others is definite. The last link of the kinematic chain is attached to the first link.

The following relationship holds for a kinematic chain having lower pair only :

$$L = 2P - 4$$

$$J = \frac{3}{2}L - 2$$

where, L = number of binary links
P = number of lower pairs
J = number of binary joints

If LHS > RHS, then chain is called locked or redundant chain
LHS = RHS, then chain in constrained
LHS < RHS, then chain is unconstrained

For a kinematic chain having higher pairs, each higher pair is taken equivalent to two lower pairs and an additional link.

$$J + \frac{H}{2} = \frac{3}{2}L - 2$$

where, H = number of higher pairs

1.6.3 Mechanisms

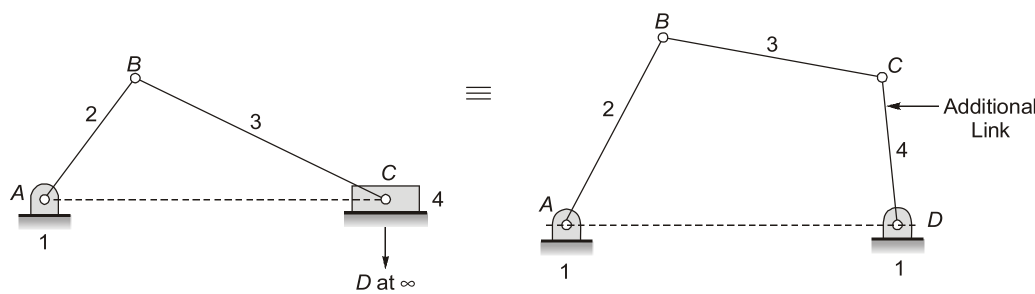
When one of the links of a kinematic chain is fixed, the chain is known as mechanism. It is used for transmitting or transforming the input motion.

The mechanisms are of following types:

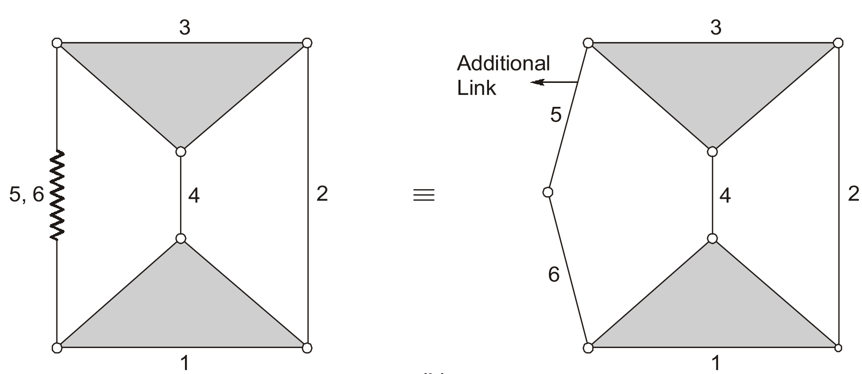
- **Simple mechanism :** A mechanism having four links.
- **Compound mechanism :** A mechanism which has more than four links.
- **Complex mechanism :** It is formed by the inclusion of ternary or higher order floating link to a simple mechanism.
- **Planar mechanism :** When all the links of the mechanism lie in the same plane.
- **Spatial mechanism :** When the links of the mechanism lie in different planes.

Equivalent Mechanisms

- A sliding pair is equivalent to a turning pair (Fig. (a) and (c)).
- A spring can be replaced by two binary links (Fig. (b)).
- A cam pair can be replaced by one binary link together with two turning pairs at each ends Fig. (d).



(a)



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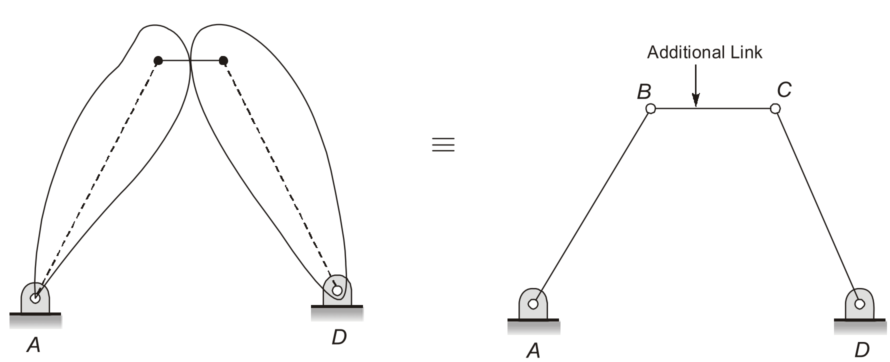
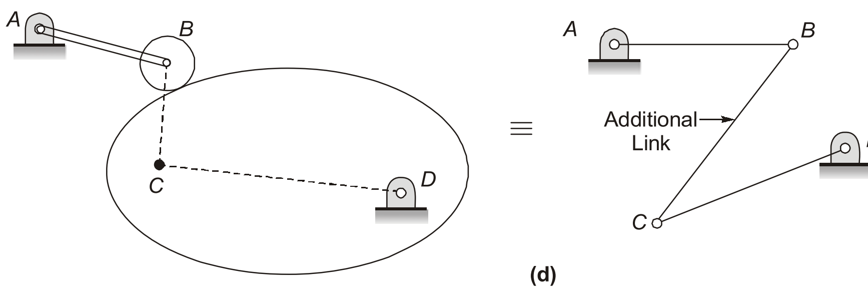


(c)



(d)

Fig. *Equivalent Linkages*

1.7 Mobility of Mechanism

The mobility of a mechanism is defined as the number of degree of freedom it possesses. In other way, the minimum number of independent parameters required to specify the location of every link within a mechanism.

1.7.1 Mobility of Space Mechanisms

Let N = total number of links in a mechanism

F = degree of freedom

P_1 = number of pairs having one degrees of freedom

P_2 = number of pairs having two degrees of freedom

P_3 = number of pairs having three degrees of freedom

P_4 = number of pairs having four degrees of freedom

P_5 = number of pairs having five degrees of freedom

As we know, DOF = 6 – number of restraints

∴ Number of restraints = 6 – DOF

Therefore,

Number of movable links = $N - 1$

Number of degrees of freedom if $(N - 1)$ movable links = $6(N - 1)$

Each pair having one degree of freedom imposes 5 restraints on the mechanism which reduces its degree of freedom by $5P_1$. Similarly by $4P_2$, $3P_3$, $2P_4$, and P_5.

So, $$F = 6(N - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 \quad ...(1.1)$$

1.7.2 Mobility of Planar Mechanisms

Based on the derivation of the Equation (1.1), the number of degree of freedom or mobility of planar mechanism is given by following relation which is also known as the Kutzbach equation

$$F = 3(N - 1) - 2P_1 \quad ...(1.2)$$

If $F = 1$, the mechanism is said to be constrained. Practically, most mechanisms used in machinery are constrained, for this condition, putting $F = 1$ in Equation (1.2),

$$2j - 3n + 4 = 0 \quad \text{or} \quad 2P_1 - 3N + 4 = 0 \quad ...(1.3)$$

The simple estimate of constrained movement expressed by Equation (1.3) is known as Gruebler's criterion for plane mechanism. The degrees of freedom of a mechanism having higher pairs can also be written as

$$F = 3(n - 1) - 2j - h \quad \text{or} \quad F = 3(N - 1) - 2P_1 - P_2 \quad ...(1.4)$$

where h and P_2 is the number of higher pairs.

Quite often, one or more links of a mechanism may have a redundant degree of freedom. If a link can be moved without causing any movement in the rest of the mechanism, then the link is said to have a redundant degree of freedom. The effective degree of freedom of a mechanism can be expressed as

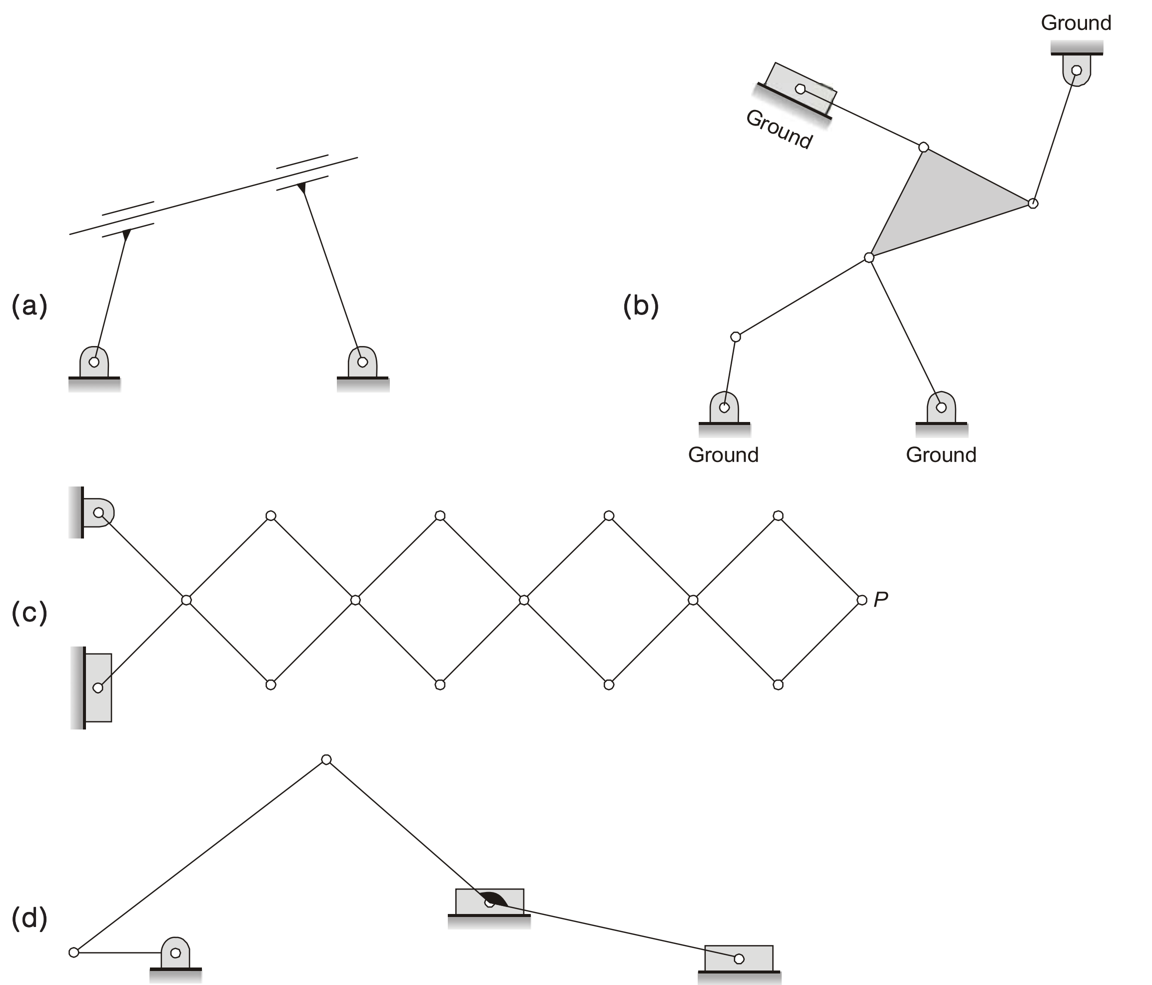
$$F_e = 3(n-1) - 2j - h - F_r \qquad \text{...(1.5)}$$

where F_r is the number of redundant degree of freedom.

A system may posses one or more links which do not introduce any extra constraint. Such links are redundant. If a plane mechanism has redundant links and redundant kinematic pairs, the Equation (1.5) gets modified as

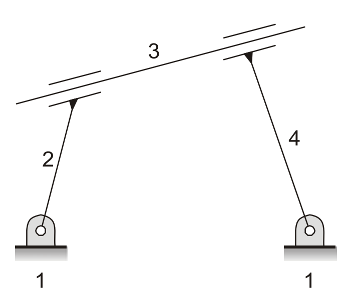
$$F_e = 3(n - n_r - 1) - 2(j - j_r) - h - F_r \qquad \text{...(1.6)}$$

Example 1.1 Determine the degree of freedom of the following mechanism:



Solution :

(a) In this mechanism, rod 3 can slide without causing any movement in the rest of the mechanism.



Number of links = 4

Number of joints = 4

Redundant degree of freedom = 1

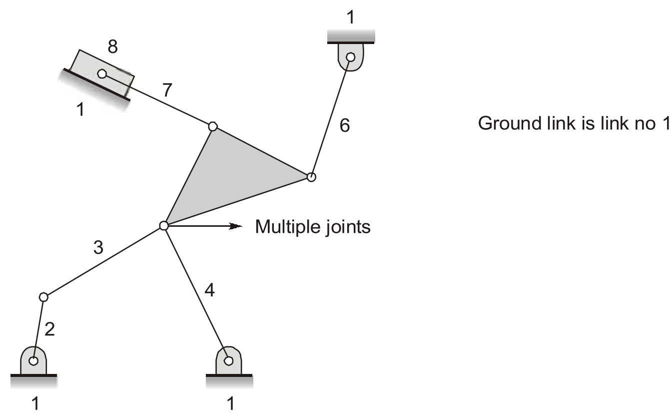
$\therefore$ $F_e = 3(n-1) - 2j - F_r$ (∵ There is no higher pair)

$F_e = 3(4-1) - 2 \times 4 - 1$

$F_e = 3 \times 3 - 8 - 1 = 0$

As the effective degree of freedom is zero, it is a locked system. In this case, Gruebler's criterion is NOT APPLICABLE.

(b)



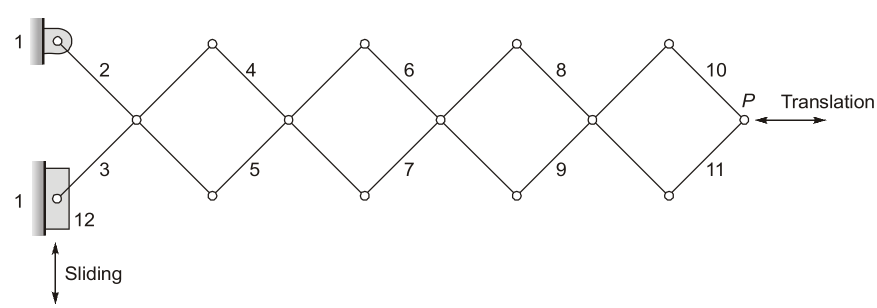
Number of links = 8

Number of joints or lower pairs = 10

$F = 3(n-1) - 2j - h = 3(8-1) - 2 \times 10 = 1$

There is no higher pair in this mechanism and this mechanism is constrained with DOF = 1.

(c)



link 1 is grounded link

Number of links, $n = 12$

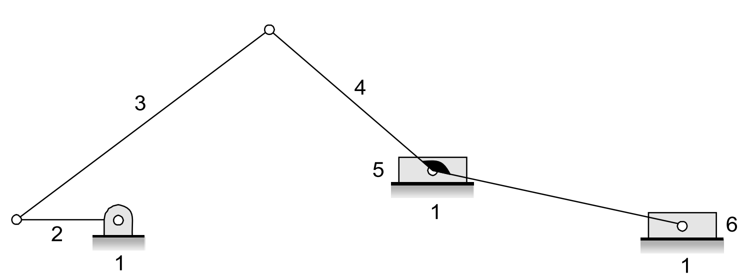
Number of lower pairs, $j = 16$

$F = 3(n-1) - 2j - h = 3(12-1) - 2 \times 16 = 1$

$F = 33 - 32 = 1$

As the degree of freedom is one, this is constrained mechanism.

(d)



Link 4 is the link connecting link numbers 5 and 6

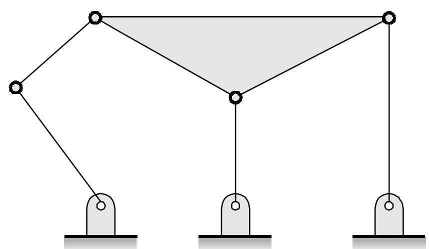
Link 4 is only one link (it is not two)

Number of link, $n = 6$

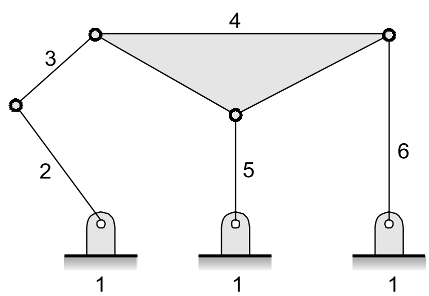
Number of lower pairs, $j = 7$

$$F = 3(n-1) - 2j - h = 3(6-1) - 2 \times 7 - 0$$
$$= 15 - 14 = 1$$

Example 1.2 What is the degree of freedom of the following mechanism?



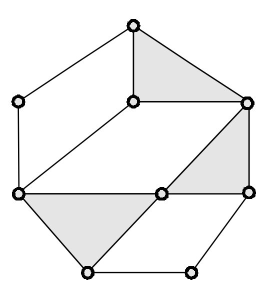
Solution:



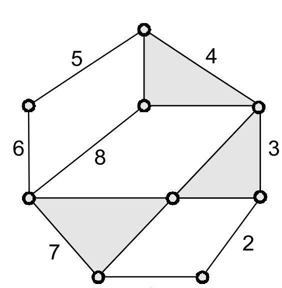
Number of links = 6
Number of lower pairs = 7
Number of higher pair = 0

Degree of freedom, $F = (n-1) - 2j - h = 3(6-1) - 2 \times 7 - 0$
$= 1$

Example 1.3 What is the degree of freedom for the linkage shown below?



Solution:



Number of links = 8

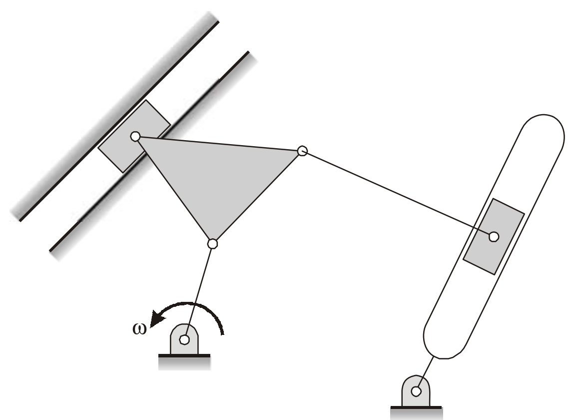
Number of ternary links = 3

⇒ Links 6, 7 and 8 constitute a double pair so that total number of pairs is equal to 10.

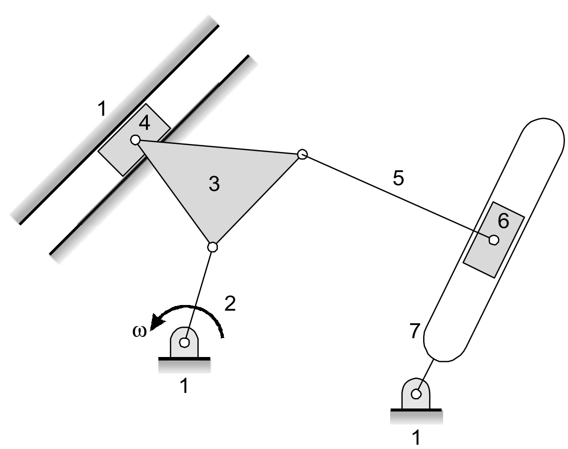
Degree of freedom, $F = 3(n - 1) - 2j - h = 3(8 - 1)\ (2 \times 10 - 0)$

$= 1$

Example 1.4 If possible coordinate is available for all points in the planar linkage shown below, can all of the velocities be determined uniquely if the value of ω is given? Justify your answer.



Solution :



Number of links, $n = 7$

Number of lower pairs, $j = 8$

$F = 3(7 - 1) - 2 \times 8 - 0 = 3 \times 6 - 16 - 0$

$= 2$

∴ Mobility is 2. Therefore, the answer to the problem is No. This mechanism has two degrees of freedom, and two independent input variables must be specified before all of the velocities must be determined.

1.7.3 Planar Mechanism with Lower Pairs

The degree of freedom of some of the planar mechanism have been listed in following table :

Table: *Degree of Freedom of Planar Mechanisms*

S.No.	Mechanism	n	j	h	$F = 3(n-1) - 2j - h$
1.	Three - bar	3	3	0	$F = 3(3-1) - 2 \times 3 - 0 = 0$
2.	Four - bar	4	4	0	$F = 3(4-1) - 2 \times 4 - 0 = 1$
3.	Five - bar	5	5	0	$F = 3(5-1) - 2 \times 5 - 0 = 2$
4.	Five - bar	5	6	0	$F = 3(5-1) - 2 \times 6 - 0 = 0$
5.	Six - bar	6	8	0	$F = 3(6-1) - 2 \times 8 - 0 = -1$
6.	Four - bar	4	5	0	$F = 3(4-1) - 2 \times 5 - 0 = -1$
7.	Three - bar	3	2	1	$F = 3(3-1) - 2 \times 2 - 1 = 1$
8.	Four - bar	4	3	1	$F = 3(4-1) - 2 \times 3 - 1 = 2$
9.	Five - bar	5	11/2	0	$F = 3(5-1) - 2 \times 11/2 - 0 = 1$
10.	Six - bar	6	7	0	$F = 3(6-1) - 2 \times 7 - 0 = 1$

As per Gruebler's criterion, $F = 1$ for constrained motion which leads to

$$1 = 3(n-1) - 2j - h$$

or
$$2j + h - 3n + 4 = 0 \qquad \text{...(1.7)}$$

Equation (1.7) represents the Gruebler's criterion

If $h = 0$, then

$$j = \frac{3n}{2} - 2 \qquad \text{...(1.8)}$$

Therefore, a planar mechanism with $F = 1$ and having only lower pairs, cannot have odd number of links. As j and n are to be whole numbers, the relation can be satisfied only if n is even.

For possible linkages made of binary links only,

$n = 4$, $j = 4$: No excess turning pair

$n = 6$, $j = 7$: One excess turning pair

$n = 8$, $j = 10$: Two excess turning pair

We can see that the number of excess turning pairs increase as the number of links increase. To get the required number of turning pairs from the same number of binary links is not possible. Therefore, the additional pairs or joints can be obtained only from the links having more than two joining points, i.e., ternary or quaternary links etc.

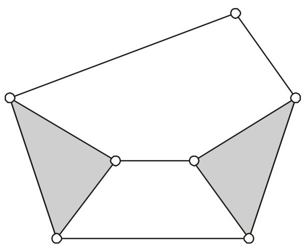
NOTE

(i) If the DOF is positive, it will be a mechanism.
(ii) If the DOF is exactly zero, then it will be a structure.
(iii) If the DOF is negative, then it is a preloaded structure.
(iv) The minimum number of binary links is four i.e., the four bar linkage is the simplest mechanism.
(v) There are few mechanism, where the direct application of Gruebler's criterion gives a wrong result are known as paradoxes.

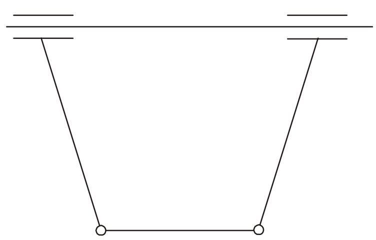
Student's Assignments

1. Calculate the number of degrees of freedom of the mechanism shown in below figure.

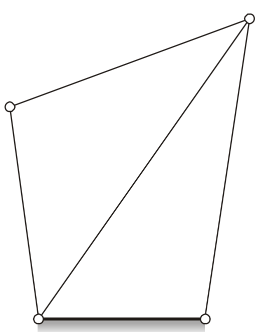
(a)



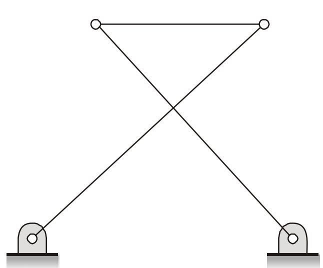
(b)



(c)



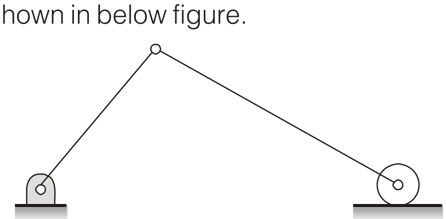
(d)



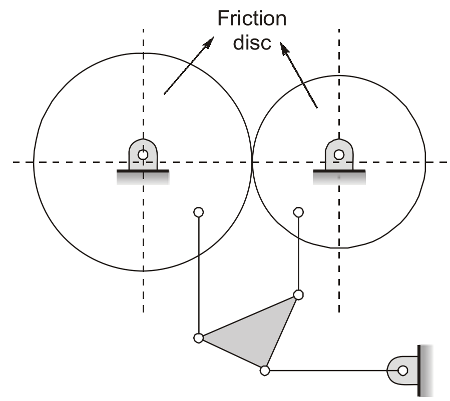
Ans. (a) 1 (b) 0
(c) 0 (d) 1

2. Find the degree of freedom of the mechanism shown in below figure.

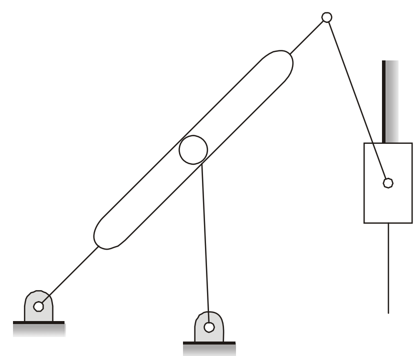
(a)



(b)



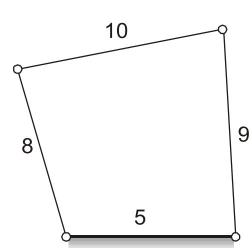
(c)



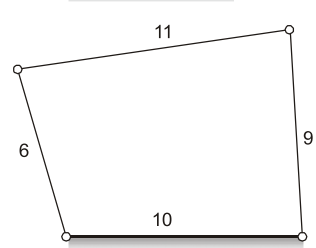
Ans. (a) 2 (b) 1
(c) 1

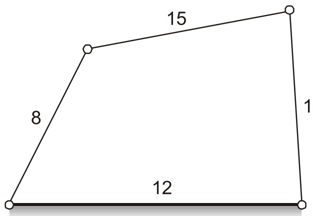
3. Figure shows some four-link mechanism in which the figures indicate the dimension in standard units of length. Determine the type of each mechanism.

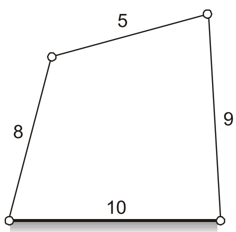
(a)



(b)



(c) 

(d) 

Ans. (a) Double crank
(b) Crank-rocker
(c) Double rocker
(d) Double rocker

4. In a quick-return motion mechanism of the oscillating link type, the distance between the fixed centres is 80 mm and the length of the driving crank is 20 mm. Determine the time ratio of the working stroke to the return stroke.

Ans. 1.38

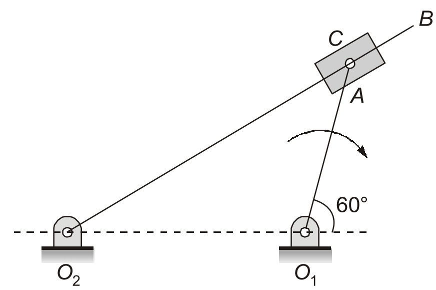
5. In an offset slider crank mechanism, the eccentricity is 50 mm, determine the quick return ratio.

Ans. 1.293

6. The distance between the parallel shafts of an Oldham's coupling is 15 mm. The driving shaft revolves at 160 rpm. Calculate the maximum speed of sliding of the tongue of the intermediate piece along its groove.

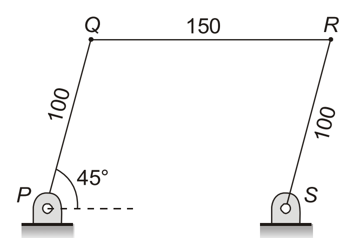
Ans. 0.251 m/s

7. An oscillating quick-return mechanism consists of a 3 cm crank O_1A rotating clockwise at 5 rad/s. The end A is pinned to a collar C which can slide along an oscillating bar O_2B 15 cm long. At the instant shown in given figure, determine
(i) the velocity of B
(ii) the angular velocity of the bar O_2B
(iii) the angular acceleration of the bar O_2B.



Ans. (i) $2.27\, i - 10\, j$ m/s
(ii) $-0.87\, k$ rad/s
(iii) $-95.3\, j$ rad/s^2

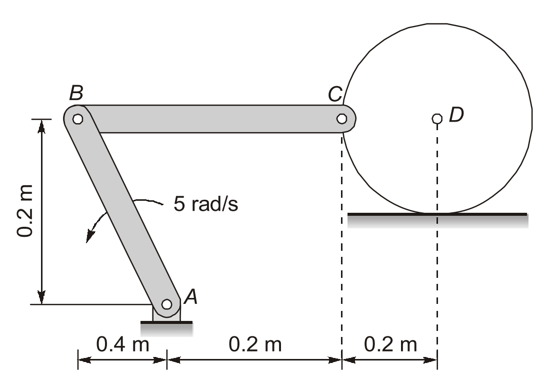
8. In a four-bar mechanism *PQRS* shown in given figure, link *PQ* rotates anticlockwise at 5.25 rad/s and accelerates clockwise at 23 rad/s^2. What is the angular velocity and angular acceleration of the link *RS*?



(All dimensions in mm)

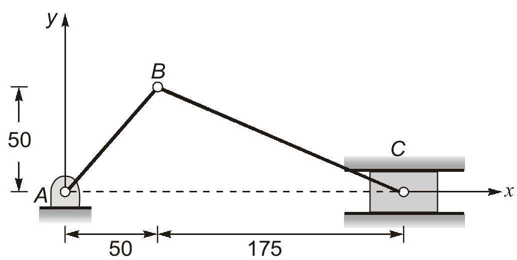
Ans. (i) 4.05 rad/s CCW
(ii) 17 rad/s^2 CCW

9. Bar *AB* is rotating in the counter clockwise direction at 5 rad/s. The disk rolls on the horizontal surface. Determine the angular velocity of bar *BC*.



Ans. $\omega_{BC} = 1.67$ rad/s CCW

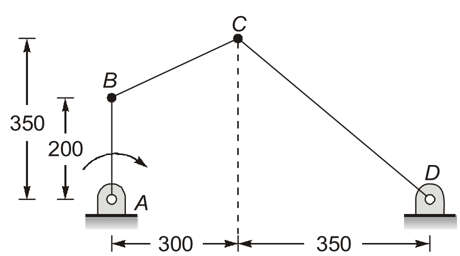
10. The crank AB (as shown in given figure) is rotating in the clockwise direction at 2000 rpm.
    (a) At the instant shown, what are the coordinates of the instantaneous centre of the connecting rod BC?
    (b) Use instantaneous centres to determine the angular velocity of the connecting rod BC at the instant shown.



(All dimensions are in mm)

Ans. (a) x = 225 mm, y = 225 mm
(b) ω_{BC} = 59.8 rad/s, v_c = 13.5 m/s

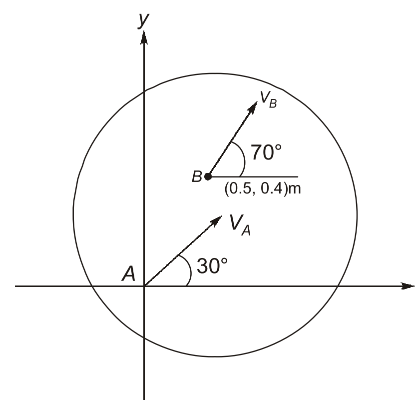
11. Bar AB rotates at 12 rad/s in the clockwise direction. Use instantaneous centres to determine the angular velocities of bars BC and CD.



(All dimensions are in mm)

Ans. ω_{BC} = 5.33 rad/s (CCW)
ω_{CD} = 4.57 rad/s (CW)

12. The disk is in planar motion. The direction of the velocities of points A and B are shown in given figure. The velocity of point A is v_A = 2 m/s.
    (a) What are the coordinates of the disk's instantaneous centres?
    (b) Determine the velocity v_B and disk's angular velocity.



Ans. (a) x_C = – 0.425 m, y_c = 0.737 m
(b) v_B = 2.31 m/s, ω = 2.351 rad/s (CCW)

■■■■