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Civil Services Main Examination Previous Solved Papers : Mechanical Engg. (Paper-I)

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Preface

B. Singh (Ex. IES)

Civil Service is considered as the most prestigious job in India and it has become a preferred destination by all engineers. In order to reach this estimable position every aspirant has to take arduous journey of Civil Services Examination (CSE). Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.

I feel extremely glad to launch the revised edition of such a book which will not only make CSE plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years papers of CSE. The book aims to provide complete solution to all previous years questions with accuracy.

On doing a detailed analysis of previous years CSE question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Service and State Services exams. Hence, this book is a one stop shop for all CSE, ESE, IFS and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh (Ex. IES)

CMD, MADE EASY Group

Previous Years Solved Papers of

Civil Services Main Examination

Mechanical Engineering : Paper-I

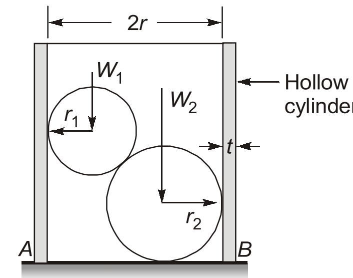
CONTENTS

1 Engineering Mechanics

1. Equations of Equilibrium and Statics

Q.1 A smooth hollow cylinder of radius r open at both ends rests on a smooth horizontal plane. Two smooth spheres of weights W_1 and W_2 and radii r_1 and r_2, respectively are placed inside the cylinder, with the larger sphere (radius r_2) resting on the horizontal plane as shown in figure.

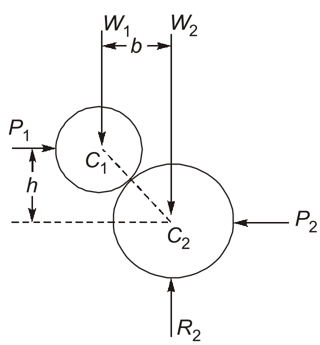
Determine the minimum weight W of the cylinder that will prevent the cylinder from tipping over.

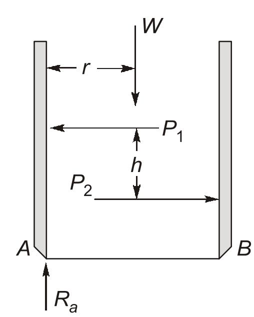


[CSE (Mains) 2009 : 12 Marks]

Solution:

Let us consider first a free-body for the two sphere, assuming that they are joined together at their point of contact as shown in figure. By virtue of the assumptions of smooth surfaces, we conclude that the reactive forces P_1 and P_2 exerted on the spheres by the walls of the cylinder are horizontal forces as shown and likewise that the reaction R_2 on the bottom of the lower sphere is a vertical force.





Thus the two spheres are in equilibrium under the action of the five coplanar forces shown in figure.

By equilibrium equation in horizontal axis,

$$P_1 - P_2 = 0 \quad \text{...(i)}$$

Taking moment about point C_2,

$$W_1 b - P_1 h = 0 \quad \text{...(ii)}$$

From equations (i) and (ii), $P_1 = P_2 = \dfrac{W_1 b}{h}$

Under equilibrium condition at the verge of tipping, there will be no contact at point B. Force P_1 and P_2 constitute a couple, $M = P_1 \times h = \dfrac{W_1 b}{h} \times h = W_1 b$

Taking moment about point A, $Wr = W_1 b$

From which, $W = W_1 \dfrac{b}{r}$

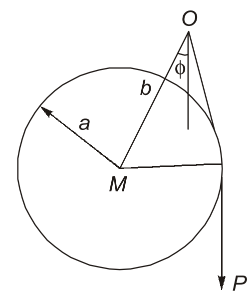
$$b = 2r - r_1 - r_2$$

Expressions becomes, $$W = W_1 \frac{(2r - r_1 - r_2)}{r}$$

Minimum weight of cylinder, $$W = W_1\left(2 - \frac{r_1 + r_2}{r}\right)$$

Q.2 A homogeneous ball of weight Q and radius a as well as a weight P are suspended by cords from a point O as shown in figure. The distance OM is b. Find the inclination ϕ of OM with the vertical when the system is in equilibrium.

[CSE (Mains) 2009 : 8 Marks]

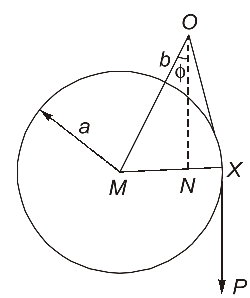


Solution:

Given: Weight of ball $= Q$, Radius $= a$, weight $= P$, $OM = b$.

Let inclination of OM with vertical when the system is in equilibrium be (ϕ).

$$OM = b$$
$$MN = b \sin\phi \text{ and } ON = b\cos\phi$$
$$MX = a$$
$$NX = MX - MN = a - b\sin\phi$$

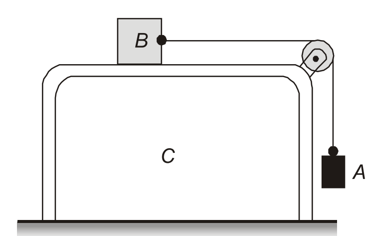


Taking moments about point O.

$$P \times (NX) = Q \times (MN)$$
$$P(a - b\sin\phi) = Q(b\sin\phi)$$
$$Pa - Pb\sin\phi = Q\,b\sin\phi$$
$$Pa = Pb\sin\phi + Qb\sin\phi = (Pb + Qb)\sin\phi$$
$$\sin\phi = \left(\frac{Pa}{Pb + Qb}\right)$$
$$\phi = \sin^{-1}\left(\frac{Pa}{Pb + Qb}\right) = \sin^{-1}\left(\frac{Pa}{b(P + Q)}\right)$$

Q.3 A body A of weight 10 kN is connected to an another body B of weight 50 kN, resting on a smooth table of weight 200 kN through an inextensible thread, passing over a freely rotating pulley mounted on a corner of the table. Find the vertical component of the reaction of the ground on the table when the bodies A and B are in motion. Does the reaction change with time? The system is shown in figure.

[CSE (Mains) 2009 : 12 Marks]



Solution:

Given data:

Weight of A, $$W_A = 10 \text{ kN} = 10000 \text{ N}$$

Weight of B, $$W_B = 50 \text{ kN} = 5000 \text{ N}$$

Weight of table, $$W_C = 200 \text{ kN} = 200 \times 10^3 \text{ N}$$

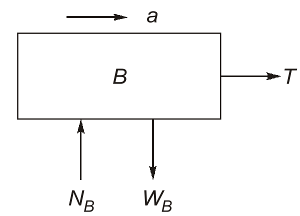
$$m_A = \frac{W_A}{g} = \frac{10000}{9.81} = \frac{10 \times 10^3}{9.81} = 1019.36 \text{ kg}$$

$$m_B = \frac{W_B}{g} = \frac{5000}{9.81} = \frac{50 \times 10^3}{9.81} = 5096.84 \text{ kg}$$

$$m_C = \frac{W_C}{g} = \frac{200 \times 10^3}{9.81} = 20387.36 \text{ kg}$$

Let T be the tension in string and N be the normal force, and acceleration of the system is a.

For body 'B'



$$T = m_B a = (5096.84)a \quad \text{... (i)}$$
$$N_B = W_B = m_B g \quad \text{... (ii)}$$

For body 'A' $\quad m_A g - T = m_A a$

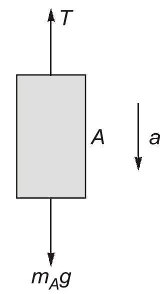
$\Rightarrow \quad (1019.36)9.81 - T = (1019.36)a \quad \text{... (iii)}$

From equations (i) and (ii),

$$(1019.36)(9.81) - (5096.84)a = (1019.36)a$$

$$a = \frac{1019.36 \times 9.81}{6116.19} = 1.634 \text{ m/sec}^2$$

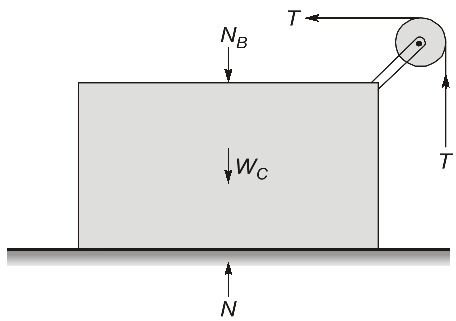
$$T = m_B a = (5096.84)(1.634) = 8328.23 \text{ N} = 8.328 \text{ kN}$$



Considering forces on table

Let vertical component of the reaction of ground on table is 'N'.

Considering equilibrium of forces in vertical direction on block



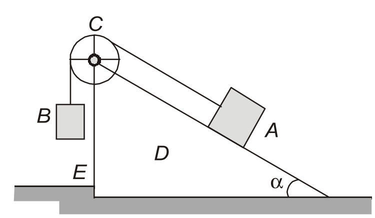
$$N - N_B - W_C - T = 0$$

$$N = N_B + W_C + T = m_B g + W_C + T = 50 + 200 + 8.328 = 258.328 \text{ kN}$$

Vertical component of the reaction of ground on table $N = 258.328$ kN.

The reaction is constant and will not change with time.

Q.4 **A body A weighing P_1 descends down an inclined plane D which makes an angle α with the horizontal and pulls a load B that weighs P_2 by means of a weightless and inextensible string passing over a pulley C as shown in figure. Determine the horizontal component of the pressure with which the inclined plane D acts on the floor rib E.**
[CSE (Mains) 2010 : 20 Marks]

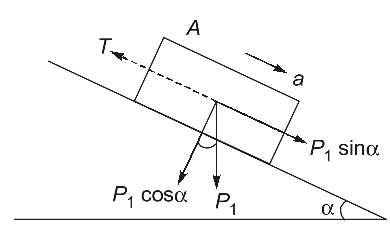


Solution:

Given: $\quad$ Weight of body $A = P_1$

$\quad$ Weight of body $B = P_2$

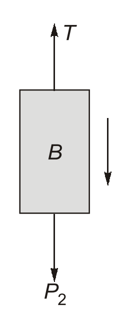
For body A



$$P_1 \sin\alpha - T = \left(\frac{P_1}{g}\right)a \quad \text{... (i)}$$

$$N = P_1 \cos\alpha \quad \text{... (ii)}$$

For body B,



$$T - P_2 = \left(\frac{P_2}{g}\right)a \quad \text{... (iii)}$$

From equations (i) and (ii), we get

$$P_1 \sin\alpha - \left(P_2 + \left\{\frac{P_2}{g}\right\}a\right) = \left(\frac{P_1}{g}\right)a$$

2 Theory of Machines

1. Analysis of Plane Mechanism

Q.1 What do you understand by inversions of a mechanism? Show two inversions of a slider crank mechanism with the help of neat sketches.

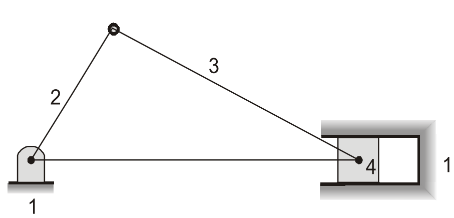
[CSE (Mains) 2001 : 20 Marks]

Solution:

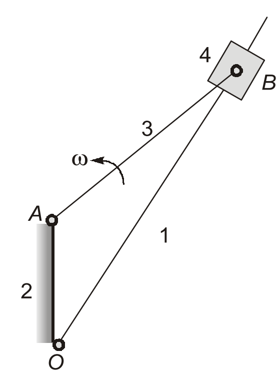
Inversion of Mechanism: When one of the links is fixed in a kinematic chain, it is called a mechanism. We can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as inversion of the mechanism.

It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically.

First Inversion of Slider-Crank Mechanism:
Applications:
(i) Reciprocating engine
(ii) Reciprocating compressor



Second Inversion of Slider-Crank Mechanism:
Applications
(i) Whitworth quick-return mechanism
(ii) Rotary engine



Two inversions of a slider crank mechanism. A slider crank mechanism consists of 4 links, i.e., link 1, link 2, link 3 and link 4, out of which one pair is sliding and other three are turning pairs.

First inversion: The inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and the slider respectively. Here, link 2 (crank) is the driver.

Applications:
(i) Reciprocating engine.
(ii) Reciprocating compressor

Second inversion: Fixing of the link 2 of slider-crank chain results in the second inversion. Here, link 2 is fixed, link 3 becomes the crank and link1 rotates about *O* with the slider reciprocating on it.

Applications:
(i) Whitworth quick return mechanism
(ii) Rotary engine

Q.2 What is a quick-return mechanism? Give its types and applications. How is the ratio of time of cutting stroke to return stroke calculated for a slotted lever and crank type of quick-return mechanism? Explain with the help of a neat sketch,

[CSE (Mains) 2010 : 15 Marks]

or

Draw a cranked-slider quick return mechanism and explain its principle with figure.

[CSE (Mains) 2015 : 10 Marks]

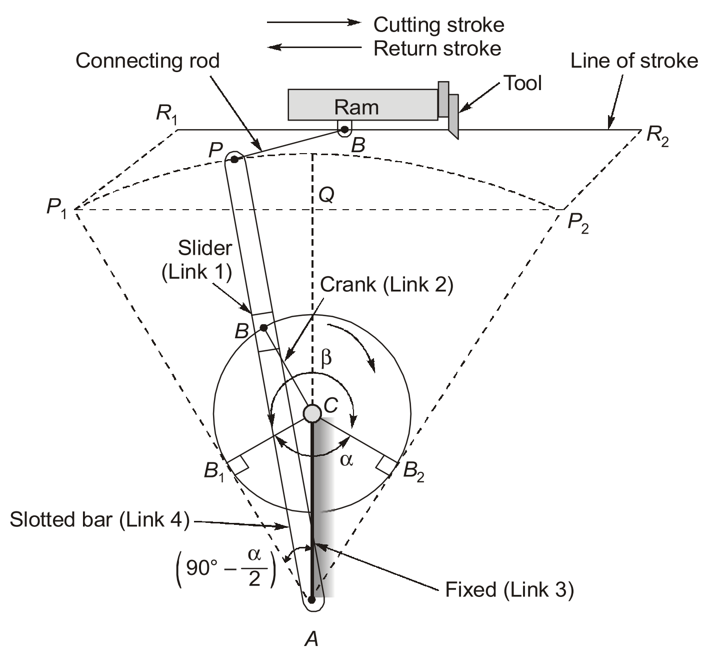
Solution:

A quick return mechanism is a mechanism which converts circular motion (rotating motion following a circular path) into reciprocating motion (repetitive back-and-forth or to-and-fro linear motion) in presses and shaping machines.

There are three types of quick return mechanism

1. Hydraulic shaper drive
2. Crank and slotted link mechanism
3. Whitworth mechanism

Following are the applications of quick-return mechanism:



Crank and slotted lever quick return motion mechanism

- Shaper
- Power-driven saw
- Revolver mechanisms
- Screw press
- Mechanical actuator

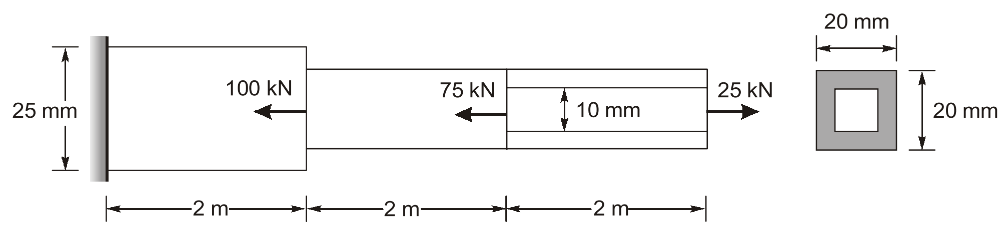
In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 and CB_1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \text{ or } \frac{360° - \alpha}{\alpha}$$

3 Strength of Materials

1. Stress, Strain and Elastic Constants

Q.1 A steel rod of square cross-section is loaded as shown in the figure.

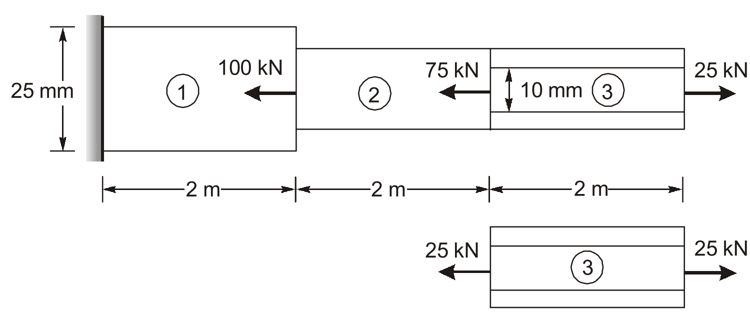


Find the section which is subjected to maximum stress, its magnitude and nature. What will be total change in its length? Take E = 200 GPa.

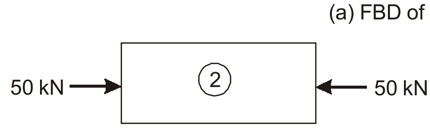
[CSE (Mains) 2005 : 20 Marks]

Solution:

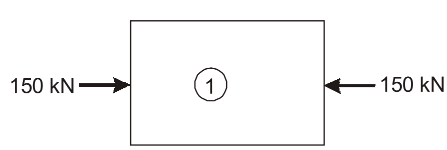
The free body diagrams of the sections shown in the below figure. Forces in sections 1 and 2 are compressive and in section 3 is tensile.



(a) FBD of section 3



(b) FBD of section 2



(c) FBD of section 1

The cross-section areas of three sections are:

$$A_1 = 25 \times 25 = 625 \text{ mm}^2$$
$$A_2 = 20 \times 20 = 400 \text{ mm}^2$$
and
$$A_3 = 20 \times 20 - 10 \times 10 = 300 \text{ mm}^2$$

Stresses induced in three sections are:

$$\sigma_1 = \frac{F_1}{A_1} = -\frac{150}{625} = -0.24 \text{ kN/mm}^2 = 240 \text{ MPa (compressive)}$$

$$\sigma_2 = \frac{F_2}{A_2} = -\frac{50}{400} = -0.125 \text{ kN/mm}^2 = 125 \text{ MPa (compressive)}$$

and $$\sigma_3 = \frac{F_3}{A_3} = \frac{25}{300} = 0.08333 \text{ kN/mm}^2 = 83.33 \text{ MPa (Tensile)}$$

Hence, maximum compressive stress is subjected in section 1,

$$\sigma_{max} = \sigma_1 = -0.24 \text{ kN/mm}^2$$

$$E = 200 \text{ GPa} = 200 \times 10^6 \text{ kPa} = 200 \text{ kN/mm}^2$$

$$\text{Total change in length} = \frac{1}{E}(\sigma_1 l_1 + \sigma_2 l_2 + \sigma_3 l_3)$$

$$= \frac{l}{E}(\sigma_1 + \sigma_2 + \sigma_3) \qquad (\because l_1 = l_2 = l_3 = l = 2 \text{ m} = 2000 \text{ mm})$$

$$= \frac{2000}{200}(-0.24 - 0.125 + 0.0833)$$

$$= -2.817 \text{ mm}$$

$$\text{Total change in length} = 2.817 \text{ mm (Decrease)}$$

Q.2 A circular disc 50 cm outside diameter has a central hole and rotates at a uniform speed about an axis through its centre. The diameter of the hole is such that the maximum stress due to rotation is 85% of that in thin ring whose mean diameter is also 50 cm. If both are of the same material and rotate at the same speed, determine the diameter of the central hole and speed of the disc for the data given below:

Allowable stress = 900 kg/cm^2

Specific weight = 7.8 gm/cm^3

Poisson's ratio = 0.3

[CSE (Mains) 2006 : 30 Marks]

Solution:

Note: (Not in Civil Services Syllabus)

In a thin disc, $\sigma_{C\,max}$ occurs at inner radius, R_1

Outer radius of disc, $R_2 = 250$ mm

$$\sigma_{C\,max} = \frac{\rho\omega^2}{g}\left[k_1\left(2R_2^2 + R_1^2\right) - k_2 R_1^2\right] \qquad \text{(Rotational stress)}$$

where, $$k_1 = \frac{3+\nu}{8} = \frac{3.3}{8} = 0.4125$$

$$k_2 = \frac{1+3\nu}{8} = \frac{1.9}{8} = 0.2375$$

$$\therefore \quad \sigma_{c\,max} = \frac{\rho\omega^2}{g}\left[0.4125\left(2\times 250^2 + R_1^2\right) - 0.2375 R_1^2\right]$$

Also $$\sigma_{c\,max} = 0.85\frac{\omega^2 \times 250^2}{g} \times \rho \qquad \left[\text{Stress in ring} = \frac{\omega^2 r^2 \rho}{g}\right]$$

$$\therefore \quad \frac{0.85\omega^2 \times 250^2 \times \rho}{g} = \frac{\rho\omega^2}{g}\left[0.4125\left(2\times 250^2 + R_1^2\right) - 0.2375 R_1^2\right]$$

$$0.85 \times 250^2 = 0.825 \times 250^2 + 0.175\, R_1^2$$

$$0.025 \times 250^2 = 0.175\, R_1^2$$

$$R_1 = 94.5 \text{ mm}$$

Allowable stress $= 900 \text{ kg/cm}^2 = 900 \times 9.81 \text{ N/cm}^2 = 8829 \text{ N/cm}^2 = 88.29 \text{ N/mm}^2$

$$\sigma_{c\max} = 0.85 \times \frac{\omega^2 \times (250)^2}{g} \times \rho$$

where specific weight, $\rho = 7.8 \text{ gm/cm}^3 = \dfrac{7.8}{1000 \times 10^3} \text{ kg/mm}^3$

$$= 7.8 \times 10^{-6} \text{ kg/mm}^3 = 7.8 \times 10^{-6} \times 9.81 \text{ N/mm}^3$$

$$= 76.518 \times 10^{-6} \text{ N/mm}^3$$

$\therefore$ $$88.29 = \frac{0.85 \times \omega^2 \times (250)^2 \times 76.518 \times 10^{-6}}{9810}$$

or $$\omega^2 = 213067.87$$

or $$\omega = 461.59 \text{ rad/s}$$

Also $$\omega = \frac{2\pi N}{60}$$

$\therefore$ $$461.59 = \frac{2\pi \times N}{60}$$

or $$N = 4407.88 \text{ rpm}$$

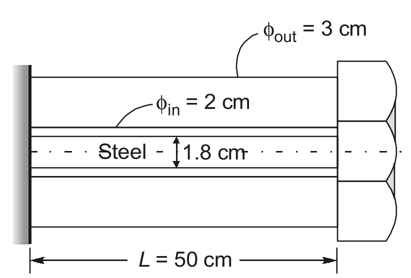
Q.3 A steel bolt of diameter 1.8 cm passes coaxially through a copper tube of inner diameter 2 cm and outer diameter 3 cm. The length of the tube is 50 cm. Washers are placed at both ends of the tube. The bolt has threads at one end having pitch equal to 0.24 cm. The nut is turned on the bolt through 45° against the washer to tighten the assembly. Determine the stress developed in the bolt and the tube. Assume the modulus of elasticity of steel to be 2×10^5 N/mm^2 and modulus of elasticity of copper is half that of steel.

[CSE (Mains) 2007 : 20 Marks]

Solution:

Given: $d_s = 1.8$ cm $= 18$ mm, $(d_c)_i = 2$ cm $= 20$ mm, $(d_c)_0 = 3$ cm $= 30$ mm, $l = 50$ cm $= 500$ mm,

$E_s = 2 \times 10^5$ N/mm^2, $E_c = \dfrac{1}{2} \times E_s = 1 \times 10^5$ N/mm^2



Let the compressive force developed in copper be P_c. Then, the force developed in steel is P_s.
From equilibrium condition;

$$P_s = P_c$$

$$\sigma_s \times A_s = \sigma_c \times A_c$$

$$\sigma_s \times \frac{\pi}{4} \times (18)^2 = \sigma_c \times \frac{\pi}{4} \times \left[(30)^2 - (20)^2\right]$$

$$\sigma_s = 1.543\, \sigma_c$$

As, nut is turned through 45° against the washer and pitch is 0.24 cm

$\therefore$ $$\text{Displacement} = \frac{45°}{360°} \times 0.24 = 0.03 \text{ cm} = 0.3 \text{ mm}$$

$$\frac{PL}{E_1 A_1} + \frac{PL}{E_2 A_2} = \text{Displacement}$$

$$P \times 500 \times \left(\frac{1}{2 \times 10^5 \times \pi \times \frac{18^2}{4}} + \frac{1}{10^5 \times \frac{\pi}{4}(30^2 - 20^2)}\right) = 0.3$$

or, $$P = 13299.77 \text{ N}$$

$$\sigma_s = \frac{P}{A_s} = \frac{13299.77}{\pi \times \frac{18^2}{4}} = 52.26 \text{ N/mm}^2 \quad \text{(Tensile)}$$

Now, $$\sigma_c = \frac{\sigma_s}{1.543} = 33.869 \text{ MPa} \quad \text{(compressive)}$$

∴ Due to tightening of nut,

$$0.3 \text{ mm} = \text{Extension of bolt} + \text{Contraction in tube}$$

$$0.3 \text{ mm} = \frac{\sigma_s}{E_s} \times L + \frac{\sigma_c}{E_c} \times L = \left[\frac{(1.543\sigma_c)}{2 \times 10^5} + \left(\frac{\sigma_c}{1 \times 10^5}\right)\right] \times 500$$

$$0.3 = (885.75 \times 10^{-5}) \times \sigma_c$$

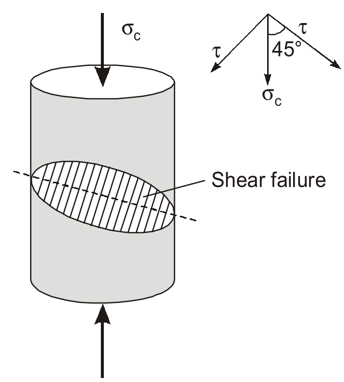
$$\sigma_c = 33.869 \text{ N/mm}^2 \text{ (compressive)}$$

and $$\sigma_s = 1.543 \times \sigma_c = 52.26 \text{ N/mm}^2 \text{ (Tensile)}$$

where, σ_c and σ_s are the compressive stress in copper tube and tensile stress in steel bolt respectively.

Q.4 A cast iron sample when tested in compression fails along approximately 45° plane from its axis while when tested in torsion also fails along a 45° (approx.) helical plane from its axis. Explain the reason for such failure and mention about the dominating stresses causing failure.

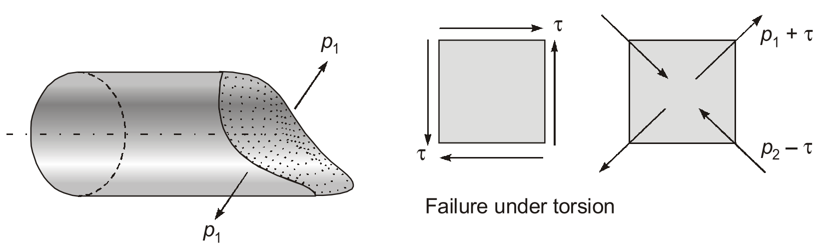
[CSE (Mains) 2011 : 10 Marks]



Solution:

Cast iron is weak in tension and shear, but strong in compression. Its compressive strength is about three times its tensile strength.

When cast iron sample is tested in compression, but shear stress τ is maximum on planes at ±45° to the axes of the sample. So sample breaks under shear on inclined plane at approx 45° to the axis as shown.



Failure under torsion

When a specimen is subjected to twisting moment, angular twist θ and shear stress τ act on the sample. Shear stress is maximum at outer radius and gradually reduces to zero at the centre. Principal stress $p_1 = +\tau$ acts on a plane about 45° to the axis of the sample. Due to angular twist, fractured surface becomes helical.

Q.5 A bar of steel is 50 mm in diameter and 600 mm long. A tensile load of 150 kN is found to stretch the bar by 0.23 mm. The same bar, when subjected to a torque of 1.4 kNm is found to twist through 1°. Find the values of the four elastic constants.

[CSE (Mains) 2016 : 10 Marks]

Solution:

Given: $D = 50$ mm, $L = 600$ mm, $P = 150$ kN, $\delta = 0.23$ mm, Torque (T) = 1.4 kNm = 1.4×10^3 kN-mm, $\theta = 1°$

∴ $$\delta = \frac{PL}{AE}$$

$$0.23 = \frac{150 \times 600}{\frac{\pi}{4}(50)^2 \times E}$$

4 Machine Design

1. Static and Fluctuating Loading

Q.1 A hollow shaft is subjected to a steady bending moment of 40 Nm and twisting moment of 50 Nm. Outer diameter of the shaft is twice the inside diameter. Calculate the diameters of the shaft using ASME Code for transmission shafting on the basis of maximum shearing stress theory of failure.
Take: Yield point stress in tension of shaft material = 280 MPa
Factor of safety = 2.0
Combined bending and fatigue shock factor = 1.5
Combined shock and fatigue factor for twisting = 1.0

[CSE (Mains) 2010 : 15 Marks]

Solution:

Given: Bending moment, $M = 40$ Nm, Twisting moment, $T = 50$ Nm, $\dfrac{D_0}{D_i} = 2$, $\sigma_{yt} = 280$ MPa, FOS = 2, $k_b = 1.5$, $k_t = 1.0$

Using ASME Code, permissible shear stress,

$$\tau_{max} = 0.30\,\sigma_{yt} = 0.30 \times 280 = 84 \text{ MPa}$$

$$(\tau_{permissible}) = \frac{\tau_{max}}{FOS} = \frac{84}{2} = 42 \text{ MPa}$$

Also, Equivalent torque, $T_e = \sqrt{(k_b M)^2 + (k_t T)^2} = \sqrt{(1.5 \times 40)^2 + (1 \times 50)^2} = 78.102$ Nm

On the basis of maximum shear stress theory,

$$\tau_{max} = \frac{T_e}{Z_P}$$

Z_P = Polar section modulus

$$= \frac{\pi}{16} D_0^3 \left[1 - \left(\frac{D_i}{D_0}\right)^4\right] = \frac{\pi}{16} D_0^3 \times \left[1 - (0.5)^4\right] = 0.1839 D_0^3$$

$\because$

$$\tau = \frac{T_e}{Z_P}$$

$$42 = \frac{78.102 \times 10^3}{Z_P} \Rightarrow Z_P = 1859.57 \text{ mm}^3$$

$$Z_P = 0.1839\, D_0^3 = 1859.57 \text{ mm}^3$$

$$D_0 = 21.62 \approx 22 \text{ mm}$$

and

$$D_i = \frac{D_0}{2} = 11 \text{ mm}$$

Q.2 Stress concentration factor is not considered harmful for ductile materials in static loading but for brittle materials it has damaging effect in both static and dynamic loading.
Justify the above statement giving illustrations.

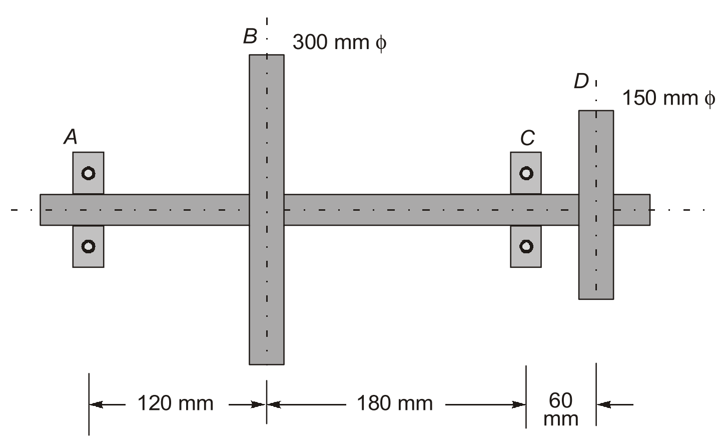
[CSE (Mains) 2011 : 15 Marks]

Solution:

Stress concentration is the localized stress considerably higher than average, even in uniformly loaded cross-section of uniform thickness due to abrupt change in the geometry or localised loading. Stress concentration factor is not considered harmful for ductile materials in static loading because of the phenomenon of local yielding in ductile materials which relieves the stress concentration. When the stress in the vicinity of the discontinuity reaches the yield point there is a plastic deformation, resulting in re-distribution of stresses. This plastic deformation prevents the harmful effects of stress concentration in ductile materials.
While in Brittle materials, stress concentration factor is important in both static and dynamic loading. Brittle materials fail due to fracture. So there is little deformation to relax the concentrated stresses and thus has damaging effects.

Q.3 A shaft is to transmit 2 kW at 750 R.P.M. The shaft is supported in bearings *A* and *C*, 300 mm apart. Two pulleys of 300 mm diameter and 150 mm diameter are located at *B* and *D* as shown in figure. Assume that the belt tensions are vertical for both pulleys. Ratio of belt tensions for both pulleys is 3. Neglect weight of pulleys and shaft. Take combined fatigue and shock factor in bending and twisting equal to K_b = 1.5, K_t = 1.0 respectively.

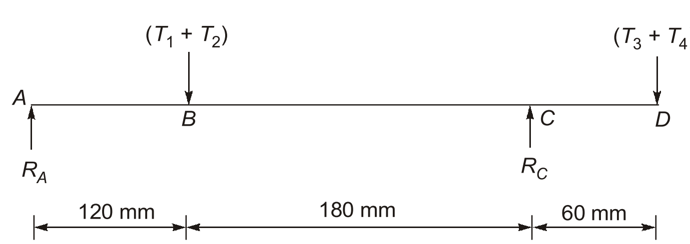
Determine uniform diameter of the shaft, if allowable tensile stress is 110 MPa and allowable shear stress is 65 MPa. (Take shaft diameter in steps of 5 mm.)



[CSE (Mains) 2014 : 20 Marks]

Solution:

Given: Shaft power = 2 kW at N = 750 rpm, K_b = 1.5, K_t = 1.0, $\sigma_{t\,(allow)}$ = 110 MPa, $\tau_{(allow)}$ = 65 MPa
Let the tensions in belt of pulley *B* be T_1, T_2.
Then tensions in belt of pulley *D* are T_3, T_4.
M_t = torque transmitted



$$\text{Power, } P = \frac{2\pi N M_t}{60}$$

or $$M_t = \frac{60 \times P}{2\pi N}$$

$$= \frac{60 \times 2 \times 10^3}{2\pi(750)} = 25.47 \text{ N-m}$$

Also, $$M_t = (T_1 - T_2)R_B$$
$$= (T_3 - T_4)R_D$$
$$25.47 \times 10^3 = (T_1 - T_2) \times 150$$

and $$\frac{T_1}{T_2} = \frac{T_3}{T_4} = 3 \text{ (Given)} \quad \text{(i)}$$

$$25.47 \times 10^3 = (2T_2) \times 150$$
$$T_2 = 84.90 \text{ N}$$

From equation (i), we get $$T_1 = 254.7 \text{ N}$$
$$T = (T_3 - T_4) \times R_D$$
$$25.47 \times 10^3 = (2T_4) \times 75$$
$$T_4 = 169.8 \text{ N}$$

From equation (i), we get $$T_3 = 509.4 \text{ N}$$

R_A and R_C be the reaction forces,

$$R_A + R_C = (T_1 + T_2) + (T_3 + T_4) = 1018.80 \text{ N} \quad \text{...(ii)}$$

Taking moment about point A

$$(T_1 + T_2) \times 120 + (T_3 + T_4) \times 360 = R_C \times 300$$
$$40752 + 244512 = R_C \times 300$$
$$R_C = 950.88 \text{ N}$$

From (ii) we get $$R_A = 67.92 \text{ N}$$

Bending moment at B, $$M_B = R_A \times 120$$
$$= 8150.4 \text{ N-mm}$$

Bending moment at C, $$M_C = (T_3 + T_4) \times 60$$
$$= 40752 \text{ N-mm}$$

Maximum bending moment will be at 'C'.

Let $$\text{Equivalent torque, } T_e = \sqrt{(K_b M_C)^2 + (K_t M_t)^2}$$

$$= \sqrt{(1.5 \times 40752)^2 + (1 \times 25.47 \times 10^3)^2}$$
$$= 66{,}222 \text{ N-mm}$$

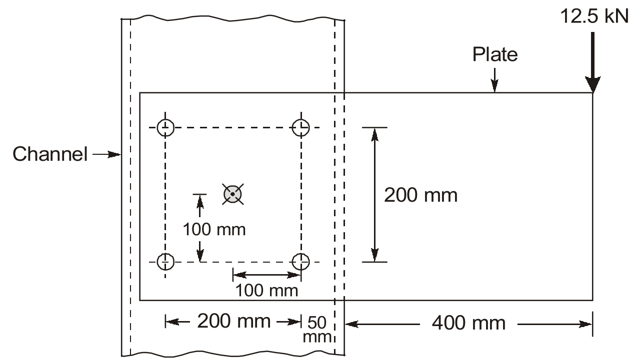
Also, $$\tau = \frac{16 T_e}{\pi d^3}$$

$$65 = \frac{16 \times 66{,}222}{\pi d^3} \Rightarrow d = 17.31 \text{ mm}$$

Diameter of shaft = 20 mm

2. Bolted, Riveted and Welded Joints

Q.1 A plate is riveted to a channel section in a structure as shown in figure. An eccentric load of 12.5 kN acts as shown on the plate. Determine the rivet diameter so that the maximum shear stress in any rivet is not to exceed 40 MPa. Diameter of the rivet should be chosen from preferred series.



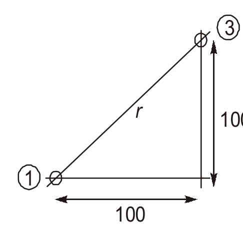
Preferred rivet diameters (in mm) : 12, 14, 16, 18, 20, 22, 24, 27, 30, 33, 36, 39, 42, 48.

[CSE (Mains) 2009 : 40 Marks]

Solution:

Given: Eccentric load, P = 12.5 kN, τ_{max} = 40 MPa = 40 N/mm^2

Primary shear force on each rivet, $P' = \dfrac{P}{5} = \dfrac{12.5}{5} = 2500$ N



$$r = \sqrt{100^2 + 100^2} = 141.42 \text{ mm}$$

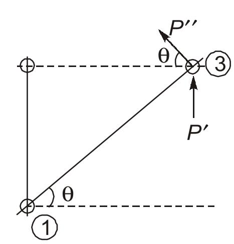
$$e = 100 + 50 + 400 = 550 \text{ mm}$$

Secondary shear force on rivet (3)

$$P'' = \frac{Per_3}{r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2} \qquad [\because r_1 = 0]$$

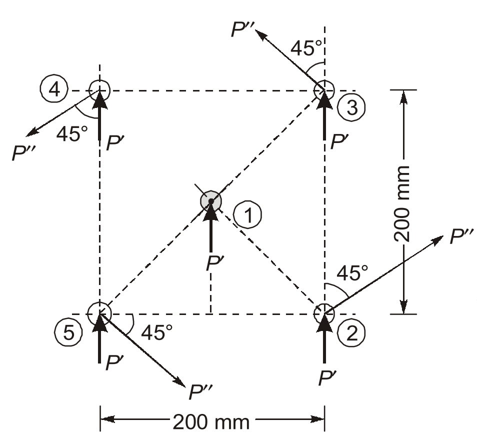
$$= \frac{(12.5\times10^3)\times550\times141.42}{4\times(141.42)^2}$$

$$= 12153.28 \text{ N}$$



$$\tan\theta = \frac{100}{100} = 1$$

$$\theta = 45°$$



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Hence rivet (2) and (3) are critical to maximum force. Hence rivet (2) and (3) are critical rivets.
So, resultant force on rivet (3),

$$P_{\text{resultant}} = \sqrt{(P' + P''\sin\theta)^2 + (P''\cos\theta)^2}$$

$$= \sqrt{(2500 + 12153.28\sin 45°)^2 + (12153.28\cos 45°)^2}$$

$$= \sqrt{(123069440.9) + (73851122.57)} = 14032.83 \text{ N}$$

Let, A = Cross sectional area of rivet

$$\tau_{max} = \frac{P_{\text{resultant}}}{A}$$

$$A = \frac{14032.83}{40}\text{mm}^2 = 350.82 \text{ mm}^2$$

As, $$A = \frac{\pi}{4}d^2 = 350.82$$

$$d = 21.134 \text{ mm}$$

From preferred series, d = diameter of rivet = 22 mm

3. Shafts

Q.1 A hollow shaft whose internal diameter is half of the external diameter is subjected to maximum bending moment of 2 kN-m at a section and constant torque of 4 kN-m all along its length. If yield stress of the shaft material is 280 MPa and factor of safety is 3.0, what should be the minimum safe diameters of the shaft?

[CSE (Mains) 2010 : 9 Marks]

Solution:

Given: $D_i = \frac{D_0}{2}$, BM (M_0) = 2 kNm, Torque (M_t) = 4 kNm, σ_{yt} = 280 MPa, FOS = 3

Let T_e be the equivalent torque, $T_e = \sqrt{(M_o)^2 + (M_t)^2} = \sqrt{(4)^2 + (2)^2} = 4.472$ kN-m

Also, $$\tau = \frac{16T_e}{\pi D_0^3(1-k^4)} \quad \text{...(i)}$$

where, $$k = \frac{D_i}{D_0} = \frac{1}{2} = 0.5$$

and $$\tau_{ys} = \frac{\sigma_{yt}}{2(\text{FOS})} = \frac{280}{2\times(3)} = 46.67 \text{ MPa}$$

Substituting the above values in equation (i), we get

$$46.67 = \frac{16\times 4.472\times 10^3 \times 10^3}{\pi D_0^3(1-0.5^4)}$$

$$D_0 = 80.442 \text{ or } 82 \text{ mm}$$

Minimum outer diameter, D_0 = 82 mm

Minimum inner diameter, $D_i = \frac{D_0}{2} = 41 \text{ mm}$